Chapter 0: **Math review**

- notation and units
- significant digits
- order of operations
- exponents and logarithms
- coordinates
- angles, geometry, trigonometry
- conic sections
- lines and interpolation
- averaging and percentages
- some algebra mistakes and how to avoid them
- sample problems

Notation and units

In what's called "scientific notation" we write numbers as using exponents, usually of 10. As an easy example, consider the number 100. It is 10 times 10, and we can write it as 10^2. Here is a table of a few numbers, both in their decimal form and in scientific notation:

decimal	scientific
13 billion = 13,000,000,000	$1.3 \cdot 10^{10}$ or $13 \cdot 10^9$
2,504,312	$2.504312 \cdot 10^6$
– 4.3	$-4.3 \cdot 10^0$
0.00135	$1.35 \cdot 10^{-3}$

Many calculators will readily swap their displays between decimal and scientific. You probably want to find one that does this and learn how to use that function.

We will be using numbers that vary wildly in size (which is another reason to be comfortable with scientific notation) and often we will use shorthand notation for units. For instance, the standard unit of measurement in the metric system is the meter. Using the meter as an example, let's look at some prefixes with which you should be familiar (and a few more that are fun simply because they are so extreme). You probably already know the centimeter, abbreviated cm, which is 1/100 of a meter. Most of the interesting units come in multiples of 10^3. Here's a table of units:

millimeter (mm)	10^{-3} m	kilometer (km)	10^3 m
micrometer or micron (μm)	10^{-6} m	megameter (Mm)	10^6 m
nanometer (nm)	10^{-9} m	gigameter (Gm)	10^9 m
picometer (pm)	10^{-12} m	terameter (Tm)	10^{12} m
femtometer (fm)	10^{-15} m	petameter (Pm)	10^{15} m
attometer (am)	10^{-18} m	exameter (Em)	10^{18} m
zeptometer (zm)	10^{-21} m	zettameter (Zm)	10^{21} m
yoctometer (ym)	10^{-24} m	yottameter (Ym)	10^{24} m

Significant digits

Your calculator will give you answers with quite a few decimals places if you let it. Those digits may not be meaningful. For instance, if we multiplied 3 times 4, we expect to get 12. On the other hand, if it were 3.0 times 4.0, it would be correct to say 12.0. How come? Think back to when you learned to round off numbers. For instance, 3.3 is, rounded to only one digit, equal to 3, and 4.2, equal to 4. But if you multiplied 3.3 times 4.2, you'd expect to get a different answer than just 12; in fact, you get 13.9. If you are told that a particular value is 3, that doesn't convey as much information as if you are told that the value was measured to be 3.0, or 3.3, either or which could be rounded down to 3. If you don't know your input values any more precisely than that they are 3 and 4, it would be misleading to claim that they multiply to 12.0, or, worse, 12.000, rather than 12.

A more relevant example: M 31, the galaxy in Andromeda, is 2 ½ million light years away. We do not know its distance accurately enough to say that it is $2.500000 \cdot 10^6$ light years away, in addition to which, it's an extended object, meaning that all parts of it are not exactly, precisely, the same distance from us.

Now, an example of where significance can be confusing: consider the surface area of a sphere. It is $4\pi r^2$. That 4 is exactly 4, but we rarely write it as $4.\overline{0}$ to indicate that fact. Sorry.

For working the problems in this text, keep two things in mind: First, if you do a calculation in multiple steps, keep the extra digits until the end and then round your final result down; you can lose actual significant, real, information if you round too often. Second, try not to report more digits than the number of digits given in the problem for observed quantities.

Order of operations

In complex expressions we usually add parentheses to indicate which operations belong together. The parentheses are often omitted if the order of operations is clear. For instance, in an expression involving multiplication and addition the multiplication is normally assumed to occur first. For example, an expression such as $2\cdot5 - 6\cdot3$ would be evaluated as $10 - 18 = -8$. If we meant otherwise, we'd add parentheses. For example, if we had written $2 \cdot (5 - 6\cdot3)$, that would be evaluated as $2 \cdot (5 - 18) = 2 \cdot (-13) = -26$.

Exponents and logarithms

You are probably comfortable with the idea that adding and subtracting are related operations, do and undo in some sense. For instance, if you add 6 to x, you can get back to x by subtracting 6. Written out, $(x + 6) - 6 = x$. Multiplying and dividing are similar. Sticking with 6, we could write $(x \cdot 6) / 6 = x$. Logs and exponents are a similar sort of pair of operations. You are likely to be most familiar with working in base 10, where, again with 6, we could write

$$\log(10^6) = 6.$$

In this case we don't have to stick with base 10, though. We often use e (= 2.71828. . .) or sometimes 2, but rarely anything else. Here are some rules for working with powers, roots, logs and exponents.

- Factorials: $a!$ means $1 \cdot 2 \cdot 3 \cdot \ldots$ up to a.
  Ex.: $7! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 = 5040$.
- 0! by definition = 1
- Powers: a^n means $a \cdot a \cdot a \cdot \ldots n$ times.
  Ex.: $2^3 = 2 \cdot 2 \cdot 2$ (= 8).
- If you multiply together two numbers written as powers, add the exponents: $a^n \cdot a^m = a^{n+m}$.
  Ex.: $2^3 \cdot 2^2 = 2^5$ (= 8 · 4 = 32).
- a^0 by definition = 1
- $a^{1/n}$ means the nth root.
  Ex.: $8^{1/3} = \sqrt[3]{8}$, i.e., the cube root of 8, = 2.

- $a^{-n} = 1 / a^n$.

  Ex.: $2^{-2} = 1 / (2^2) = ¼$.
- $(a^n)^m = (a^{nm})$.

  Ex.: $(2^3)^2 = 2^{3 \cdot 2} = 2^6$ $(= 8^2 = 64)$;

  Ex.: $\left(\sqrt{2}\right)^3 = 2^{3/2} = 2.8$
- if $y = a^x$, then $\log_a(y) = x$.

  Ex.: if $10^x = 1000 \rightarrow x = \log_{10}(1000) = 3$.

  Ex.: if $e^x = 6 \rightarrow x = \log_e(6)$ [which may be written ln (6)] = 1.8

  Ex.: if $\log_{10}(y) = 1.4 \rightarrow y = 10^{1.4} = 25.1$

  Ex.: if $\log_2(y) = 17 \rightarrow y = 2^{17} = 1.3 \cdot 10^5$.
- $\log_a(x \cdot y) = \log_a(x) + \log_a(y)$

  Ex.: $\log_{10}(2 \cdot 5) = \log(2) + \log(5) = 0.3 + 0.7 = 1.0$ (check: $\log(10) = 1$)
- $\log_a(x/y) = \log_a(x) - \log_a(y)$

  Ex.: $\log_{10}(2/5) = \log(2) - \log(5) = 0.3 - 0.7 = -0.4$ (check: $\log(0.4) = -0.4$)
- $\log_a(y^n) = n \log(y)$

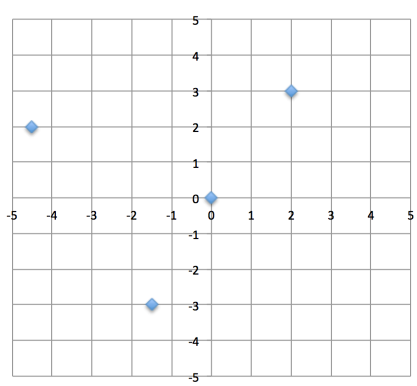
  Ex.: $\log_{10}(2^3) = 3 \cdot \log(2) = 3 \cdot 0.3 = 0.9$ (check: $\log(8) = 0.9$)
- $\log_a(y) = \log_a(b) \cdot \log_b(y)$

  Ex.: $\log_2(12) = \log_2(10) \cdot \log_{10}(12) = 3.322 \cdot 1.08 = 3.59$ (check: $2^{3.59} = 12$)

Another conversion: $e^x = 10^{0.4343x}$ and $\ln(x) = 2.3026 \log(x)$.

Coordinates

The *x-y* coordinates with which you are probably most familiar are called Cartesian coordinates, after René Descartes. In two dimensions, they look like this:



Usually, the horizontal axis will be the x axis and the y axis will be vertical. Here we've added four points:

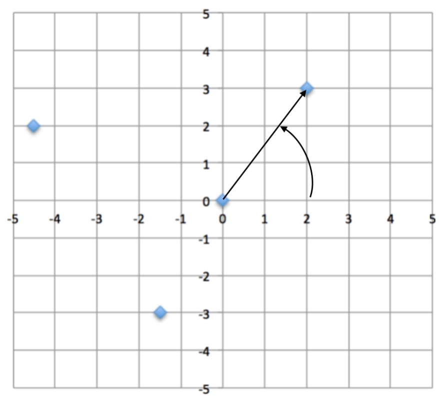
x	y
-4.5	2
-1.5	-3
0	0
2	3

E.g., $(x,y) = (2,3)$ means over 2 and up 3.

Figure 0.1

We could add a third dimension, a z axis, perpendicular to the other two, with one axis running in/out of the page. If you want to keep track of the mathematical terms, the x-value of a point is called the *abscissa* and the y-value is called the *ordinate*.

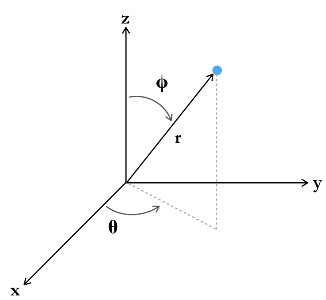
Instead of Cartesian coordinates, we might prefer in some circumstances to use polar coordinates. Let's take our Cartesian example from above and superimpose polar coordinates:



The arrow drawn from the origin indicates the direction to the point with (x, y) coordinates (2, 3). The distance from the origin to that point is 3.6, at an angle from the horizontal of 56.3º. We may thus also describe the location of this point as being at $(r, \theta) = (3.6, 56.3°)$.

Figure 0.2

There are two distinct ways of extending polar coordinates into three dimensions. In cylindrical coordinates, we take r and θ and add a vertical coordinate z, just as we did in the Cartesian case. More often in astronomy we will want spherical coordinates, where we add a second angle, so that we have an angle in the plane and an angle down from the vertical. Which angle is θ and which is φ may be flipped (physics tends to have one convention and mathematics has the other).



In spherical coordinates we would label our point as having coordinates (r, θ, φ).

Figure 0.3

Angles, Geometry, and Trigonometry

In two dimensions: The circumference of a circle = $2\pi r$ and the area of a circle = πr^2.

In three: the surface area of a sphere = $4\pi r^2$ and the volume of a sphere = $\frac{4}{3}\pi r^3$.

If you know some calculus, you can see in both of these examples that the former is the derivative of the latter expression.

There are 360 degrees in a circle, with each degree divided into 60 arcminutes and each arcminute divided into 60 seconds. We may write this as

$1° = 60' ; 1' = 60''$.

There are 2π radians in a circle. The radian is defined such that one radian has unit arc length along a unit circle. Because the circumference of the unit circle is 2π, there must be 2π radians in a full circle.

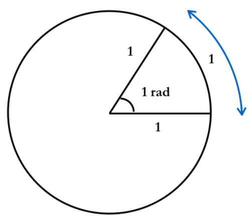


Figure 0.4 — illustration of the radian.

In terms of solid angles, there are 4π ster (or sr; steradians, for square radians) in a full sphere.

A right triangle has one angle of 90°:

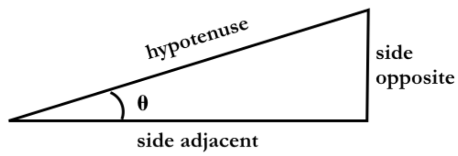


Figure 0.5 — a right triangle.

Let (side opposite) = y, (side adjacent) = x, and (hypotenuse) = r.
The trig functions are defined thusly:

$\sin\theta = y / r$

$\cos\theta = x / r$

$\tan\theta = \sin\theta / \cos\theta = y / x$

The Pythagorean theorem says that

$(r)^2 = (y)^2 + (x)^2$.

Example: suppose that we are given that (side opp) = 5 and (side adj) = 9; solve the triangle.

$$r = \sqrt{x^2 + y^2} = \sqrt{9^2 + 5^2} = 10.3 \rightarrow$$
$$\tan\theta = y / x = 5 / 9 = 0.556;$$
$$\tan^{-1}(0.556) = 29^\circ.$$

The 3rd angle in the triangle must be 180° – (90° + 29°) = 61°

Check this for consistency: sin (29°) = 0.485; y / r = 5 / 10.3 = 0.485.

Note that the inverse trig functions, e.g., $\tan^{-1}(z)$ do *not* mean "1 / tan (z)" but rather "what is the angle for which z is the tangent?" Inverse in this case means undo the trig function rather than taking its reciprocal. Find a calculator that has inverse trig functions and learn how to use them.

In this example, we could have calculated $\tan^{-1}(0.556)$ in radians rather than degrees. We would have gotten 0.507 rad. One radian is equal to 57.3°; 57.3 · 0.507 = 29, telling us that 29° and 0.507 rad are the same angle. Note that your calculator is unlikely to calculate trig functions or inverse trig functions in arcseconds. There are many applications in astronomy where we are working with very small angles and will be using arcsec. Be careful!

A related use of the term tangent is to refer to a line touching a circle in one point only, and thus being perpendicular to the radius of a circle:

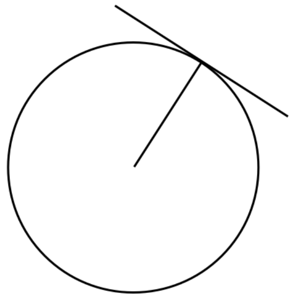


Figure 0.6 — Tangent line

Other trig functions: Some times we will want the reciprocals of the sin, cos, or tan functions. Those functions are

$1 / \sin\theta$ = cosecant (θ) $= r / y$

$1 / \cos\theta$ = secant (θ) $= r / x$

$1 / \tan\theta$ = cotangent (θ) $= x / y$

For triangles that do not have a 90° angle the relationships between angles and sides are a little bit more complicated. Consider the following triangle with vertex angles denoted with the capital letters and side lengths denoted with the lower-case letters:

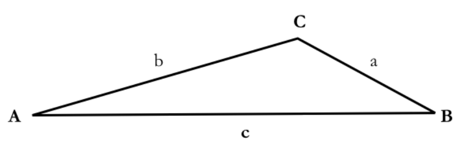


Figure 0.7 — a generic, non-right, triangle

The law of sines: [a / sin (A)] = [b / sin (B)] = [c / sin (C)].

The law of cosines: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos(A)$

This works for the other two sides as well; just rotate through the letters.

The area of an arbitrary triangle = ½ base · height.

There are a few more esoteric trig functions that might come in handy if you need, for instance, the sin of the sum of two angles. Note that $\sin^2\theta$ means $(\sin\theta)^2$.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin(\theta \pm \varphi) = \sin\theta \cdot \cos\varphi \pm \cos\theta \cdot \sin\varphi \ [\text{and} \rightarrow \sin(2\theta) = 2\sin\theta \cdot \cos\theta]$$

$$\cos(\theta \pm \varphi) = \cos\theta \cdot \cos\varphi \mp \sin\theta \sin\varphi \ [\text{and} \rightarrow \cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1]$$

$$\sin^2\theta = \tfrac{1}{2}(1 - \cos(2\theta))$$

$$\cos^2\theta = \tfrac{1}{2}(1 + \cos(2\theta))$$

Among other possibilities, the site: http://en.wikipedia.org/wiki/List_of_trigonometric_identities has a list of these and many other formulae involving trig functions.

Conic sections

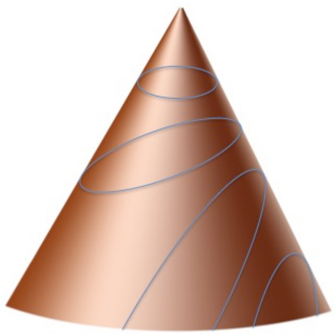


Figure 0.8: Slice a cone across, perpendicular to the vertical axis, and you get a circle.

Slice it at a slight angle, get an ellipse.

Slice it at an angle that is parallel to the side of the cone, and get a parabola.

Slice it at an angle that is steeper than the side of the cone, and get a hyperbola.

Looking down on these various figures from above, they look approximately like this:

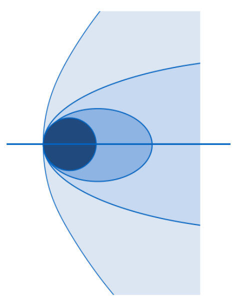
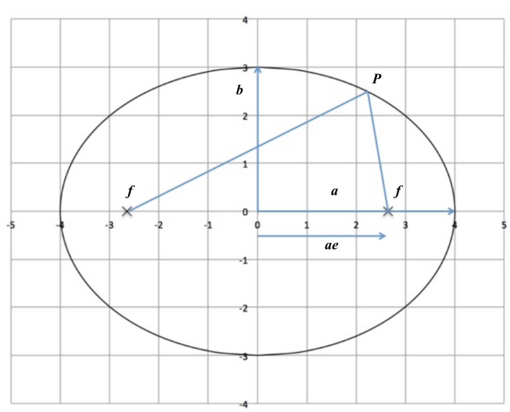


Figure 0.9

An ellipse, in particular, is useful in astronomy because orbits (e.g., of planets around the Sun or two stars around each other) follow elliptical paths.

We can describe an ellipse in either spherical or polar coordinates.



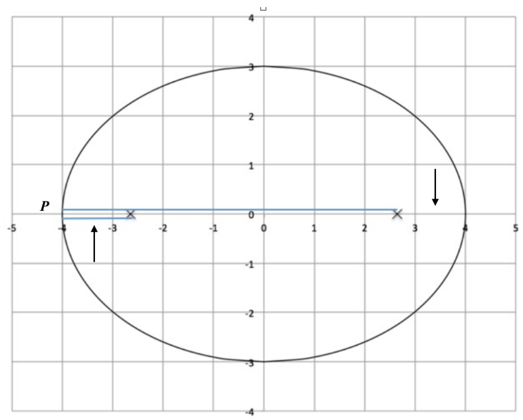
Where a circle has a radius, *r*, an ellipse is described by a semi-major axis, *a*, and a semi-minor axis, *b*.

An ellipse has two foci, *f*, such that the sum of the distance from the foci to the ellipse (at an arbitrary point *P*) is a constant.

(One focus, two foci.)

Figure 0.10

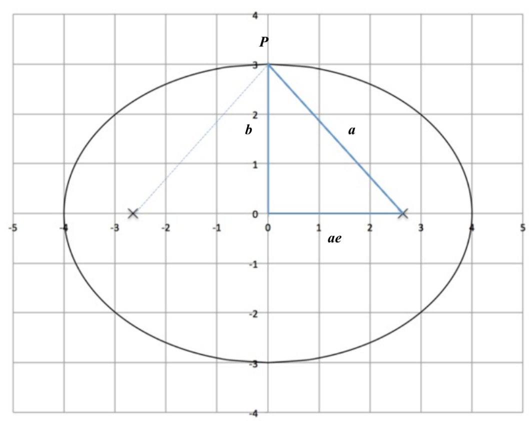
In the next figure we will draw that line representing the sum of the distance from the foci to the ellipse to the point on the ellipse at left end of the major axis. In other words, the line goes left along the axis from the right focus to the left end of the major axis and back to the left focus. The two segments of the line are offset a bit so you can see both of them:



The blue line, from focus to ellipse to other focus, is as long as the major axis, $2a$. The piece that wraps around on the left is the same length as the piece that is missing on the right, hence a total length of $2a$.

Figure 0.11

Let's use this information to determine the relationship between the eccentricity, e, and the lengths of the axes. In the following sketch, the point P is at the top of the minor axis. Only half of the line from f to P to f is emphasized.

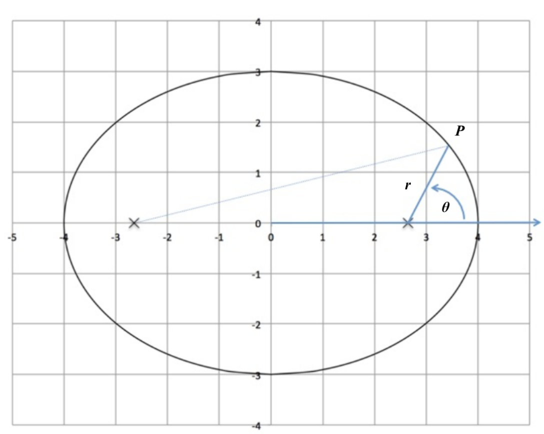


By the Pythagorean theorem, we know that $a^2 = b^2 + (ae)^2$. Rearranging,
$b^2 = a^2 \cdot (1 - e^2)$.

The total area of an ellipse is
$A = \pi ab$.

Figure 0.12

In the following diagram we are showing polar coordinates, again only emphasizing the line from one focus to the arbitrary point P:



In this case the point P is a distance r away from the focus of interest, at an angle θ from the horizontal.

Figure 0.13

The equation for the location of P in terms of (r; θ) is

$$r = a \cdot (1 - e^2) / (1 + e \cdot \cos\theta).$$

The smallest possible value for r occurs when θ is 0°. In that case, $r = a\ (1 - e)$.

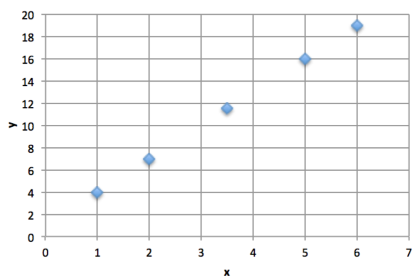
Where did that come from? Expand the $1 - e^2$ term as $(1 - e) \cdot (1 + e)$. If cosθ = 1, then

$$r = a \cdot (1 - e) \cdot (1 + e) / (1 + e) = a \cdot (1 - e).$$

The $(1 + e)$ terms cancel. Similarly, the largest possible value for r occurs when θ is 180°. In that case, $r = a\ (1 + e)$.

Lines and interpolation

If you have data that are described by a smoothly varying function you may find yourself needing to estimate the value the function would take at a point intermediate between some of your data points. Here are five data points that fall along a line:



x	y
1.0	4.0
2.0	7.0
3.5	11.5
5.0	16.0
6.0	19.0

Figure 0.14

What is the value of y at an x value of 4.0? Lines are described by a function of the form

$$y = mx + b,$$

where m is the slope of the line and b is the intercept, the value of y for which $x = 0$. The slope is given by the change in y over the change in x; i.e., $m = \Delta y / \Delta x$. The slope between the end points would be

$$m = (19 - 4) / (6 - 1) = 3.$$

We could have gotten the same answer using the points closest to 4: $(16 - 11.5) / (5 - 3.5) = 3$.

Knowing the slope, we could set up a similar expression, this time using $x = 4$ and y unknown:

$$m = 3 = (16 - y) / (5 - 4).$$

This gives us $3 \cdot 1 = 16 - y$, or $16 - 3 = y = 13$.

We have *interpolated* between our known points at $x = 3.5$ and 5 to determine the value of the function at $x = 4$. Does the answer make sense? Our x value of 4 is 1/3 of the way from 3.5 to 5; a y value of 13 is also 1/3 of the way from 11.5 to 16, so yes, our result makes sense.

In this example our function was a line; if our function had been some other shape but only slowly varying, such as a large ellipse (to describe, e.g., the orbit of a planet), and our known data points not too far apart, it would still be reasonable to approximate the ellipse by a straight line between the two known data points. If the known data points are far apart or the function varies rapidly, then using linear interpolation is not justified. We would then need to use some more complicated function than a line to join our known points together. If you work with spreadsheet software (such as Excel) the software may have the ability to do this interpolation for you, e.g., by adding trendlines with various functional forms (lines, polynomials, exponentials, etc.) that will predict the values of a function for you. It's good to know how to do a simple linear interpolation yourself, though.

Averaging and percentages

There are many ways of calculating an average, or mean, of a set of data points or a function. You are probably most familiar with the *arithmetic mean*:

$$\bar{x}_{AM} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{x_1 + x_2 + ... + x_n}{n} .$$

The bar over the x indicates that this is the average.
Example: The average of 1 and 100 would be (1+100)/2 = 50.5.
If you have a set of data points having varying levels of reliability you might want to use a weighted mean:

$$\bar{x}_{WAM} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i} .$$

Example: Suppose we have four data points, 2.1, 4.1, 4.5, 7.2, where we have reason to trust the values of 4.1 and 4.5 twice as much as we trust the outlying values of 2 and 7. In taking an average we could weight those more trustworthy values twice as heavily:

$$\bar{x} = \frac{2.1 + (2 \cdot 4.1) + (2 \cdot 4.5) + 7.2}{6} = 4.42 .$$

If we hadn't done the weighting we would have calculated an average of 4.48, which in some cases could be enough of a difference to matter.

Sometimes we want an order of magnitude estimate, in which case it might make more sense to use a *geometric mean*:

$$\bar{x}_{GM} = \left(\prod_{i=1}^{n} x_i\right)^{1/n} = (x_1 \cdot x_2 \cdot ... \cdot x_n)^{1/n} .$$

Example: I teach in a building that is has three floors. How high is it? It's obviously not 1 meter high and 100 meters is too much. How about 10 meters? That might not be enough, but it's undoubtedly closer than either of my bounding guesses of 1 or 100 meters. Why not just take the arithmetic mean of those two guesses? That would be 50.5 meters, which differs from 1 meter by a factor of 50 and differs from 100 meters by a factor of 2. The geometric mean of 1 and 100 is $\sqrt{1 \cdot 100} = 10$. By definition, that differs from both of the guesses, 1 and 100, by a factor of 10. As an order of magnitude estimate between the two guesses of 1 and 100, 10 is closer to the actual answer than 50.5.

There are some problems where we want the *harmonic mean*:

$$\bar{x}_{HM} = n\left(\sum_{i=1}^{n} \frac{1}{x_i}\right)^{-1} = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + ... + \frac{1}{x_n}} .$$

One example where this is useful is a problem you might have encountered in a physics class. Suppose that you travel a distance of 100 km and back; on the outbound leg of the trip the traffic is light and you can travel at 100 km/hour, but on the return trip you have to slow to 50 km/hour. Over the whole trip the average speed is not 75 km/hour, the arithmetic mean of the two speeds (it's also not the geometic mean of the two speeds, which is 70.7 km/hr).

Here's where the harmonic mean comes in: $\dfrac{2 \cdot 100 \text{ km}}{\frac{100 \text{ km}}{100 \text{ km/hr}} + \frac{100 \text{ km}}{50 \text{ km/hr}}} = \dfrac{2}{\frac{1}{100} + \frac{1}{50}} \text{ km/hr} = 66.7 \text{ km/hr}.$

In terms of time, the outbound leg of the trip took 1 hour and the return took 2 hours. The total distance, 200 km, divided by the total time, 3 hours, gives an average speed of 66.7 km/hr.

These three types of average, the arithmetic, geometric, and harmonic means, are collectively known as the Pythagorean means because they were studied by the Pythagoreans.

You may also encounter the *median* of a set of data points. The median is the middle value in the set, meaning that half the data points are larger and half smaller. In the set used above, 2.1, 4.1, 4.5, 7.2, the median would be (4.1 + 4.5)/2 = 4.3, which is not that different from any of the averages we calculated. But if our data set had been 2, 4, 4, 47, then the arithmetic average would be 14.25, very much skewed by the fact that 47 is much larger that the other three values. The median would be 4, much more representative of the majority of the data points.

Calculus alert: Here are two examples involving integrals:

Suppose we have a continuous distribution function *f(x)* which gives the probability that the variable *x* will have a particular value. The mean or the *expected value* of *x* is calculated this way:

$$E(x) = \int_{-\infty}^{+\infty} x f(x)\,dx\,.$$

We might also a function *f(x)* that is continous over some domain that's not necessarily infinite. The mean of that function over an interval [a,b] is given by

$$\bar{f} = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

Percentages are numbers expressed as a fraction of 100. For example, the number 8 is 80% of the number 10. We get that by taking $100 \cdot (8/10)$ and tacking on the % symbol.

Manipulating percentages can be a bit tricky. For example: 8 is 100% of 8, i.e., of itself; half of 8, or 50% of 8, is 4. If we took that number 8 and increased it by 50%, we would have 12. But if we take 12 and decrease it by 50%, we are now subtracting 6, because that's half of 12, leaving us with 6 rather than getting back to our starting value of 8. Be careful.

Often we will want the percentage difference between observational data and theoretical values. For example: Suppose that the average of our data measurements in a particular problem is 52 km/sec. Our prediction from basic physics for this problem is that we would have expected to measure 50 km/sec. The percentage difference between our measurement and the expected value is given by:

$$100 \cdot \frac{\text{observed-expected}}{\text{expected}}, \text{which in this case} = 100 \cdot \frac{52-50}{50} = 4\%.$$

Some algebra mistakes and how to avoid them

If you are rusty using algebra you may forget some of the ways of handling fractions.

1) Cross-multiplying. Suppose you have the following ratio and want to solve for *x*:

$$\frac{3}{7} = \frac{22}{x}.$$

To solve for *x*, multiply both sides by *x* **and** by 7/3:

$$x \cdot \frac{7}{3} \cdot \frac{3}{7} = x \cdot \frac{7}{3} \cdot \frac{22}{x};$$

On the left-hand side the 7/3 and 3/7 cancel and on the right-hand side the *x*'s cancel, leaving you with $x = (7 \cdot 22) / 3 = 51.3$.

2) Inverting a fraction with a negative exponent in the denominator. Suppose you have an expression that involves using the mass of one hydrogen atom in the denominator: $\frac{1}{1.67 \cdot 10^{-27}\,\text{kg}}$.

If you want to get that into the numerator you have to invert both the 1.67 and the 10^{-27}, *not* just the exponent. You should get $6 \cdot 10^{26}\ \text{kg}^{-1}$.

3) Adding or subtracting fractions. Here what you need to remember is that you *don't* add denominators. Suppose you had an expression such as $\frac{2}{3}-\frac{4}{7}$ and you want to combine that into one fraction. You evaluate this as follows:

$\frac{2}{3}-\frac{4}{7}=\frac{2\cdot 7-3\cdot 4}{3\cdot 7}=\frac{14-12}{21}=\frac{2}{21}$ or 0.095.

You could have gotten the same result by converting each fraction to a decimal before subtracting: 0.667 – 0.571 = 0.095.

More math may take up residence here at some point in the future.

Sample problems

1. Express 172 in scientific notation.

2. Express $2.67\cdot 10^{4}$ in decimal notation.

3. Express $3.69\cdot 10^{-4}$ in decimal notation.

4. Evaluate $(2.512)^{3}$ and express the result in decimal notation.

5. Evaluate $(3^{2})\cdot(3^{4})$ and express the result in scientific notation.

6. Evaluate $(3^{-2})^{3}$.

7. Find the ratio of the area of a circle with a radius = 4 m to that of a circle with a radius = 8 cm.

8. Find the volume of a sphere with a radius = 7 µm.

9. Density is mass per unit volume. The density of liquid water under normal ground-based Earth atmospheric conditions is ~1 g/cm^{3}. Convert this to kg/m^{3}.

10. Consider a 3 - 4 - 5 right triangle, i.e., a triangle in which one angle = 90°. Find the values of the other two angles in this triangle; express the results in degrees. If you want more practice converting units, express the values in radians as well.

11. Consider an ellipse with a semi-major axis a = 14 cm and an eccentricity e = 0.15. Find the largest and smallest possible values of r for this ellipse.

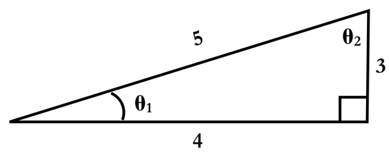
12. The general expression for a line is $y = mx+b$. In the example on page 9 we determined that the slope = 3 and used that to interpolate to find the value y for $x = 4$. Extrapolate, i.e., use the same procedure as interpolation but go outside the bounds of the given points, to determine the value of the intercept b.

13. Solve for x: 4 / 9 = 13 / x.

14. Solve for x: $^3\!/_4 - {}^1\!/_5 = {}^7\!/_x$.

The solutions are on the next page.

1. $1.72 \cdot 10^2$
2. 26,700
3. 0.000369
4. 15.85
5. $3^{2+4} = 3^6 = 7.29 \cdot 10^2$ or $9 \cdot 81 = 7.29 \cdot 10^2$
6. $3^{(-2 \cdot 3)} = 3^{-6} = 1.37 \cdot 10^{-3}$ or $(1/9)^3 = 1.37 \cdot 10^{-3}$
7. area = πr^2 so ratio = $\left(r_1^2 / r_2^2 \right) = \left(r_1 / r_2 \right)^2 = \left(400\text{cm} / 8\text{cm} \right)^2 = 2.5 \cdot 10^3$
8. volume = $4/3 \, \pi r^3 \rightarrow \dfrac{4\pi (7 \cdot 10^{-6}\text{m})^3}{3} = 1.44 \cdot 10^{-15}\,\text{m}^3$
9. $\dfrac{1\text{g}}{\text{cm}^3} \cdot \dfrac{1\text{kg}}{1000\text{g}} \cdot \left(\dfrac{100\text{cm}}{\text{m}} \right)^3 = \dfrac{10^3\text{kg}}{\text{m}^3}$
10.



$\tan \theta_1 = 3/4 \rightarrow \theta_1 = \tan^{-1}(3/4) = 36.87°$ or 0.644 rad
$\theta_2 = 180 - 90 - 36.87 = 53.13°$ or 0.927 rad

11. $r_{\min} = a(1-e) = 14\text{cm}(1-0.15) = 11.9\text{cm}$
$r_{\max} = a(1+e) = 14\text{cm}(1+0.15) = 16.1\text{cm}$

and checking, 11.9 + 16.1 = 28 = $2a$ = entire major axis.

12. Using the slope, already determined to be 3, and the first data point (1,4) we have $y = mx + b \rightarrow 4 = 3 \cdot 1 + b$; solving this shows that $b = 1$.

13. $\dfrac{4}{9} = \dfrac{13}{x} \rightarrow x \cdot 4 = 9 \cdot 13 \rightarrow x = \dfrac{9 \cdot 13}{4} = 29.25$

14. $\dfrac{3}{4} - \dfrac{1}{5} = \dfrac{7}{x} \rightarrow \dfrac{15-4}{20} = \dfrac{11}{20} = \dfrac{7}{x} \rightarrow x(11) = 140 \rightarrow x = \dfrac{140}{11} = 12.73$ or
$0.75 - 0.2 = 7/x \rightarrow x(0.55) = 7 \rightarrow x = 7/0.55 = 12.73$