Chapter 11: **Distances to stars, star motions, and the magnitude system.**

- equatorial coordinates
- parallax and the parsec
- motions of stars
- magnitudes
- sample problems

Equatorial Coordinates

Maps of the sky are often going to be displayed in coordinates called *Right Ascension* and *Declination*. These are the celestial equivalents of longitude and latitude on the ground. Imagine first extending the equator and north and south poles out into space; the resulting line and points are called our celestial equator and celestial poles. Imagine extending lines of latitude out into space; these become lines of declination. The celestial equator is 0° declination and the celestial poles are ± 90° Dec., just as the terrestrial equator and poles are 0° and ± 90°, respectively, latitude. Right Ascension is not quite as easy to visualize; first, it's measured in hours, 0 to 24, rather like having celestial time zones; second, because RA and Dec. are tied to the stars, as the Earth rotates the RA of stars, for instance those that are above you, will continually be changing. RA starts with 0^h at the point in the sky (on the equator, in Pisces) where the Sun is on the day of the March equinox, and it increases toward the east among the stars. Lines of RA are perpendicular to the celestial equator, just as lines of longitude are perpendicular to the terrestrial equator. As time passes, successively larger right ascensions will rise, or ascend, which helps in remembering which way it goes around the equator. Degrees of declination are broken up into arcminutes and arcseconds (denoted ′ and ″, respectively). Hours of right ascension are divided into minutes and seconds (m and s), rather like hours of time would be divided. Minutes of RA and arcminutes of Dec are not the same! Coordinates are covered in more detail in Chap. 3.

Distances, parallax, and the parsec

Distances to nearby stars are more than 10^5 AU (the Earth-Sun distance) and 1 AU is 150 million km, so you can see that the *meter* is rapidly going to become an unwieldy unit to use to express distances. We will sometimes use the light year for distances between stars (and the Mly, i.e., 10^6 ly, for intergalactic distances); a light year is the distance light travels (in a vacuum) in a time of 365.25 days. A light year is almost 10^{16} m (more precisely, it's $9.461 \cdot 10^{15}$ m). It takes light ~ 8 1/3 minutes to reach us from the Sun. The light year does not, however, come with any handy relationship with observations of stars. The parsec, on the other hand, does.

Nearby stars appear to move against the background of more distant stars over the course of the year as the Earth moves around the Sun.

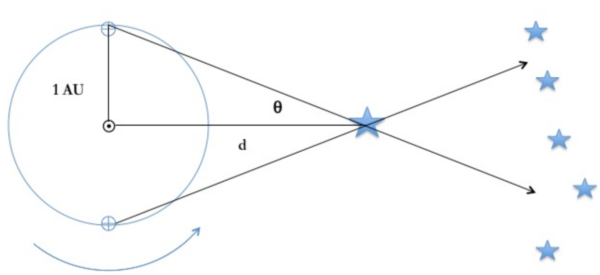

Figure 11.1: Trigonometric parallax

Note that the symbol ⊙ is shorthand for the Sun and ⊕ stands for the Earth. In this diagram – *not to scale!* – we have the Earth in orbit around the Sun at two positions six months apart, a relatively nearby star in the same plane as the Earth's orbit, and several more distant stars. In this most simple configuration the only object moving is the Earth. The angle θ is the parallax we observe for that nearby star due to the fact that we are observing from different points. If that star were close enough to have a parallax of one second of arc (1″) it would be at a distance of 206,265 AU. Why:

$\tan(1/3600\ °) = 1\ AU / d\ AU = 4.8481 \cdot 10^{-6}$; solve for d:

$d\ AU = 1\ AU / 4.8481 \cdot 10^{-6} = 1\ AU \cdot 206{,}264.81 = 206{,}265\ AU$.

Define the *parsec* (pc) as the distance to a hypothetical star with a parallax of 1″. The "hypothetical" bit is because there aren't actually any stars with a parallax that large. The closest system is the triple of stars known as α Centauri, which has a parallax of ~3/4″. This means that the distance to α Cen is ~4/3 pc. Note that all we have to do to determine the distance in parsecs is flip over the parallax. Handy, but why?

Recall the small-angle approximation: if our angle is tiny, *and in radians*, then sin θ and tan θ are both ≈ θ. In other words, in our diagram above,

$\theta_{rad} = 1\ AU / d\ AU$.

Note that there are 206,265 arcsec in a radian. Multiply both sides of this equation by 206,265:

$\theta_{rad} \cdot (206{,}265\ ''/\mathrm{rad}) = 1\ AU / [d\ AU \cdot (1\ pc / 206{,}265\ AU)]$.

In other words,

$$\theta'' = 1\ AU / d\ pc.$$

Learn this! This looks like nutty units: arcsec = AU / pc. It works because of the small-angle approximation and the units conversions that we just performed. There are, for comparison, 3.26 ly / pc.

As an historical note, the observations of stellar parallax were not made reliably until the 1830s, two centuries after the telescope showed up on the scene. The accepted first parallax accurate enough to be considered successful was reported in 1838 by Friedrich Bessel for the star 61 Cygni. In part this is because the angles are tiny and very hard to measure. (That parallax for the α Cen system? That's like the angle a dime would subtend 4.8 km away. . .) Ground-based observations, using modern CCD (charge-couple device) cameras, can measure star positions with accuracies of ~1 mas (milliarcsecond). That is still not very many stars, in the grand scheme of things. Things changed in 1989, when the European Space Agency (ESA) launched the Hipparchos spacecraft. Hipparchos, named for the ancient Greek astronomer, operated for nearly four years, totally dedicated to doing precision astrometry, i.e., measuring stellar positions (and motions) as precisely as possible. Hipparchos obtained high-precision results for more than 10^5 stars, with lower-precision measurements for over 2 million more. The Gaia spacecraft, launched by ESA in 2013, should be able to catalog ~ 10^9 stars over its intended five-year mission lifetime. That's impressive, and it will take us out to some very faint stars, but it's still only a tiny fraction of the ~ 10^{11} stars in our Milky Way Galaxy.

Examples:

1. 61 Cygni is 3.498 pc away; what's its parallax?

   $\theta'' = 1 / d_{pc} = 1 / 3.498 = 0.''286$

2. 61 Cygni is a binary, a pair of stars separated by ~24″. At that distance of 3.498 pc, what's the approximate separation of these two stars in AU?

   In this case the base of the triangle with ″ = AU / pc isn't the radius of the orbit of the Earth, it's the separation between the two stars. Same basic idea, though, because it's still a fairly small angle. Therefore

   $x_{AU} = \theta'' \cdot d_{pc} \rightarrow x_{AU} = 24'' \cdot 3.498_{pc} = 84\ AU$

Nearby stars are not of course all lying in the plane of the solar system (the *ecliptic*). Imagine a star directly above the center of the Earth's orbit; in that case, we would see the star trace out a tiny circle (ever so slightly elliptical) on the sky mimicking the orbital motion of the Earth. At some angle between the ecliptic and directly above the Sun, we will get a path that's a distinct ellipse. The semi-major axis of the ellipse is the parallax angle to use in the distance calculations.

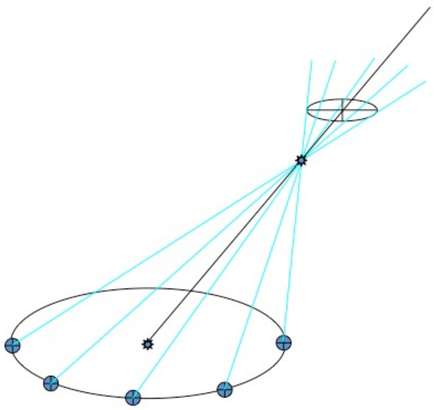

Figure 11.2: Parallax for a star lying off the ecliptic

Motions of stars

We made a simplifying, and incorrect, assumption, stating that nobody but the Earth moves, in the figure above. In other words, another other reason it is hard to measure parallaxes is because stars *do* move. The Sun, along with all the other nearby stars, is in orbit around the center of the galaxy. The stars are going to have slightly different velocities, though, meaning that nearby stars will often have measureable motions with respect to the Sun. In the following figure, ignore the motion of the Earth around the Sun, let the Sun stand still, and let's look at one possible trajectory for a nearby star over the course of one year:

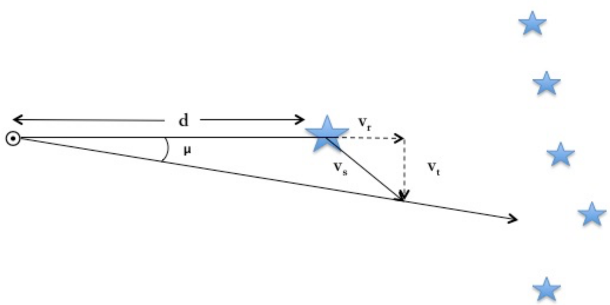

Figure 11.3: Motion of stars

First, note that the figure is not to scale!; among other things, this means that the distance to the star at the end of the year is really not noticeably different from its distance at the beginning of the year. Second, nomenclature on the velocity arrows: v_r represents the velocity in the radial direction, either toward or away from the Sun; v_t is velocity perpendicular to the radial direction, i.e., along a line tangent to a circle of radius *d* centered on the Sun; v_s is the star's space velocity. Using Pythagoras,

$$v_s^2 = v_r^2 + v_t^2.$$

The angle μ is called the star's *proper motion*. That's slightly archaic sounding, but it does make sense in that μ is what we see because of motion that is "proper" to the star itself, unlike the parallax due to the motion of the Earth. Proper motion is measured in units of ″ / yr. We would like to have the star's actual tangential velocity, though, not just the angle through which it changes its position among the background stars. You can probably see that the relationship between μ and v_t is going to involve the distance: if, for example, we had a second star with the same tangential velocity but twice the distance of the one drawn, the proper motion angle we would measure for this second star would only be half the angle for the first star.

Pause for an analogy: Imagine that you are driving along a straight stretch of highway through a fairly open rural setting. Personally I visualize being on the east side of the Cascade mountains in Washington State, far enough down in the foothills that I can see for several miles in any direction. There are a few trees and farmhouses nearby and tree-covered hills in the distance. If you look to the side (quickly, because you are driving) you see the nearby bushes seem to fly past while the distant hills barely seem to move. We clearly have the same tangential velocity with respect to both the nearby houses and the distant trees, but the *angular* change we see in the nearby fenceposts is much much larger than the angular shift in the distant hills. For a given tangential velocity, we are going to measure a larger proper motion for a nearby star.

Here's the math that goes with the proper motion – distance – tangential velocity equation:

$$v_{t,\,AU/yr} = \mu_{''/yr} \cdot d_{pc}.$$

This works because, recall, ″ · pc = AU. We might, though, want to express our tangential velocity in km/sec rather than in AU/yr. For that we need a conversion factor, namely how many (km/sec) / (AU/yr) . The conversion factor is 4.74, giving us

$$v_{t,\,km/s} = 4.74\,\mu_{''/yr} \cdot d_{pc}.$$

Next, the radial velocity. Here we make use of the fact that waves tell us about the relative radial motion of the source and us by way of the Doppler shift. A source that is moving towards us, say, emits light waves of a certain wavelength (λ) but because it is moving towards us while emitting that light, the waves get scrunched together, so that we see shorter wavelengths than we would have seen if there were no relative radial motion. The faster the motion, the more the shift in wavelength. Sideways motion won't have any effect—only the radial component of the motion matters. Below is a sketch showing a star moving toward the right at a constant velocity while emitting light, shown as circles expanding about the points from which the light was emitted (spherical bubbles, if we considered three dimensions). If the circles represent the crests of waves, we can see that the wavelength looks shorter to an observer on the right, longer to an observer on the left, and unchanged to an observer viewing the system tangentially.

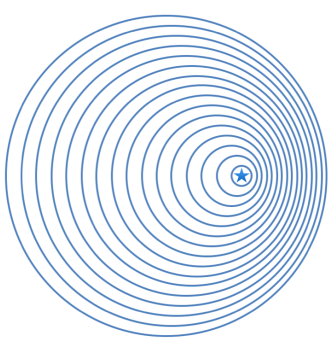

Figure 11.4: Doppler effect

The math looks like this:

$$\frac{\Delta\lambda}{\lambda_0} = \frac{\lambda_{\text{observed}} - \lambda_0}{\lambda_0} = \frac{\lambda_{\text{observed}}}{\lambda_0} - 1 = \frac{v_r}{c}.$$

The relativistic expression looks like:

$$\frac{\Delta\lambda}{\lambda_0} = \sqrt{\frac{1+v_r/c}{1-v_r/c}} - 1,$$

which, you can check, reduces to the non-relativistic expression in the limit of small velocity. In frequency units,

$$\frac{v_r}{c} == \frac{f_0 - f_{\text{obs}}}{f_{\text{obs}}} = \frac{f_0}{f_{\text{obs}}} - 1.$$

Why do we care? How much mass lies at the center of a galaxy? Assuming that the speeds of the objects in that galaxy are directly related to the amount of mass that they orbit, Doppler shifts are what will give us the answer. How fast is a nearby star coming toward us? Doppler shifts. How fast is a distant galaxy receding from us? Similar math. . .mostly. We'll look at that case later.

Examples:

1. Where did the 4.74 come from? You can almost do this in your head, especially if you approximate the number of seconds in a year as $\pi \cdot 10^7$ and write 150 million as $15 \cdot 10^7$:

$$\frac{1\text{ AU}}{\text{yr}} \cdot \frac{1\text{ yr}}{\sim\pi \cdot 10^7\text{ s}} \cdot \frac{15 \cdot 10^7\text{ km}}{\text{AU}} \approx \frac{15\text{ km}}{\pi\text{ s}} \approx 4.74\ \text{km}/\text{s}.$$

2. The star system 61 Cygni has a radial velocity of ~ – 64 km/sec, a proper motion of ~5.28 ″/yr, and a distance of 3.498 pc. What is its space velocity?

$v_{t,\,km/s} = 4.74\ \mu_{''/yr} \cdot d_{pc} \rightarrow 4.74 \cdot 5.28_{''/yr} \cdot 3.498_{pc} = 87.5$ km/sec;

$v_s = \sqrt{(v_t^2 + v_r^2)} \rightarrow \sqrt{[(-64_{km/s})^2 + (87.5_{km/s})^2]} = 108.5$ km/sec

Let's mention here that we might want to be a bit more picky about the proper motion. Stars are moving in a three-dimensional space and so far we've only considered two directions, radial and tangential. That tangential motion really could be broken up into two components at right angles to each other, some sort of cross-ways and up-down. We can use this three-dimensional picture to get an idea of how the Sun is moving around the galaxy. We'll want to use this idea, later, when we start discussing the structure of the Milky Way. Here's a figure to help visualize this. Step back from the Sun so that you can see it and its motion. A nearby star has its proper motion indicated and also the components of that proper motion, one, the υ (upsilon) component, that is parallel to the solar motion, and the other, the τ component, that is perpendicular to the solar motion. In this view, the radial component of the star's motion would be in/out of the page.

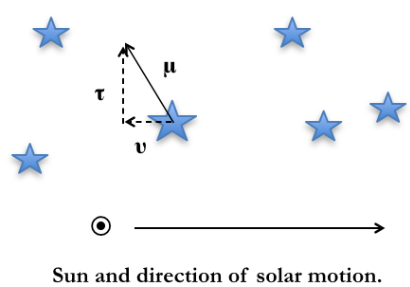

Figure 11.5: Components of proper motion.

But given that all the stars around us are moving it's not trivial to figure out what the solar motion is. The brightest part of the Milky Way galaxy is flattened rather like a fried egg. Bright stars, young clusters of stars, dust, and the solar system lie pretty much in the disk of the galaxy. Stars in the solar neighborhood are orbiting the center of mass of the galaxy in roughly the same direction that the Sun is, although orbits in the galaxy are not as neat and tidy as orbits of planets in the solar system, where nearly all the mass is in the Sun. If we look at stars within ~100 pc of the Sun the space velocities that we observe should mostly be due to slight orbital differences—a bit more eccentric, a bit more inclined, etc.—on what's basically the same orbit around the galaxy. For stars within that bubble the average of all the space velocities ought to be zero. We call that bubble the Local Standard of Rest. Make sense?: Everybody in the LSR bubble is going around the galaxy together; everybody in the LSR bubble has small random motions with respect to the framework of the bubble. By definition the motions of the LSR stars average to zero. The fact that in reality they don't average to zero is thus a reflection of the Sun's own peculiar motion with respect to the LSR bubble.

The Sun is moving toward a point called the *solar apex* and away from a point called the antapex. We are, on average, moving toward stars at the apex and away from stars at the antapex. That means ahead of us we see stars a bit blueshifted and behind we see them a bit redshifted. Stars to the sides appear to be moving backwards as we move through them; this is reflected, on average, in the upsilon components of their proper motions. Roughly, we're moving nearly toward the star Vega and away from the star Sirius. The solar motion with respect to the LSR is ~20 km/s; the LSR moves at ~220 km/s around the galaxy.

Radial velocities and proper motions are easier to measure than parallaxes for most stars. This has led, over the years, to some creative statistical methods for obtaining distances, if not to individual stars, at least to groups of stars. Even pre-Hipparchos satellite, no one was satisfied with these distances, but they were a starting point. Post-Hipparchos, consider the following distance methods as providing a bit of historical perspective.

- Statistical parallax. Recall that we can divide the stars' proper motions into υ and τ components that are parallel and perpendicular to the direction of the solar motion, respectively. The υ components are going to reflect the Sun's motion, but the τ components are by definition independent of the solar motion. We can write $v_\tau = 4.74\ \tau\ d$ and if we knew v_τ for a star we could find its distance. We don't know v_τ for *a* star, but we do know it *on average* for stars in the local standard of rest. Their motions are random, remember. That means that the magnitudes of their radial velocities are, on average, *the same as* the magnitudes of their tau velocities, on average. *If* we have a large enough sample of stars, we can use the average *radial* speed as if it were the average *tau* speed and determine the average distance to the stars in our sample. It's possible to narrow this down a bit by taking as your sample stars that you suspect are at roughly the same distances; e.g., take a group that have similar spectra (we'll discuss this below, but basically you are assuming that if the lines in the spectra look similar then the bulk properties of the stars are pretty similar) and roughly similar brightnesses. This method of statistical parallaxes will then give you an average distance to the stars in your sample.
- Secular parallax. The fact that the Sun moves ~4 AU/yr means that over time we have a much longer baseline for parallax measurements than simply the 2 AU across the Earth's orbit. The upsilon components of the proper motions of the stars in the LSR are on average going to be "backwards" since they are a reflection of the Sun's motion toward the solar apex. Remember that, were they standing still, nearby objects would have larger proper motions than distant objects. If we have a group of objects for which, as above, we have cause to expect have similar distances, and which are spread around the sky so that their motions with respect to the LSR by definition average to zero, then we can use the average υ components of the stars' proper motions and the solar velocity (it's relative—it might as well be the case that we were standing still and the stars moving) and use ~4 AU/yr = υ ("/yr) · *d* (pc) to get the average distance to our group of stars.
- The Moving Cluster method. Stars form in clusters, from a cloud of material moving around the galaxy. The stars will have their own peculiar motions with respect to the cluster, but by and large they will retain the overall velocity of the original cloud. One of the best-studied clusters is the Hyades, which form most

of the head of the constellation Taurus. The distance to the center of the cluster is ~47 pc and the cluster is several parsecs across. For an analogy, imagine looking along railroad tracks heading off straight into the distance; the rails will seem to converge. If you were looking across the tracks, there'd be no converging. If we look at the proper motions of the stars in the Hyades, which aren't moving straight away from us but at an angle, they seem to converge toward a point in Orion, a bit east of Betelgeuse. In the mid-20th century, this was a crucially important step in determining the distances across the Milky Way and to nearby galaxies.
Here's the geometry as seen from the side:

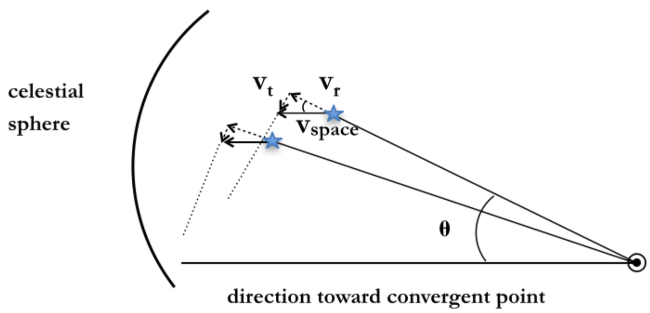

Figure 11.6: Motion of cluster stars in space.

Given the large distances to the stars, the lines drawn here from the Sun to the stars will all be close to parallel, and the lines drawn extending the stars' tangential velocities will all look as though they converge at one point. In other words, if you change your point of view 90° and look from the Sun toward the sky, the stars' motions look as though they are all moving toward the convergent point:

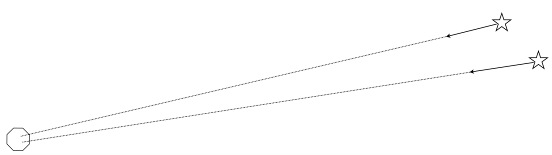

Figure 11.7: Motion of cluster stars on the sky.

For a more familiar example, think about the fact that if you were looking straight ahead and straight down a pair of straight railroad tracks (which might not be a smart thing to do, since that implies you are standing in the middle of the tracks!), they would appear to converge in the far distance toward a point in between the tracks; if you were looking at the tracks at a bit of an angle, e.g., they aren't going straight away from you nor are they perpendicular to your line of sight but at something like 45°, they would still appear to converge off in the distance. Now. . .imagine removing most of the track, just leaving a few meters of track and those bits being a few hundred meters away from you. It would be a lot harder to tell where those short stretches of track would appear to converge but you know that they would because they still represent recognizeable segments of parallel lines. The stars in a cluster are vastly farther away and their velocity vectors are not all exactly segments of parallel lines like the bits of railroad track, but the idea is similar: if we could extend the stars' velocity vectors off into space they would look as if they were eventually converging to a point; how long it would take them to converge depends on the stars' distances, speeds, and the direction the cluster is moving relative to our line of sight.

We can rewrite the stellar velocity equations to include the angle θ to the convergent point: $v_{space} = v_r / \cos \theta$ and $v_t = v_{space} \sin \theta$ (and as usual, $v_t = 4.74\mu d$).

On a map of that region of the sky, with many of the cluster member stars' proper motions indicated, we see something like this:

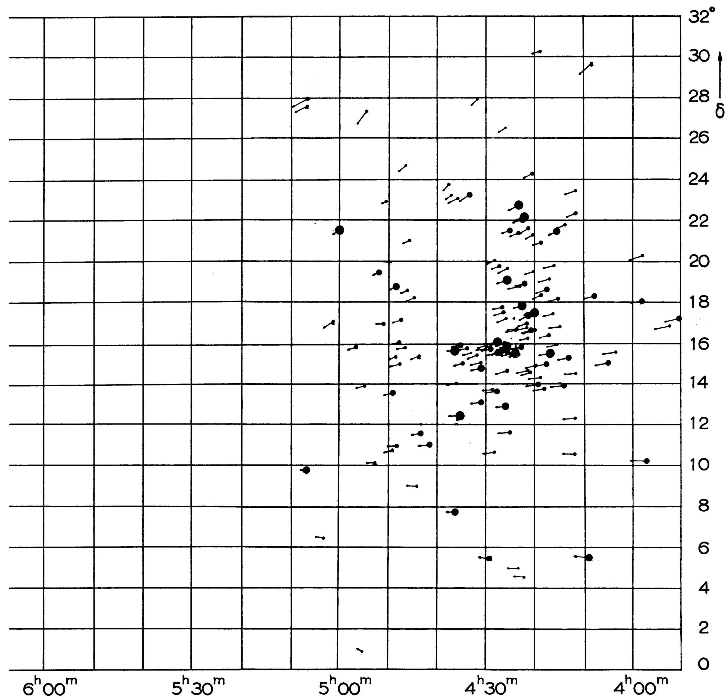

Figure 11.8: Motion of stars in the Hyades cluster.
The vertical axis is degrees of declination and the horizontal scale is hours and minutes of right ascension.

Credit: H. G. van Bueren 1952; Bulletin of the Astronomical Institutes of the Netherlands; "On the Structure of the Hyades Cluster", figure 4.

Distance matters. If we don't know the distance to a star or galaxy, we are going to have difficulty knowing what it is really like, e.g., how luminous, how large.

Magnitudes

Stars (and galaxies, and planets, etc., etc.) emit a certain amount of light per second; this is called their luminosity and it's measured in units of Joules / second. The amount of light that an observer will receive from a distant star falls off as the square of the distance between star and observer. Usually. This assumes that the light is spreading out spherically symmetrically from the source (some light sources are beamed, not symmetrical), in which case it crosses successively larger and larger surface areas as time passes so any unit-sized detector receives less and less as the distance from the source increases and the light gets spread out. The area of the surface that the light is crossing goes up as $4\pi d^2$ so the flux of light we receive with our detector must go down by $4\pi d^2$. As we get to more distant objects we will often find ourselves estimating the luminosity of the object and comparing the amount of light emitted with the amount we receive. Assuming that we know what's happened to the light on the way to us (a big "if", since interstellar dust will scatter light) we can work out the distance. Or if we think we can estimate the distance, perhaps because we can estimate the size of an object and compare that with its apparent angular size, then we can use the estimated distance and the amount of light we receive to estimate the luminosity. Recap that paragraph: f (J s^{-1} m^{-2}) $=$ L (J / s) / (4 π d^2 m^2). If you are really fuzzy about light, wavelengths, colors, blackbodies, etc., etc., you might want to go to the introductory chapter and read the section on light.

The amount of energy that we receive from a given star varies, though, over the day, with the weather, depending on where on Earth you are. The principal variable is how much air (what "air mass") the light is traversing. You've probably noticed that when the Sun is on the horizon it is a bit redder and a bit dimmer than when it is high in the sky or that the stars seem brighter when you are at high elevations than when you are at sea level. Particles in the atmosphere scatter and absorb sun and starlight. We say that there is "atmospheric extinction". And the reddening?— the extinction is color dependent.

We'll come back to extinction in a bit, because the same thing happens to light out in space, but first let's talk about one way to deal with it. Hipparchus, the chap of the early star catalog, established a magnitude scale for stars. He assigned stars to six brightness categories, one through six, with one being the brightest, and six the faintest, stars that he could see. It looks to your eyes, more or less, as though this is five equal steps in brightness. These five steps in magnitude corresponded to a factor of 100 in actual energy received as was verified by Norman Pogson, who in 1856 was attempting to quantify the magnitude. In other words, a first magnitude star is $\sqrt[5]{100}$ = 2.512... times brighter than a second magnitude star. (Our eyes, as detectors of light intensity, are not linear in their response.) Telescopes take you a lot fainter than your eyeballs, and the magnitude scale has simply been extended. Today we would call these *apparent* magnitudes, because they are based on how the stars appear to us.

Now, why this might be even remotely practical: variations in extinction and variations between detectors mean that the actual amount of energy you record as having been received from a given star keeps changing. But if your target star is near an already known star, i.e., a comparison star for which a magnitude has already been established, those variations won't matter (well, properly, they will matter much less). You can observe both your target star and the nearby comparison star and the magnitude difference between the two will not change (much). If your star is half a magnitude fainter than the comparison star it will be half a magnitude fainter whether you are at sea level or 5,000 feet, whether the stars are at their highest in the sky or near the horizon, whether you are observing with binoculars or using the spiffiest CCD (charge-couple device, i.e., digital) camera on a meter-scale telescope. As long as you observe both stars with the same equipment, with the starlight coming through the same amount of air, as close in time as possible, you can determine the magnitude of your target star in a straightforward fashion.

We do still need to be able to convert, at some point, between magnitudes and fluxes. To recap the words again: each step in magnitude corresponds to a difference in flux of a factor of the 5th root of 100. Translation into algebra:

$$\frac{f_a}{f_b} = 100^{(m_b - m_a)/5} = 10^{\frac{2}{5}(m_b - m_a)},$$

where m_a and m_b are the magnitudes of stars *a* and *b*. Take the base 10 log of both sides of this equation, and recall that $\log x^n = n \log x$; this gives us:

$$m_b - m_a = 2.5 \cdot \log \frac{f_a}{f_b}.$$

Examples.

1. How many times more energy do we receive from a 3rd magnitude star than from a 6th magnitude star?

   3 magnitudes is $\left(\sqrt[5]{100}\right)^3$ = 15.85 ;

   by the full equation,

$$\frac{f_a}{f_b} = \frac{f_{3^{rd}\text{mag}}}{f_{6^{th}\text{mag}}} = 10^{\frac{2}{5}(6-3)} = 15.85.$$

2. With a 16-inch diameter telescope we receive 4 times as much light as with an 8-inch diameter telescope. How many magnitudes fainter can we see?

$$\Delta m = 2.5 \cdot \log 4 = 1.5$$

magnitudes fainter. Check to see if this makes sense: We have a factor of 4 times more light; one magnitude would be 2.512 times more light, two magnitudes would be 6.25 times, so yes, an answer in between 1 and 2 makes sense.

Warning: if you have two stars, e.g., in a close binary system, and you want to *add* their light you have to add fluxes, you cannot add magnitudes! But the deal with magnitudes is that we frequently don't know what the fluxes are, right, so how can we add them? There are ways around this problem.

Suppose you have a binary star with known magnitudes m_a and m_b. You can split them in a telescope but in binoculars they are too close together and look like one star. What's the magnitude of that run-together star, i.e., what's the combined magnitude of the two stars? One way to approach the problem is to compare each, individually, to a hypothetical star of magnitude zero:

$$10^{0.4(0-m_{a+b})} = \frac{f_{a+b}}{f_0} = \frac{f_a}{f_0} + \frac{f_b}{f_0} = 10^{0.4(0-m_a)} + 10^{0.4(0-m_b)}.$$

What you want is m_{a+b}, which you can express in terms of f_{a+b}/f_0. That ratio can be split into ratios involving the individual fluxes and those expressed in terms of the individual magnitudes, which you know. An equivalent way to tackle the problem is to use the fact that the magnitude difference means that you know the ratio f_a/f_b:

$$10^{0.4(m_b-m_{a+b})} = \frac{f_{a+b}}{f_b} = \frac{f_a}{f_b} + 1 = 10^{0.4(m_b-m_a)} + 1.$$

Same basic algebra, just a slightly different approach. You're still going to have to take a power of ten, do some addition, and take a $\log_{10}$ to extract the m_{a+b} term. This isn't just an exercise in gratuitous math: overall about 1/3 of the stars in the Milky Way are in multiple systems (more among the massive stars, about half of solar-mass stars, fewer of low-mass stars).

Example.

Suppose we have a binary with stars of magnitudes 7.2 and 8.5. What is the combined apparent magnitude of the system? First consider what sort of answer would make sense: the combined magnitude should be brighter than the brightest of the two stars, but not by a lot since they are not wildly different; 7, maybe, or 6.8-9 would make sense as answer. Let's do it twice, using both of the above equations; let star *a* be the 7.2 magnitude star and star *b* the 8.5 magnitude star.

$$10^{0.4(0-m_{a+b})} = \frac{f_{a+b}}{f_0} = \frac{f_a}{f_0} + \frac{f_b}{f_0} = 10^{0.4(0-m_a)} + 10^{0.4(0-m_b)} \rightarrow$$
$$10^{0.4(-m_{a+b})} = 10^{0.4(-7.2)} + 10^{0.4(-8.5)} = 1.318 \cdot 10^{-3} + 3.981 \cdot 10^{-4} = 1.716 \cdot 10^{-3}$$
$$\log\left[10^{0.4(-m_{a+b})}\right] = \log(1.716 \cdot 10^{-3}) = -2.765$$
$$m_{a+b} = -2.765 / -0.4 = 6.9.$$

$$10^{0.4(m_b-m_{a+b})} = \frac{f_{a+b}}{f_b} = \frac{f_a}{f_b} + 1 = 10^{0.4(m_b-m_a)} + 1 \rightarrow$$
$$10^{0.4(8.5-m_{a+b})} = 10^{0.4(8.5-7.2)} + 1 = 3.311 + 1 = 4.311$$
$$\log\left[10^{0.4(8.5-m_{a+b})}\right] = \log(4.311) = 0.6346$$
$$0.4(8.5 - m_{a+b}) = 0.6346 \rightarrow$$
$$0.4 \cdot 8.5 = 0.6346 - [0.4 \cdot (-m_{a+b})]$$
$$m_{a+b} = \frac{0.4 \cdot 8.5 - 0.6346}{0.4} = 6.9.$$

Does this agree with our estimate? Yes. Why 6.9 instead of 6.914 or whatever your calculator gave you? If you are looking at significant digits, notice that we carried a couple of extra decimal places in doing the logs and exponents and then rounded back down in the last step. Rounding in the middle would decrease the accuracy of our result. But 6.914 would not be right, either, because those last two digits don't mean anything; we couldn't justify keeping them with only two places in the initial magnitude data.

Let's get the distance into the problem. Two stars with similar luminosities but different distances are going to have different fluxes. To compare stars in terms of their actual energy output, i.e., their luminosities, we need to ask what magnitudes they would have if they were at a standard distance. The distance that we use is 10 parsecs. The magnitudes based on fluxes, i.e., on what we see, are called apparent magnitudes; the magnitudes that stars would have if they were at 10 pc, i.e., magnitudes that are directly related to luminosities, are called absolute magnitudes and denoted by capital *M*. The flux and the luminosity are related by the inverse square of the distance. In other words, we can write

$$\frac{f_{10\,\mathrm{pc}}}{f_{\mathrm{actual\ d}}} = 10^{0.4(m-M)} \text{ and } \frac{f_{10\,\mathrm{pc}}}{f_{\mathrm{actual\ d}}} = \frac{4\pi d^2}{4\pi 10^2} = \frac{d^2}{10^2}.$$

Take the $\log_{10}$ of the left- and right-hand sides of this equation and divide through by 0.4:

$$10^{0.4(m-M)} = \frac{d^2}{10^2} \rightarrow \frac{2}{5}(m-M) = 2\log(d) - 2\log(10) \rightarrow$$

$$\boxed{m - M = 5\log d - 5.}$$

This equation is called the "distance modulus" and it is to be learned!

Now let's add color to the mix. The original magnitude definition was based on how bright stars appear *to the eye*. As detectors, our eyes, in addition to not responding linearly to changes in the intensity of light, are not flat (or "gray") in terms of their response to different colors of light. We're best with greens and yellows, not as good with reds or blues. The first photographic emulsions were more sensitive to blue than our eyes are. (And they were downright lousy with red—if you've seen any very old movies, you might have noticed that an actress' red lipstick looked totally black!) The grains in a photographic emulsion respond to the intensity of light in a way that's very similar to our eyes' response; in other words, the diameter of a star image on a photograph is linearly related to the magnitude of the star. But they don't respond to color the same way our eyes do. Stars that we might think look similar in brightness, in the "visual" colors, could be more than a magnitude apart in the blue. "Visual" magnitudes needed to be distinguished from "photographic" magnitudes.

Today we are more likely to use digital detectors, but the problem still remains; we need to specify a magnitude by color. One way to cope with this is to use filters designed to transmit blue light, say, or red, or IR, and block out all other colors. Multiple observations of a star, using a set of several transmission filters of different colors, allow you to sort out which stars are bluer and which redder. In other words, we're beginning to "dissect" the starlight and learn something about the physical properties of the stars. The downside is that this takes more time at the telescope than collecting all the light from a star, since you're making multiple observations, each one of which takes longer since there's less light per observation.

The most widely used broad-band photometric filter system is named for H.L. Johnson, who developed the standards for its visible wavelength filters. Johnson's initial filters were called *U, B*, and *V*, for ultraviolet, blue, and visual bandpasses. The system has been extended into the red and infrared— the Cousins *R* and *I* for red and near-IR and then *J, H, K, L, M, N*, out into the IR. CCD detectors tend to be sensitive well out into the infrared, much more so than photographic emulsions. (And yes, it can look as though astronomers are alphabetically challenged!). Here's a sketch of the transmission profiles of the *UBVRI* filters and a table with data about the bandpasses of all the Johnson - Cousins system filters.

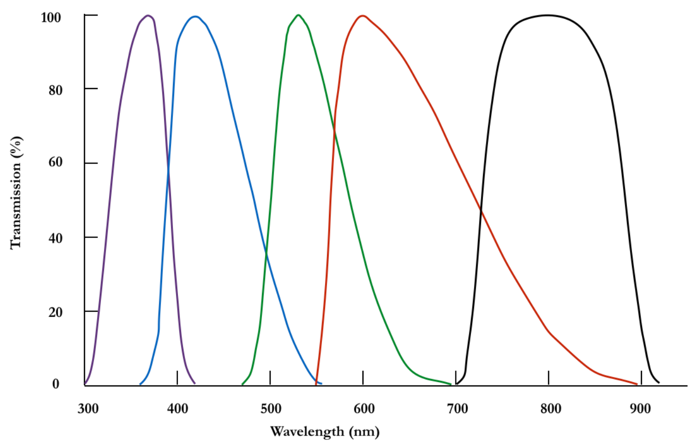

Figure 11.9: Johnson - Cousins filters

filter	average wavelength	width
U	365 nm	66 nm
B	445 nm	94 nm
V	551 nm	88 nm
R	658 nm	138 nm
I	806 nm	149 nm
J	1.22 µ	0.21 µ
H	1.63 µ	0.31 µ
K	2.19 µ	0.39 µ
L	3.45 µ	0.47 µ
M	4.75 µ	0.46 µ
N	10.5 µ	2.5 µ

Astronomers have established a set of calibration stars which are defined to have zero magnitude for a given filter (Vega is good, because it's very white) and standard stars around the sky which have established magnitudes in the various colors for observers to use as comparison stars.

We use a subscript to indicate that a magnitude refers to a specific color; e.g., m_V would indicate an apparent visual magnitude. The Johnson system filters are used so extensively that it is usually more convenient simply to use V or B rather than m_V or m_B. If we were talking about V-band magnitudes, we would write the distance modus as

$$m_V - M_V = 5\log d - 5.$$

A *color index* for a star is the difference in magnitudes taken in two different color bandpasses. Recall the comment, above, about how two stars might look similarly bright in the visual but be very different in the blue. The values for color indices, calculated as blue – visual, will allow us to quantify how much bluer one star is than the other. The most common color index, as you might expect, uses the Johnson blue and visual filters. $B - V$ is the difference in magnitudes between a star's brightness in the blue and in the visual. But if you recall the relationship between magnitudes and fluxes, you can see that a magnitude difference is equal to a flux ratio. In algebra,

$$(B-V) = 2.5 \cdot \log\left(\frac{f_V}{f_B}\right) + \text{ constant.}$$

(You need the constant because the filter bandpasses are not identical.)

Here is a sketch showing blackbody spectra for objects at 30,000 K and 3,000 K as well as the locations of the *UBVRI* filter passbands. The 30,000 K object is brighter in *B* than in *V* and has a *negative (B – V)* value; the cooler object is brighter in *V* and has a *positive (B – V)* value. The arrows, offset for clarity, are aligned with the *B* to *V* slope of the spectra. Remember: brighter means a *smaller* value for the magnitude!

The Sun has a *(B – V)* value of 0.62; a 10,000 K star will have a *(B – V)* ~0. Hotter stars will have negative *(B – V)* values, down to ~ – 0.3; the coolest stars, at the other end, will have *(B – V)* values up to ~3.

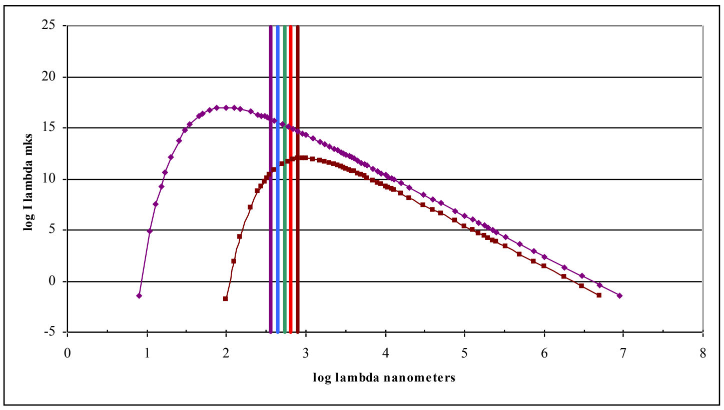

Figure 11.10: Blackbody spectra

Stars are not exactly blackbodies but *if they were* then a star's *(B – V)* value would give us the slope of its spectrum which would be inversely related to its surface temperature. Define a star's *effective temperature* (T_{eff}) as the temperature of a blackbody with the same total flux as the star is actually emitting. Calculus alert: in other words, the area under the star's spectrum and the integral of the blackbody spectrum of temperature T_{eff} are equal. What we would like is a relationship between (*B – V*) and T_{eff} , preferably one that would apply to all stars. Unfortunately it's not that simple. It will, for instance, depend on a star's composition: stars with lower metallicity (e.g., old stars, formed before there was much enrichment of heavy elements, when the universe was mostly still just hydrogen and helium) will look a bit bluer than solar type stars because heavy elements tend to absorb a lot in the blue and green; if more blue and green gets out, then your star will look bluer. It will also depend on whether the star is a giant or not, and it won't be the same for hot stars as for cool stars. Very annoying, but there it is. You can, at least, still state that, most other things being equal, hotter stars are going to be bluer and thus have smaller (*B – V*) values than cooler stars.

Asking about a relationship between a star's color index and its temperature presupposes that we can observe the actual color index. We mentioned extinction, above, and the fact that it is color dependent. On its way from the star to us, starlight is likely going to encounter interstellar dust. Dust will scatter starlight. One result is that the star will look dimmer than it would in the absence of dust. When we look in detail at the electromagnetic interaction between the light and the dust particles, we find that the really high energies, X-rays and Gamma-rays where the light is basically a very high energy particle, blast their way through the dust. The lowest frequencies, i.e., out in the radio regime, don't tend to interact with particles the size of the typical interstellar dust particle (micron-ish). In between, i.e., UV – visible – IR, we find that the shortest wavelengths are the most likely to be scattered by the dust. (Physics note: It's called Rayleigh scattering; the interaction cross-section goes as λ^{-4}.) Blue light is more likely to get scattered than red light, meaning that by the time the starlight reaches us, it will appear reddened to us compared to the light as it left the surface of the star.

Make sense? The star will look dimmer and redder. Let's see how that affects the mathematical relationships involving magnitudes and colors.

First, being dimmer than it would be without dust is equivalent to saying that the star looks at though it is farther away than it really is. The distance modulus equation becomes:

$$m_V - M_V = 5\log d - 5 + A_V.$$

A_V is the number of magnitudes of extinction that the starlight has experienced, in this case in the *V* bandpass. Near the Sun, in the disk of the galaxy, the extinction in the visual bandpass is on average about 1 magnitude per kiloparsec. E.g., if a star is about 500 pc away, then it is likely to appear about 0.5 magnitudes fainter in *V* than it would if there were no dust. Dust is clumpy, though, so this is only an estimate. And you can see that, algebraically, it is a tough rule of thumb to use:

$$m_V - M_V = 5\log d - 5 + (d/1000).$$

This is not an easy equation to work with, since the distance is in both a log term and a linear term.

The star will look extra red, or, equivalently, its $(B-V)$ value will be larger, or, equivalently, the slope of its spectrum (from blue toward red) will increase. We define the *color excess* as the difference between the observed color index and the color index that the star would have in the absence of dust. In $(B-V)$ the notation looks like this:

$$E_{(B-V)} = (B-V)_{\text{observed}} - (B-V)_{\text{intrinsic}}.$$

We could rearrange the magnitudes to express the color excess slightly differently:

$$E_{(B-V)} = (B_{\text{obs}} - B_{\text{int}}) - (V_{\text{obs}} - V_{\text{int}}) = A_B - A_V.$$

Notation alert: you may see this written as $E\,(B-V)$, which can look confusingly as though we are multiplying something rather than describing a function of $(B-V)$.

You can also talk about an excess in other color indices, e.g., $E\,(U-B)$, as well.

As you might expect, the amount of extinction and the amount of reddening tend to be related. Another rule of thumb is that

$$A_V \approx 3.1\, E_{(B-V)}.$$

Here is a sketch of a 3,000 K blackbody and that same blackbody reddened (lower curve), with 1^{mag} extinction in the visual.

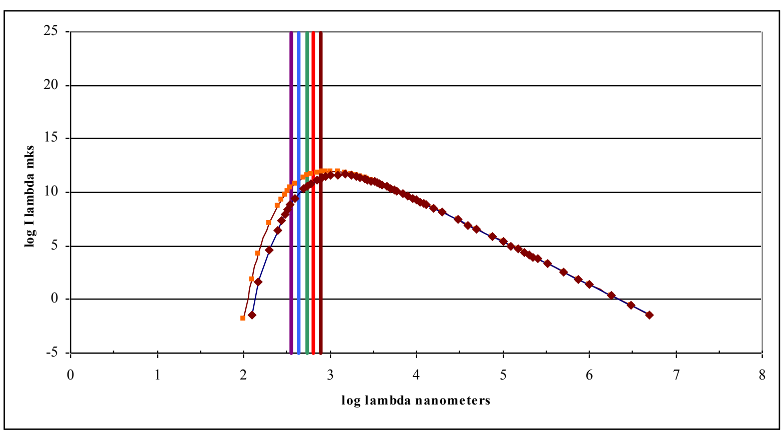

Figure 11.11: Blackbody curve with extinction

The arrows, again, are offset for clarity and aligned with the *B* to *V* slope of the spectra. The reddened slope is steeper; this object has a color excess $E_{(B-V)}$ of ~0.32.

If you are interested in a bit more detail about how to quantify the extinction as a function of wavelength, the following table shows the extinction A_λ relative to A_V for the other filters in the Johnson-Cousins system.

λ (nm)	filter	A_λ / A_V	λ (nm)	filter	A_λ / A_V
365	U	1.53	1220	J	0.28
445	B	1.32	1630	H	0.18
551	V	1	2190	K	0.11
658	R	0.75	3450	L	0.06
806	I	0.48	4750	M	0.02

Ideally we'd also like to be able to observe the brightness of the star over its entire spectrum; that's not easy to do, since detectors tend to be sensitive over limited wavelength regions. When we *can* estimate the brightness over all wavelengths we have what's called a *bolometric* magnitude. Since a star's bolometric magnitude is brighter than its visual magnitude, the *bolometric correction*, $M_{\text{bol}} - M_V$, is going to add a negative number to the distance modulus equation. Bolometric corrections are usually given as negative anyway, so solving the distance modulus for absolute magnitude we have:

$$M_V = m_V - 5\log d + 5 - A_V \rightarrow$$
$$M_{bol} = m_V - 5\log d + 5 - A_V + B.C.$$

For example, the Sun's M_V is 4.83 and its $M_{\text{bolometric}} = 4.75$; the Sun's bolometric correction is –-0.08.

Let's catch up on vocabulary:

- apparent magnitude: how bright a star appears, on a log scale where 6 is ~ naked-eye limit; we receive 2.512 times more light from a 6th magnitude star than from a 7th magnitude star
- absolute magnitude: the magnitude the star would have if it were at a distance of 10 parsecs.
- magnitudes are wavelength dependent; the Johnson *UBVRI* are standard filters; bolometric magnitudes mean over all wavelengths
- extinction: how many magnitudes fainter a star looks because of the dimming effect of interstellar dust.
- color index: a difference in magnitudes through two different-wavelength filters; for example, $(B - V)$.
- color excess: the difference between the observed color index and the actual color index.
- reddening: starlight looks redder than it would in the absence of interstellar dust.

Examples: Suppose that we have a star that we know (for instance, based on its spectrum) to be a twin to the Sun, meaning the same M_V and $(B - V)$. This star has a distance of 400 pc.

a) Estimate the extinction for this star.
In the absence of other evidence, we can assume that $A_V \sim 1$ mag/kpc; at a distance of 400 pc, that's $A_V \sim 0.4^{\text{mags}}$.

b) Estimate the color excess for this star.
We can use $A_V \approx 3.1\ E_{(B-V)}$ to estimate the color excess: 0.4 / 3.1 = 0.13.

c) Estimate the observed $(B - V)$ color index.
$E_{(B-V)} = (B - V)_{\text{observed}} - (B - V)_{\text{intrinsic}} \rightarrow 0.13 = (B - V)_{\text{obs}} - 0.62$
$(B - V)_{\text{obs}} = 0.75$

d) What apparent magnitude would this star have if there were no extinction?
$m_V = 5 \log d - 5 + M_V = 5 \log (400\text{ pc}) - 5 + 4.8 = 12.8$

e) What apparent magnitude would this star have accounting for the extinction?
$m_V = 5 \log d - 5 + M_V + A_V = 5 \log (400\text{ pc}) - 5 + 4.8 + 0.4 = 13.2$

f) If we didn't know about the dust and used that value of 13.2 for the apparent magnitude, what would we, incorrectly, estimate the star's distance to be?
$5 \log d = m_V - M_V + 5 = 13.2 - 4.8 + 5 = 13.4 \rightarrow d = 479$ pc.

g) What if we had known that the star has an apparent magnitude of 13.2 but didn't know how far it actually is; how would we estimate its distance?
$m_V - M_V = 5 \log d - 5 + A_V \rightarrow 13.2 - 4.8 = 5 \log d - 5 + d/1000$
$13.4 = 5 \log d + d/1000$.
This isn't easy. You can't solve this analytically, not with a d in a log and in a linear term. We know the real distance must be *less* than 479 pc, because we know that the dust makes the star appear to be farther away than it really is. You could use an online site such as WolframAlpha (at http://www.wolframalpha.com/) or you could just guess and check. As an example of the guess and check, let's do a few rough calculations to see how we might iterate to the real distance (which, recall, we expect to be 400 pc):

estimate distance (pc)	calculate $5 \log d + d/1000$	
479	5 log(479) + 479/1000 =	13.88
350	5 log(350) + 350/1000 =	13.07
400	5 log(400) + 400/1000 =	13.41

We see that 400 pc gives us the expected distance modulus of ~13.4.

Sample problems

1. Suppose that someday there are astronomers observing from Mars, which orbits the Sun at an average distance of 1.52 AU. Alpha Centauri is 1.3 pc away from our solar system. What's the parallax of α Cen as seen from Mars?

2. Consider a binary star system whose two stars we can both see, the two having an apparent angular separation of 34″; the parallax of the system is $7.5 \cdot 10^{-3}$″. Assume that we are seeing this system edge-on, i.e., that the two stars are the same distance from us. How far apart are the two stars from each other in AU?

3. Consider a star whose spectrum we have recorded. The "K" line, one of the two resonance lines of singly ionized calcium, has a rest wavelength of 3933.6614 Å (1 Ångstrom = 0.1 nm). We measure the K line in the observed spectrum of the star to have a wavelength of 3933.3729 Å. Calculate the star's radial velocity.

4. Barnard's Star is a relatively close, faint, high-proper motion star. It has the following observed properties:
$m_V = 9.51$ $d_{now} = 1.827$ pc $v_{rad} = -110.5$ km/sec
$B - V = 1.73$ $\mu_{now} = 10.37$ ″/yr

a) Assuming it keeps moving with its current space velocity, how far will Barnard's Star be when it is at its closest to the Sun? Hint: distances and velocity vectors make similar triangles.

b) How many years from now will it be at that closest point?

5. Suppose a variable star changes its luminosity periodically by a factor of 3.7. What is the corresponding change in bolometric magnitude?

6. The star β Centauri has a parallax of $8.32 \cdot 10^{-3}$ ″ and an apparent visual magnitude of 0.61.

   a) Assuming no interstellar extinction, calculate β Cen's absolute visual magnitude.

   b) If there were no extinction, how far away could a star such as β Cen be and still be visible to the unaided eye (i.e., be at least ~ 6^{th} mag)?

   c) Now, dust: Assuming we have an average of 1 mag extinction per kpc, how far could such a star still be visible to the unaided eye?

7. Consider two stars both having apparent magnitudes $V = 6.8$. Their blue magnitudes differ: $B_1 = 7.2$, $B_2 = 8.3$.

   a) Which star is bluer and how do you know?

   b) What is the ratio of the flux we receive in the blue from these two stars?

8. For the binary star β Cygni (Albireo) we have the following:
   star 1: $m_V = +3.09$ $(B - V) = +1.08$
   star 2: $m_V = +5.11$ $(B - V) = -0.10$

   a) What is the ratio of the flux we receive in the blue from these two stars? Hint: the bluer star might not be brighter in the blue.

   b) What is the combined visual apparent magnitude of these two stars?

9. The choice of 10 parsecs in the definition of absolute magnitude is arbitrary. What would the distance modulus be if we'd used 251 pc?

10. Reading carefully? Explain / sketch / define:
   a) parallax
   b) light year
   c) parsec
   d) proper motion
   e) space velocity
   f) local standard of rest
   g) flux
   h) luminosity
   i) apparent magnitude
   j) absolute magnitude
   k) reddening
   l) redshift
   m) color index
   n) color excess
   o) effective temperature
   p) bolometric correction

Answers to problems are on the next page:

1. 1.17″
2. 4530 AU
3. −22 km/sec

4. tangential velocity = 89.8 km/sec
   space velocity = 142.4 km/sec
   a triangle of the velocity vectors has angle at Barnard's Star of 39.1 degrees
   distance to go to closest point: 1.42 pc
   a) closest distance: 1.15 pc
   b) years until closest distance: ~9735

5. 1.42

6. a) −4.79
   b) 1,438 pc
   c) ~ 935 pc

7. a) star 1 is bluer; its *(B − V)* is smaller
   b) ~2.8

8. a) $f_1/f_2 = 2.2$; star 2 is bluer but star 1 is brighter in the blue.
   b) 2.93

9. $m - M = 5\log d - 12$