Chapter 12: **Binary stars**

- detection methods
- determination of binary star masses
- eclipsing binaries and stellar diameters
- measuring stellar diameters for single stars
- comments on statistics on multiple star systems
- sample problems

Detection methods

A binary, shorthand for binary star system, is a pair of stars gravitationally bound together, in orbit around their mutual center of mass. If we can determine the properties of that orbit — distances, speeds — we can calculate the masses of the stars. Observing the gravitational influence of a star on another object is about the most straightforward to measure stellar masses. If single stars behave similarly to stars in multiples, then we can use what we learn about masses of binary stars to infer masses of individual stars. Some binary orbits will be aligned so that the stars periodically eclipse each other; timing the eclipses can provide information about the stars' diameters. Again, if single stars behave the way binaries do, we can use what we learn from binary stars to infer diameters for single stars. Binaries are very important for measuring sizes, although we will consider at the end of the chapter possibilities for determining diameters of individual stars during occultations or by direct imaging. If the stars are of unequal masses or brightnesses, it is customary to call the larger star the primary and the smaller the secondary. (Yes, this can be confusing if the fainter star is more massive!)

Binary systems are often classified according to the method by which they are detectable as binaries; some may be detectably binaries by more than one method. Lots of stars in the sky look, to our eye, as if they are close together even though they may in reality be at vastly different distances and simply lie along the same line of sight. These are called optical doubles or apparent binaries. It takes careful measurement of the stars' distances and / or motions to determine that they are not actually physically bound. We'll introduce the types of actual binaries briefly and then discuss how to use them in more detail.

Visual binaries are, as you might expect, pairs of stars that can be seen in images, often collections of images taken over many years, to be moving around each other. In an *astrometric binary* we can only see one of the stars in the pair but we can over time see its motion around its unseen companion. It's akin to a visual binary but you only see one star.

Two detection methods involve spectroscopy, meaning the study of the spectrum of the stars. A *spectrum binary* refers to a pair that can't be resolved visually but whose spectrum shows clear evidence of two stars, for instance of different temperatures. *Spectroscopic binaries* are like spectrum binaries in that we see two sets of lines in the spectrum but in this case the stars have sufficiently large radial velocities to produce a detectable Doppler shift in the spectral lines. The lines shift back and forth in wavelength as the stars move to and fro.

Eclipsing binaries, mentioned above, are binaries whose mutual orbit lies perpendicular, or near enough, to the plane of the sky that one star periodically passes in front of or behind its companion. The combined light we receive from the pair drops when one or the other star is hidden from us during an eclipse.

Determination of binary star masses

If you've read the section on orbits you know that mass shows up in two useful (related) equations. To restate,

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3$$

$$\mathrm{v}^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right).$$

Both of these equations involve the stars' separation — recall that *a* is the orbital semi-major axis — and a measure of their motion, either orbital period or orbital speed. If we can measure these quantities we will be able to determine the stars' masses.

Let's look first at visual binaries, where we are able to estimate the stars' orbit period but not their speeds. What we would like to be able to do is to reconstruct the stars' actual orbit around their mutual center of mass. That actual orbit doesn't necessarily lie in the plane of the sky, but rather at some *inclination* to the plane of the sky. Inclination in this case is defined such that 0° corresponds to an orbit in the plane of the sky and 90° corresponds to an orbit that's edge-on to our line of sight. For any non-zero inclination we are going to observe an apparent orbit that is the projection of the actual orbit on the plane of the sky. For example, imagine holding something circular and flat, such as a quarter. If you are holding the quarter face-on, so that all parts of it are equally distant from your eye, then it looks round. If you tip the quarter, for instance so that the top side is more distant than the bottom side, it will look as if it is an ellipse. If you held it edge-on, the extreme case for that tilt, the quarter looks like a line segment. A visual binary orbit is an ellipse; if it is at all inclined to the plane of the sky, the apparent orbit will will be a more eccentric ellipse than the actual orbit. While both stars are orbiting their mutual center of mass it often makes sense to draw a relative orbit, i.e., one in which the larger star is fixed and we plot the positions of the secondary star relative to the first. The following sketch shows what a relative apparent orbit might look like.

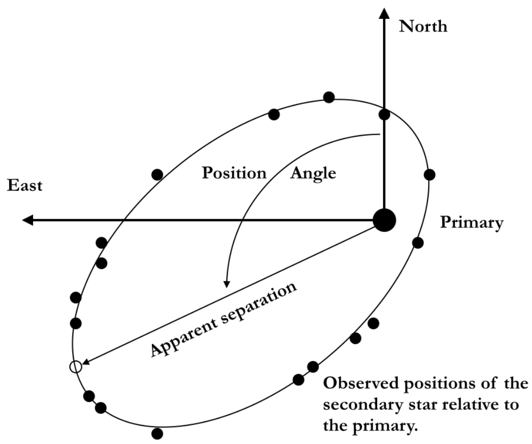


Figure 12.1: Example relative apparent orbit

Locations of the secondary star are indicated by the position angle, measured eastward from north, and the apparent separation. If we know the distance to the pair, we can convert an apparent angular separation into an apparent separation in AU. (Recall that for small angles, we can say that ″ · pc = AU.)

In a relative orbit we are assuming that the primary is at the center of mass. This means that the primary should be at one focus of the orbit, but notice that in our example relative apparent orbit, the primary is not along the major axis of the apparent ellipse. We may be able to reconstruct the actual relative orbit: the center of an ellipse stays fixed when the ellipse is tilted and we know that the major axis of the actual relative orbit must go through the center and through the primary. If we have enough observations we can also apply Kepler's 2nd rule for orbits, namely that a line connecting a star with the center of mass will sweep out equal areas (of the true relative orbit) in equal times. The apparent and actual major axes of the example apparent relative orbit are shown in the following sketch:

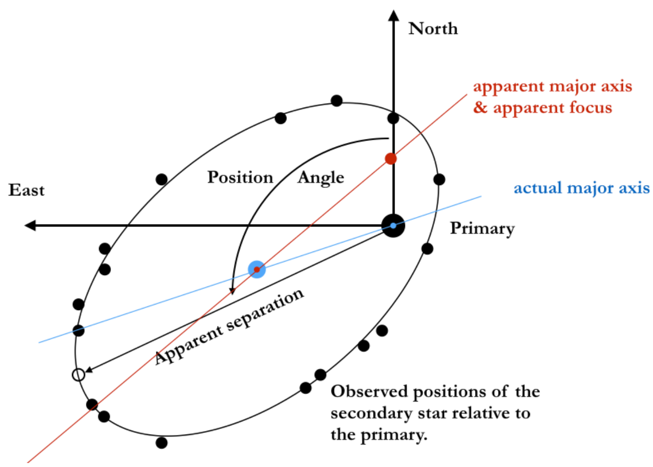


Figure 12.2: Example relative apparent orbit with true major axis.

Relative orbits are convenient in the sense that only one object is "moving", but as noted above, both stars are actually in orbit around their mutual center of mass. Here's a recap from the section on celestial mechanics: The center of mass, or barycenter, between the two stars is given by $m_1 r_1 = m_2 r_2$. The problem of m_1 and m_2 in orbit around each other at distances r_1 and r_2 from their center of mass is equivalent to the problem of an object of mass μ, where $\mu = {}^{m_1 m_2}\!/\!_{(m_1 + m_2)}$, in orbit around a mass $M = m_1 + m_2$ with a semi-major axis $a = r_1 + r_2$. In other words, we have gone from two orbits, such as on the left, to one reduced orbit, such as on the right:

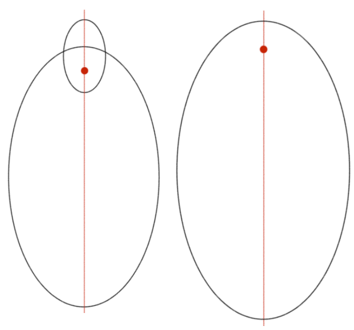


Figure 12.3: Two orbits around mutual center of mass, left; reduced orbit, right.

Another complication to keep in mind is that chances are good that the center of mass of our binary system has a proper motion; in other words, the pair of stars is moving at some direction across the sky from year to year. For example, Sirius, or α Canis Majoris, was discovered to be a binary in 1862 (Sirius B is a white dwarf, nearly 10 magnitudes fainter than Sirius A). Its relative orbit looks approximately like the sketch on the left in the following figure. Sirius is not that far away (about 2.6 pc or 8.6 ly) and has a relatively large proper motion. That relative orbit has been constructed from a path across the sky that looks roughly like the sketch on the right.

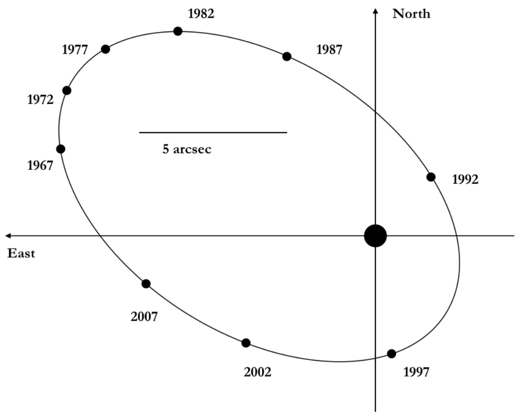


Figure 12.4: Relative orbit of Sirius.

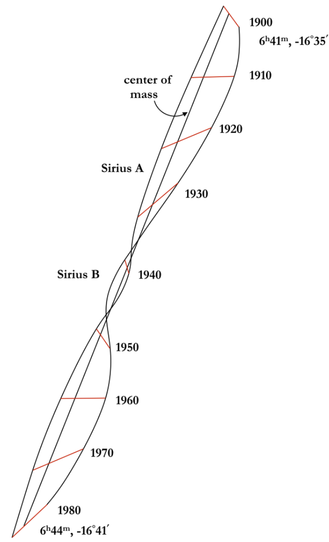


Figure 12.5: Sirius' motion across the sky.

If we are able to reconstruct the relative orbit and determine the stars' orbit period and separations (in AU) then determining the masses is simply an application of Kepler's third rule for orbits.

Example: Suppose we observe a visual binary with a parallax of 0.02″ and in the actual relative orbit the two stars have a maximum separation of 1.6″. From their individual motions we can tell that the secondary star is 5 times farther from the center of mass than the primary star. The orbit period is 146 years. What are the masses of the stars?

We need the stars' separation in AU; for this, recall that ″·pc=AU. The stars' distance is 50 pc (i.e., 1 / parallax). That means their separation is 80 AU (= 50 pc · 1.6″). Note that we could also have gotten this without calculating their distance: 80 AU = 1.6″ / 0.02″. Next, recall Kepler's 3rd rule for orbits, rearrange to solve for mass, and plug in numbers:

$$P^2 = \frac{4\pi^2}{G(m_1+m_2)}a^3 \rightarrow (m_1+m_2)\ M_\odot = \frac{(a\ \text{AU})^3}{(P\ \text{yr})^2} \rightarrow \frac{(80\ \text{AU})^3}{(146\ \text{yr})^2} = 24\ M_\odot .$$

The sum of the two stars' masses is 24 solar masses. To find their individual masses, we can see from the equation for center of mass that the mass ratio is the inverse of the ratio of the stars' distances from the center of mass:

$$m_1 r_1 = m_2 r_2 \rightarrow {}^{m_1}\!/_{m_2} = {}^{r_2}\!/_{r_1} .$$

In other words, if the separation ratio is 5:1, with the secondary being 5 times farther from the center of mass than the primary, then the mass ratio is 1:5, with the primary being 5 times more massive than the secondary. A ratio of 5:1 means we need to divide our 24 solar masses total into 6 pieces. Therefore the more massive star is 20 $M_\odot$ and the secondary is 4 $M_\odot$.

Follow this example a bit further to calculate the stars' speeds and show that the reduced mass can be used to calculate the total kinetic energy of the system. If the total separation is 80 AU, then the individual distances from the center of mass are 13.33 and 66.67 AU. Assuming that the orbits are circular, we can find the speeds:

$$2\pi r_1 = 83.76\ \text{AU in 146 y} \rightarrow v_1 = 0.57\ \text{AU/yr}$$
$$2\pi r_2 = 418.90\ \text{AU in 146 y} \rightarrow v_2 = 2.87\ \text{AU/yr}$$

and the total relative speed is the sum of the individual speeds, or 3.44 AU/yr. We get the same result if we use the *vis viva* equation for a circular orbit:

$$v_{total} = \sqrt{\frac{GM}{a}} = \sqrt{\frac{4\pi^2 \frac{\text{AU}^3}{\text{M}_\odot \text{yr}^2} 24\ \text{M}_\odot}{80\ \text{AU}}} = 2\pi\sqrt{\frac{24}{80}} = 3.44\ \text{AU/yr}.$$

The reduced mass for this system is

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{20 \cdot 4}{24} = 3.33\ \text{M}_\odot.$$

The two relative orbits look like the orbit of an object of 3.33 solar masses in an 80 AU orbit with a speed of 3.44 AU/yr. We can check to see that these two pictures give the same total kinetic energy, as they must:

$$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}\left(20\ \text{M}_\odot \cdot (0.57\ \text{AU/yr})^2 + 4\ \text{M}_\odot \cdot (2.87\ \text{AU/yr})^2\right) = \frac{1}{2}(6.50 + 32.95) = 19.7$$

$$\frac{1}{2}\mu v_{total}^2 = \frac{1}{2} 3.33\ \text{M}_\odot \cdot (3.44\ \text{AU/yr})^2 = 19.7$$

in solar system units and ignoring a fair bit of rounding error.

And for a bit more gratuitous algebra, make sure that you can convert AU/yr into km/sec; for this example,

$$3.44 \frac{\text{AU}}{\text{yr}} \cdot 150 \cdot 10^6 \frac{\text{km}}{\text{AU}} \cdot \frac{1\ \text{yr}}{3.16 \cdot 10^7 \text{s}} = 16.3 \frac{\text{km}}{\text{s}}.$$

The spectra of stars in visible wavelengths show absorption lines. This is because the atoms in the stars' outer layers are capable of absorbing specific energies, i.e., specific wavelengths. These outer atoms are relatively cool, meaning that their electrons are mostly in relatively low energy levels. The electrons can absorb wavelengths of the light coming from the interior of the star that correspond to the energies they need to jump to upper energy levels. The electrons then de-excite, reemitting that energy, but the emitted photons are in random directions. On balance, we see spectra with dark lines. If the star has a radial velocity with respect to us, we will observe the lines in the spectrum shifted by an amount that corresponds to the speed and direction of the star. In other words, we'll see a Doppler shift.

In a spectroscopic binary the stars in the pair have an orbit that is significantly inclined with respect to the plane of the sky. The stars will regularly move towards us and away from us. One will move towards as the other moves away, then the directions of their motions will be transverse to our line of sight, then the first will move away as the second moves towards, and so on. The following sketch follows two spectral lines, one from the primary star and one from the secondary, as they slide back and forth in wavelength over one orbital period.

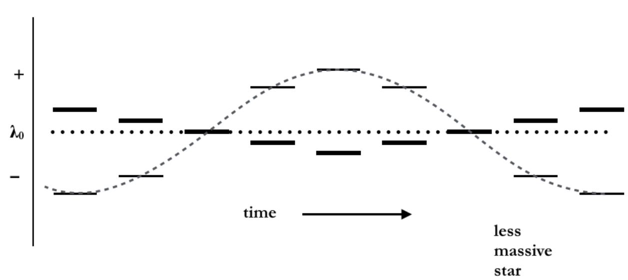


Figure 12.6: Doppler shifting lines in a spectroscopic binary.

Notice that the Doppler shift is larger for the less massive star. The less massive star is farther from the center of mass, its orbit is larger, and thus is speed must be higher than the speed of the primary. Recall that the shift in wavelength is proportional to the speed:

$$\frac{\Delta\lambda}{\lambda_0} = \frac{\lambda_{\text{observed}} - \lambda_0}{\lambda_0} = \frac{v_{\text{radial}}}{c}.$$

The rest wavelength, λ_0, is the wavelength we would observe this particular spectral line to have if we measured it from a source that's standing still. As the star moves away from us its wavelengths shift longer; toward us and the wavelengths are blue shifted. The maximum shifts corresponds to the points in the orbit when the star is moving directly away or directly toward us. The lines of the two stars are exactly out of phase because the stars must stay on opposite sides of their center of mass.

Suppose we observe a binary with a circular orbit and a 90° inclination. For circular orbits the radial velocity curves are sinusoidal, with a period equal to the orbit period. The following sketch shows an example of the radial velocities for such a circular orbit. The stars have a mass ratio of 3:1.

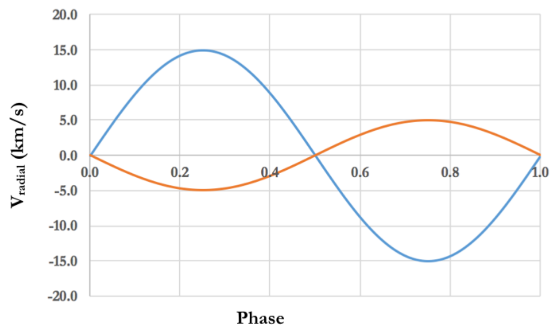


Figure 12.7: Binary star radial velocity curve.

The phase here runs from 0 to 1, as a fraction of angle around the orbit, although phase angle could run from 0 to 2π or 0 to 360°. Phase could also mean the fraction of time through the orbit period, i.e., running from 0 to *P*. For a circular orbit a star's angular location changes at a constant rate, so these two types of phase are the same thing. When the orbit is eccentric we'll have to specify whether phase means angle or period.

How do we know that the mass ratio in this example is 3:1? Because the velocity ratio is 3:1. We see the stars actual speeds when they are moving directly toward or away from us; in this case, the speeds are 5 and 15 km/sec. These orbits are circular, meaning that $v = 2\pi r / P$. Use the center of mass equation once again:

$$m_1 r_1 = m_2 r_2 \rightarrow \frac{m_1}{m_2} = \frac{r_2}{r_1} = \frac{v_2 P / 2\pi}{v_1 P / 2\pi} = \frac{v_2}{v_1}.$$

The mass ratio is the inverse of the velocity ratio; the velocity ratio is equal to the separation ratio. The orbits in this case might looks something like the following figure:

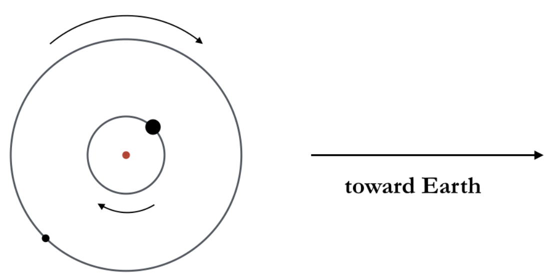


Figure 12.8a: Binary star circular orbits, 90° inclination.

Which points in the orbit correspond to which points in the radial velocity curve? The stars' radial velocities will be zero at the points where they are moving only transversely to our line of sight. In the radial velocity diagram, above, those points are at phase = 0 and 0.5. The massive star is moving toward us (the radial velocities are negative) from phase 0 to 0.5. The massive star is moving directly toward us at phase 0.25 (and, similarly, the less massive star is moving directly away from us at phase 0.25). In the following sketch we've labelled the phases around the primary star orbit to show the relationship between the phases in the radial velocity curve, above, and the positions of the stars in their orbits.

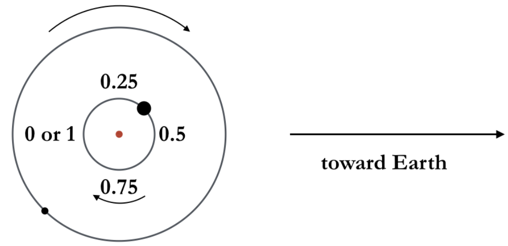


Figure 12.8b: Orbital phases corresponding to the velocity curve, above.

Binary orbits are not always going to be circular nor are they going to be at 90° inclinations. If the inclination is 90° then the maximum speeds are the actual speeds of the stars. If the inclinations are less than 90° then of the actual speed v_{star} only a component v_{star} sin(i) will be directed along our line of sight. In other words, $v_{radial} = v_{star}$ sin(i). In the following sketch, top, we have an orbit that's in our line of sight, perpendicular to the plane of the sky. When the stars are moving directly toward or away from us, their velocity vectors are directed along our line of sight. In the lower panel we have an orbit that is less inclined to the plane of the sky. When these stars are moving most directly toward or away from us, only the component of their velocities (v sin(i)) is along our line of sight.

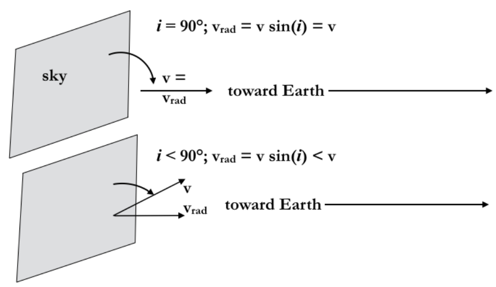


Figure 12.9: Geometry for orbits not perpendicular to the plane of the sky.

If we observe v_{star} sin(i) then we can't determine the actual separations of the stars but only a sin (i). In our attempts to determine the stars' masses, we will determine M sin(i); in other words, we will determine a lower bound on the objects' masses.

Now, what if the orbits are eccentric? The following sketch shows a binary orbit with an eccentricity of 0.714 (that's a ratio $a : b = 4 : 2.8$). Let the inclination of the orbit be 90° and consider the two simplest orientations of the orbit with respect to our line of sight, i.e., with either the major or minor axes pointing toward Earth.

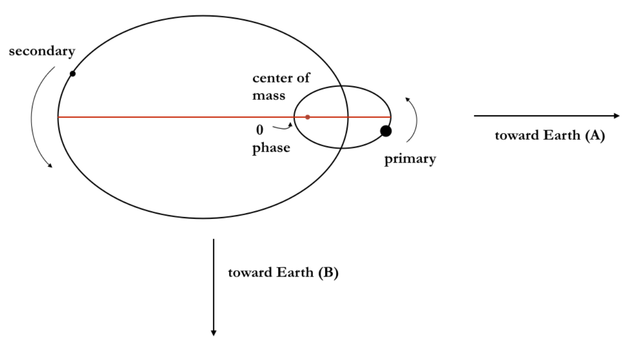


Figure 12.10: Example geometry for binary stars with eccentric orbits.

According to Kepler's second rule for orbits, about conservation of angular momentum, the stars' speeds will vary around their orbits, with higher speeds when the stars are nearer the center of mass. Let phase = 0 correspond to the primary star being at pericenter (closest to the center of mass). At phase = 0 the stars' actual speeds are highest; they are lowest half a period later, at apocenter, when the phase = 0.5. Further, the length of time it will take to get from one side of the pericenter point to the other side is less than the length of time it will take to get from one side of apocenter to the other side. These varying speeds and times are going to have an effect on the the radial velocity curve. The following two sketches show what the radial velocity curves look like for the orientations labelled (A) and (B), above.

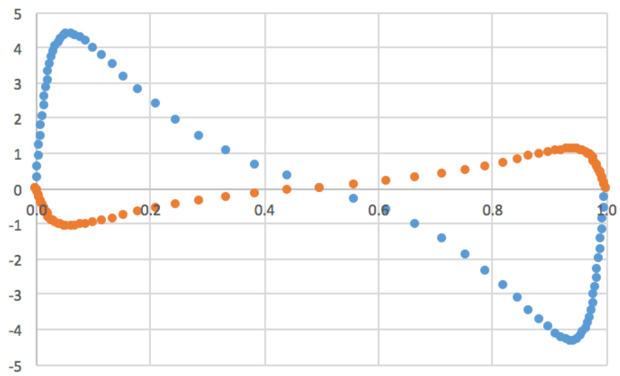


Figure 12.11a: Radial velocity curves, orbit end-on to our line of sight.

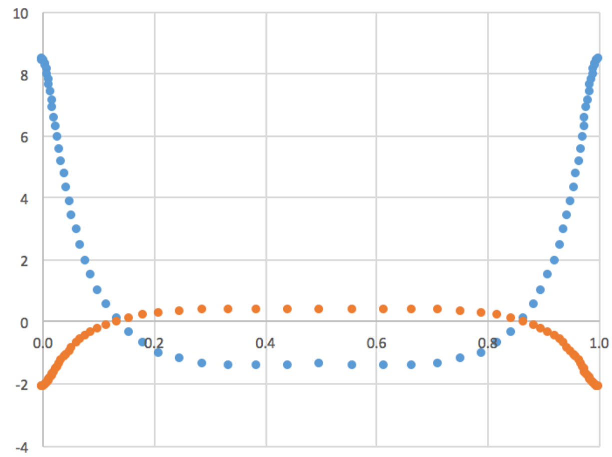


Figure 12.11b: Radial velocity curves, orbit sideways to our line of sight.

The x-axis on these two plots is time as a fraction of the total orbit period. The y-axis is radial velocity in arbitrary units, scaled the same for each plot.

Real orbits are not likely to be quite so neatly aligned. More often the orbit will be inclined at something less than 90° and rotated in such a way that we are not looking at the major axis either end-on or side-on.

We don't necessarily need to be able to see both objects to measure the Doppler shift. Observations of a single set of lines can still let us reconstruct a relative orbit and hence determine the mass, or at least a lower limit to the mass, of the second object. The determination of the mass of a star's unseen companion is particularly useful as a means of detecting the presence of a planet or a black hole.

There are theoretical reasons for expecting that a star's luminosity depends on its mass. Luminosity also depends on where the star is in its life cycle, i.e., whether, like our Sun, it is producing energy simply by hydrogen fusion or is a giant nearing the end of its life or, later still, a stellar remnant. For *main sequence* (more on that term in a later chapter) hydrogen-fusing stars like the Sun we expect the luminosity to scale with mass roughly as

$$\left(\frac{L}{L_\odot}\right)=\left(\frac{M}{M_\odot}\right)^{\alpha},$$

with α being approximately 3 and mass and luminosity both in solar units. Observationally we find that these stars are distributed as shown in the following sketch. If our power law were correct, then in a log-log plot the slope of this distribution would be 3. It's not quite, but it's close. Stars of different masses have slightly different internal structures and mechanisms for transporting energy outwards, resulting in slightly different relationships between luminosity and mass. The table below gives better approximations for the mass-luminosity relation for stars of various masses.

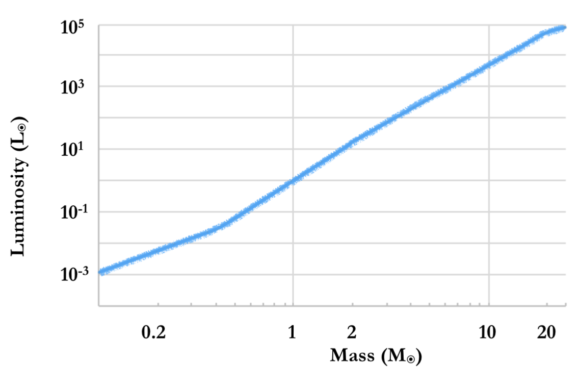


Figure 12.12: Mass-luminosity relation

Modify the expression above slightly: $\left(\frac{L}{L_\odot}\right) = C\left(\frac{M}{M_\odot}\right)^\alpha$, allowing a multiplicative factor C.

Mass-luminosity relation coefficients

$M < 0.43\ M_\odot$	$0.43\ M_\odot < M < 2\ M_\odot$	$2\ M_\odot < M < 20\ M_\odot$	$20\ M_\odot < M$
$C \sim 0.23$ $\alpha \sim 2.3$	$C \sim 1$ $\alpha \sim 4$	$C \sim 1.5$ $\alpha \sim 3.5$	$C \sim 2300$ $\alpha \sim 1$

If we have a binary system for which the two stars are well enough separated that we can measure their angular separation on the sky but close enough together that we can measure their orbit period, it's possible to use a combination of Kepler's third rule for orbits and the mass-luminosity relation to estimate the distance to the system. This process is called *dynamical parallax*. Here's how it works: First, estimate the two masses — for example, their colors or spectra may give reasonable first estimates for the masses. Second, use the estimated masses, orbit period, and Kepler's third rule to determine a first estimate for the orbital semi-major axis. Third, use that estimated semi-major axis and the observed angular separation to estimate the distance to the pair. Fourth, use that estimated distance and the observed apparent magnitudes (appropriately accounting for extinction due to dust) to estimate the stars' luminosities. Fifth, use those estimates and the mass-luminosity relation to obtain improved estimates for the stars' masses. Iterate until the mass estimates converge.

Eclipsing binaries and stellar diameters

Binaries with orbits inclined at or near 90° may show regular dips in light as the stars eclipse each other. Timing these eclipses lets us estimate the stars' diameters. Let's look at how this works. When the stars are not in eclipse we are receiving light from both of them. We use similar terminology to describe points in a stellar eclipse as we use to describe lunar or solar eclipses: *first contact* is the point at which the disks of the stars touch and begin to overlap; *second contact* is the point at which the smaller star is totally in front of or behind the larger star; *third contact*, the last point at which the smaller star is totally in front of or behind the larger; and *fourth contact*, the last point at which the disks of the stars touch. The following sketch illustrates the geometry.

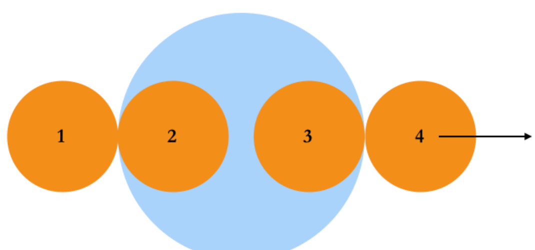


Figure 12.13:
Binary eclipse geometry
for 90° inclination.

Prior to 1st contact and after 4th contact we receive light from both stars. Between 1st and 2nd the light we receive is decreasing; then it's flat for a time and then it starts increasing again as the smaller star starts to move off the disk of the larger. Here's a sketch of an eclipse light curve, corresponding to the eclipse geometry in the previous sketch:

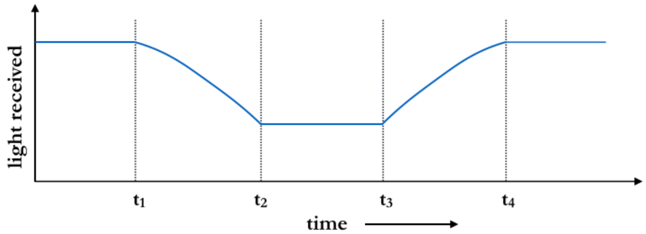


Figure 12.14: Binary eclipse light curve.

Consider the motion of the smaller star relative to the larger one; in the time between 1st and 2nd contact the small star moves its own diameter (if you aren't convinced, draw a dot on the leading edge of the star and follow that dot from contact to contact). In the time between 1st and 3rd contact the small star moves the diameter of the larger star. What would the light curve look like if the two stars were the same diameter? In this case, 2nd contact is immediately followed by 3rd contact, in a fairly pointed light curve, as shown in the following sketch.

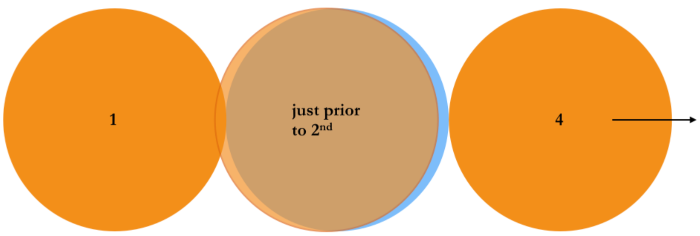
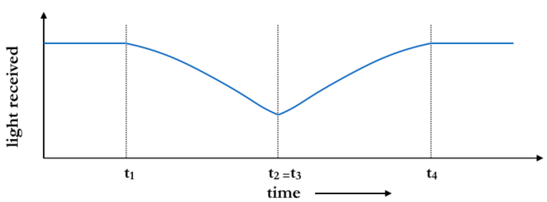


Figure 12.15a: An eclipsing binary, stars having equal diameters.



12.15b: light curve

The binary orbit doesn't have to have an inclination of exactly 90° for there to be eclipses. The eclipse won't be central if $i \neq 90°$ and the stars' separations will matter — too far apart and their disks will miss each other at conjunction. On the left in the next sketch is an eclipse for an orbit with an inclination just a bit less than 90°; on the right, an orbit that's a bit less inclined and stars a bit farther apart, such that they almost miss.

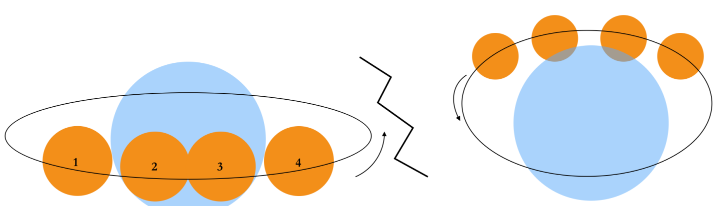


Figure 12.16: Two possible non-central eclipses.

In the eclipse on the left there are still distinct points of 2nd and 3rd contact but using the contact timings will clearly produce inaccurate estimations of the stars' diameters.

There are two eclipses (unless the geometry is really marginal, in which case there could be only one) and it is customary to refer to the deeper drop in the light curve as the primary eclipse. But which one is that, i.e., which star being eclipsed creates the largest loss of light? Return to the central eclipse and notice that in each eclipse we lose the same surface area, either of the larger or of the smaller star. In the following sketches you can see that in either case we lose an area equal to the area of the smaller star.

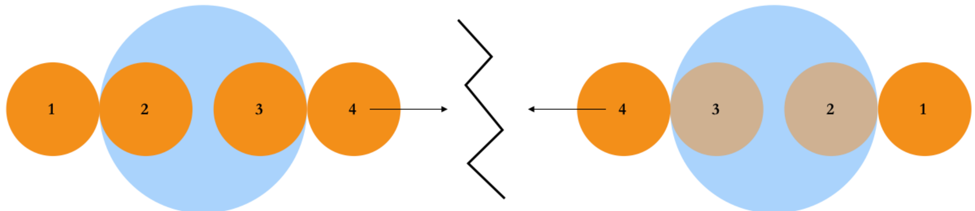


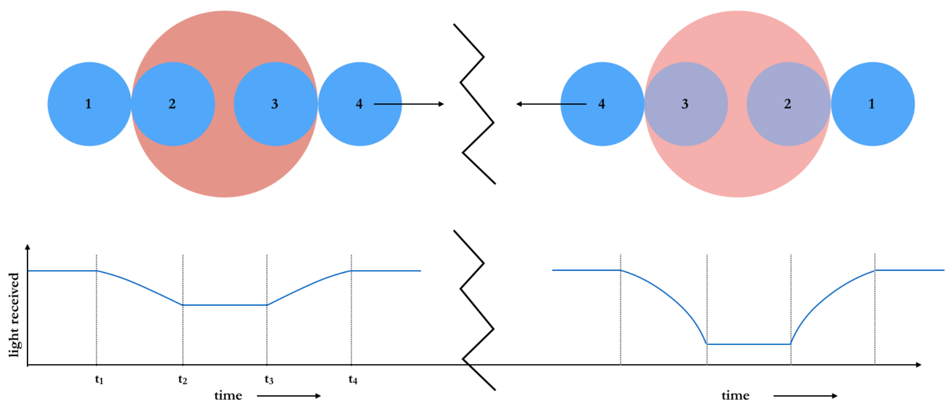
Figure 12.17: The stellar surface area eclipsed is equal to the area of the smaller star in both eclipses.

We need to distinguish between *luminosity* and *flux*: recall that luminosity is the star's total energy output per second and flux is the energy output per second *per unit area*. Because we are covering up the same area in each eclipse, we need to compare the stars' fluxes to determine which eclipse produces a deeper drop in the light curve. The relevant equation is:

$$F = \sigma T^4 .$$

The hotter surface has a higher flux. The deeper drop in the light curve occurs when the hotter star is the one being eclipsed. In the sketches above we've had a larger blue star and a smaller orange star. If we properly associate the blue star with the hotter surface, then the primary eclipse is the one shown on the left (orange star in front) and the secondary eclipse is the one on the right (orange star being eclipsed). It doesn't have to be the case, though, that the star with the larger diameter is hotter. We could have a blue star and a red giant, as in the following:

Figure 12.18: Binary pair with large cool star and small hot star. The deeper, primary, eclipse occurs when the hotter star is eclipsed.



The ratio of the depths of these eclipses is equal to the 4th power of the ratio of the stars' temperatures.

As with the spectroscopic binaries, the orbits of the eclipsing binaries need not be circular. The two eclipses might not be equally spaced in time or last for equal amounts of time. For instance, if our line of sight were parallel to the major axis of the orbits (i.e., like case A in our consideration of eccentric spectroscopic binary orbits) the eclipse that occurred at pericenter wouldn't last as long, because the stars' speeds are higher at that point, than the eclipse that occurred at the apocenter of the orbits. If on the other hand our line of sight were parallel to the minor axis of the ellipse (i.e., like case B of the eccentric spectroscopic binary orbits) we might see the primary eclipse and then the stars would rapidly pass the pericenter of their orbits and reach the secondary eclipse, after which it would be a longer period of time as the stars were slowly passing the apocenter of their orbits before we'd be back to the primary eclipse.

Eclipses give us relative stellar diameters; if our stars are both eclipsing and spectroscopic binaries then we can calculate the distances they travel during the eclipses in kilometers and obtain actual stellar diameters. Stellar radii don't vary quite as dramatically as stellar luminosities. The smallest main sequence stars have radii on the order of 1/10 the radius of the Sun. The largest main sequences stars might be 20-ish solar radii but a million times as luminous as the Sun. Red supergiants, evolved stars nearing the ends of their lives, may be several hundred solar radii.

Another distance method: if we have an eclipsing binary for which we have light curves, radial velocities and high-quality spectra, it's possible to determine the distance to the pair. The light curve and radial velocities permit the determination of all the orbital properties of the system as well as the radii of the stars and the relative temperatures and luminosities (i.e., T_B/T_A L_B/L_A). It's possible to determine a star's temperature from a good-quality spectrum — software is available to model the spectrum of a star, based on assumptions about the star's photospheric properties: temperature, metallicity, microturbulent velocity (contributing to Doppler line broadening), and surface gravity (i.e., to distinguish giants from main sequence stars). We know also that the star's luminosity is proportional to r^2 T^4 and that the flux from a star falls off as distance squared. Thus, assuming the appropriate corrections for extinction, and having determined the temperature, it's possible to calculate $(r/d)^2$ from the spectrum and the observed flux (this combination study of the spectrum and brightness is called spectrophotometry). Since we already have determined the radius from the light curve and radial velocities, we can determine the distance. This isn't trivial, particularly because high-quality spectra require large telescopes. This method has been used successfully to determine the distance to eclipsing binary stars in the Large Magellanic Cloud. An accurate distance to the LMC is a critical first step in determining distances to galaxies beyond our Milky Way.

Some binary stars are very close together, so close that they can be said to be in contact. Stars that are that close are going to distort each other tidally. In other words, each star will tug harder gravitationally on the near side of its companion than it will on the far side of the companion. Stars are fluid enough that they will be elongated. Here we need to introduce the concept of the *Roche lobe*. Consider two stars in orbit and a tiny third test mass. If the test mass is close enough to one star its motion will be controlled solely by that one star; if the test mass is far enough away from both stars, its motion will be controlled by their combined gravity, as if the two stars were one large mass. In between we need to consider all the forces acting on our test mass — gravity from each star plus the centripetal acceleration it feels because the system is rotating. The test mass could be the outer layers of one of our stars: Any bit of material in one of these stars feels a gravitational force from its own star, and from the other star, and it also experiences a centripetal acceleration because the stars are in orbit around each other. It is possible to determine mathematically where all those forces are in balance. That surface is shaped a bit like a figure 8, although its exact dimensions depend on the masses and separations of the two stars. Around each star there's a volume of space controlled by the gravity from that star; this region is called the Roche lobe. The Roche lobes of the two stars in the binary touch at a point called the first Lagrange point, or L_1. If one star expands enough to fill its Roche lobe then mass from the outer layers of that star can spill through L_1 into the Roche lobe of its companion star. We'll consider this point in more detail when we discuss the evolution of stars, because close binary stars can toss mass back and forth, changing each other's evolutionary state. For now, considering eclipsing binaries, its enough to note that a star that has filled its Roche lobe has a distended shape. If neither star has filled its Roche lobe we say that the binary stars are detached. If one has done so, it's semi-detached. If both stars fill their Roche lobes and material spills out into a common envelope around the two stars, we have a contact binary. The following sketches show a pair of stars in close but detached, semi-detached, and contact binaries.

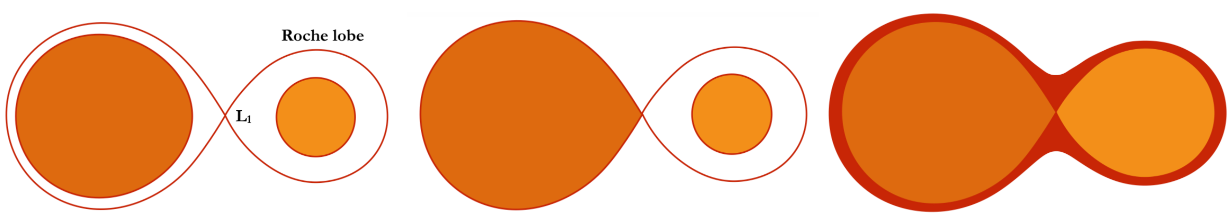


Figure 12.19a: Detached close binary; b: Semi-detached binary; c: Contact binary

Distended shapes are going to affect the stars' light curve. When we see the stars edge-on, as in the above sketches, we are seeing relatively more surface area and the light curve goes up. Just before eclipse we see nearly the end-on, round cross-section of the stars. Shape isn't the only property that changes, though. Suppose that one of these stars is considerably hotter than the other. In that case, the hot-star-facing side of the companion star could itself be heated above the normal surface temperature of the companion. A surface hot spot on the cooler star is going to have a higher flux than the rest of that star's surface, meaning that the light curve will get brighter as that hot spot comes into view just before or just after that star is eclipsed. In other words, light curves for close eclipsing binary stars can be a bit complicated but, if we can disentangle the various effects, can also be quite informative.

Measuring stellar diameters for single stars

In principle you might expect that we could just measure the diameters of stars: if a star of known distance were near enough and large enough to have a measurable angular size on the sky we could use trigonometry to determine its diameter. There are two problems with this. The first is that stars' angular sizes are almost all too small to resolve with reasonably sized telescopes and cameras. For ground-based observations, atmospheric turbulence makes it very hard to resolve distances that are less than about 1 arcsec (which, recall, would be 1 AU at a distance of 1 parsec). The second is that cameras observing point (or nearly point) sources of light produce diffraction patterns that make it difficult to image the source. In 1995 Andrea Dupree and Ron Gilliland were able to image Betelgeuse directly using the Hubble Space Telescope, as shown in the following image.

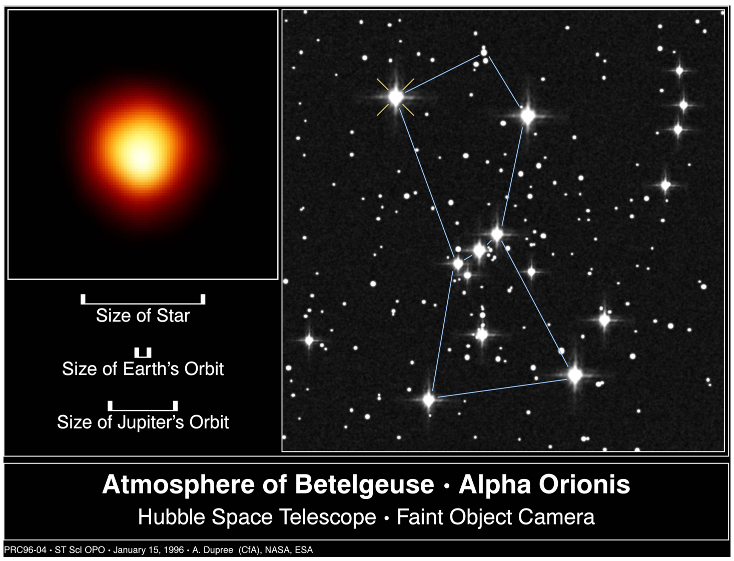


Figure 12.20: Betelgeuse
http://hubblesite.org/newscenter/archive/releases/1996/04/image/a/

It is sometimes possible to use the properties of waves — diffraction, interference — to determine the diameter of a star. For instance, a point source passing behind a straight edge produces a diffraction pattern while the light from an extended source drops smoothly as the edge moves, at a known rate, in front of it. A moderately nearby star, slightly larger on the sky than a simple point source, passing behind the limb of the Moon, which is nearly a straight edge,

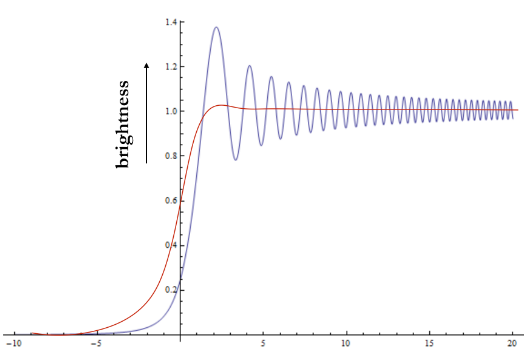


Figure 12.21: Light from an extended source (red) and a point source (purple) falling off as the source is covered by a straight edge.

will produce a cross between a diffraction pattern and a simple decrease in the light. The above sketch illustrates roughly the difference between the light drop-off for a point source and an extended object. Astronomers who observe such lunar occultations of stars model the diffraction pattern that a source of a given angular size would produce and can determine angular sizes for stars between about 1 and 50 milliarcsecs. If we know the distance to the star, we can turn the angular diameter into a linear diameter. There are on the order of 100 stars with known distances that are near enough to the path of the Moon in the sky and have large enough angular diameters to be measured this way.

Stellar interferometers make use of the fact that light waves arriving at a telescope from slightly different directions, say from the two halves of a stellar disk, interfere with each other. Covering the telescope aperture with an opaque screen with two slits near the edges of the aperture or combining the light received from two telescopes separated by many meters allows the interference pattern to be detected. Around 1920 A.E. Michelson mounted two small mirrors on the ends of a 6 meter rigid bar and positioned that across the front of the 100-inch telescope at Mount Wilson Observatory. The two small mirrors acted as the two slits and the bar provided a baseline between the slits that was longer than the diameter of the telescope. Light from the two sides of the star travels slightly different path lengths, depending on the angular size of the star in the sky, shown schematically at left. Waves traveling different path lengths will interfere. The sketch, below, shows light waves along the last section of their path; where they are in phase they will constructively interfere and where they are out of phase, destructively.

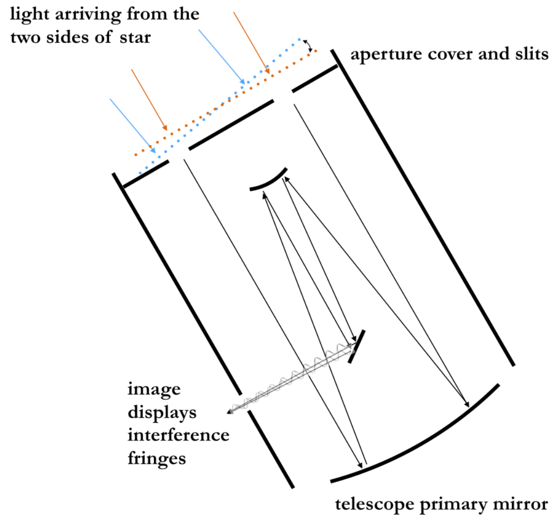


Figure 12.22: Optical stellar interferometry

The image of the star will show fringes which depend on the separation of the slits and the angular size of the star. Michelson and Pease, in 1921, reported the successful measurement of the diameter of Betelgeuse, the red supergiant star that is the shoulder of Orion. Similar modern experiments often use two separate telescopes rather than two slits on one. Regardless, this sort of measurement is technically challenging; interferometry is much more tractable at longer radio wavelengths.

By and large, sizes measured for individual stars are in good agreement with sizes measured for eclipsing binaries.

Statistics of multiple star systems

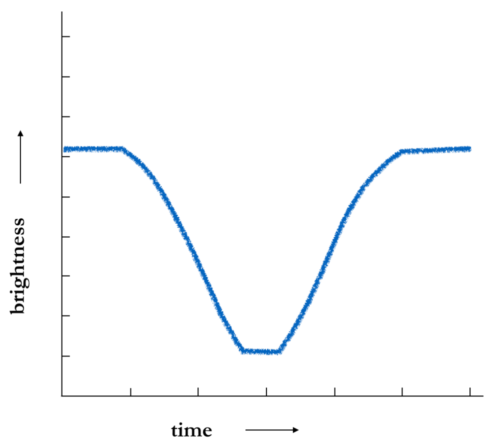
Many stars are in multi-star systems but the fractions are not the same for stars of all masses. Exactly how the multiplicity fraction varies with stellar mass is still an open topic of research. The Sun is a medium-sized star but it's also the case that the majority of stars are less massive than the Sun. The most massive stars are very rare; the most abundant, low-mass, stars are very faint and hard to observe. Recent studies suggest that approximately 50 - 60% of solar type stars are in multiple star systems. That multiplicity fraction decreases for lower-mass stars to perhaps as low as 15 - 25% for stars less than half the Sun's mass. The multiplicity fraction is high for stars more massive than ~5 $M_\odot$, likely higher than 70%, and it seems to be the case that massive stars' companions are themselves likely to be high-mass stars.

In the majority of cases being in a multiple-star system means being a binary but there are a substantial number of stars in systems with more than two members. The closest star system to us is α Centauri, a triple system in the southern hemisphere sky. Alpha Centauri A and B are roughly similar to the Sun and orbit each other in ~80 years; α Cen C (or Proxima Centauri), the third member of the system, is a less massive star (~0.12 $M_\odot$) in a much larger orbit (~15,000 AU) around the other two. The bright star Algol, in Perseus, in another triple system. Multiples don't stop at triples: Mizar and Alcor, a visual double in the handle of the Big Dipper, seems to be a sextuple system. With a small telescope Mizar resolves itself into two; each of these stars is a spectroscopic binary. Alcor, ~12 arcmin away from Mizar, is a binary that appears to be in a long orbit around Mizar, although we aren't absolutely sure that this is a bound system. Another bright star, Castor, one of the two main stars in the constellation Gemini, also seems to be a sextuple system. Castor A and B are both themselves pairs of stars, each a star of nearly 3 solar masses in a tight orbit (periods of several days) with a star of ~0.5 $M_\odot$; Castor C is a pair of low-mass stars, in close orbit with each other, that orbits Castor A+B in ~14,000 years.

These estimates are of stars in orbits with other stars. It seems to be the case that most stars have planets although the data are not yet adequate to state with confidence what fraction of stars in which mass ranges have what numbers of sub-stellar companions.

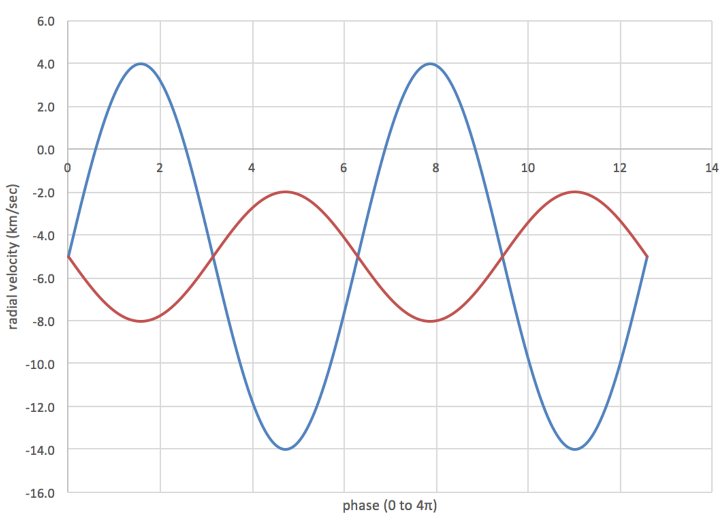
Sample problems

1. Here is a light curve for the eclipse of the secondary star in an eclipsing binary system.



Is the primary star's radius = 3 times the radius of the secondary star and how do you know?

2. Here is a plot of radial velocity data vs. phase for a hypothetical spectroscopic binary. This shows two full cycles (i.e., 0 to 4π); let the orbit period = 42 years. Assume that the orbital inclination $i = 90°$.



a) Is the orbit circular and how do you know?

b) What is the ratio of the two stars' masses and how do you know?

c) What are the distances from each star to the center of mass?

d) What are the masses of the two stars?

e) What does it mean that the velocities oscillate back and forth around −5 km/s rather than around 0?

3. Take another look at figure 12.2 and explain the principles behind reconstructing a real orbit from the apparent orbit.

4. If the orbit of a spectroscopic binary does not lie parallel to our line of sight we aren't going to measure the actual orbital speeds; is the speed we measure larger or smaller than the actual orbital speed?

5. Reading carefully? Briefly explain or define
   - a) astrometric - spectroscopic - eclipsing binaries
   - b) inclination
   - c) main sequence mass-luminosity relation
   - d) Roche lobe
   - e) contact binary
   - f) 1st - 2nd - 3rd - 4th contact

Answers to selected problems are on the next page:

1. no; review the relationship between the eclipse contact points and the diameters of the stars.

2. b) mass ratio = 3:1 because velocity ratio = 1:3

   c) circular, so $r_1 = v_1P/2\pi$ and $r_2 = v_2P/2\pi$; converting to AU we have 4.23 and 12.69 AU

   d) $M_{1+2} = a^3/P^2$ in AU, years, and solar masses. The masses are 2.06 and 0.69 solar masses

3. Think about the centers of the real and apparent ellipses and about the fact that the center of mass must lie along the major axis of the real ellipse.

4. For visualizing what's going on with measuring *v* sin *i*, it might help to think about the extreme cases; i.e., when *i* = 0° we measure no speed and we measure the actual speeds when *i* = 90°.