Chapter 15: **Stellar evolution**

- The interstellar medium and star formation
- Modelling stellar interiors
- Main sequence and Post-Main Sequence evolution
  for stars of various mass ranges and compositions
- sample questions

Interstellar medium and star formation

In the chapter on the Milky Way we'll consider the various components of the interstellar medium (ISM) and their locations in the galaxy. In this chapter we will consider the particularly dense regions of the ISM within which stars are born.

Most of the material in the ISM is in the form of gas, with only about 1% (by mass) being in dust particles. Most of the gas is hydrogen, although we also see elements such as carbon, oxygen, nitrogen, calcium, sodium, titanium, and others. Most of the gas is either hot and ionized, generally with number densities $n \lesssim 0.01$ / cm^3 and T $\gtrsim 10^5$ K, or warm and fractionally ionized, with temperatures of a few 10^3 K, $n \sim 0.1 - 1$ / cm^3. Only a small fraction of the gas is molecular, ranging from simple species such as CN, CH, and OH to longer hydrocarbons made of roughly a dozen atoms. We suspect that most of the molecules are H_2, although because they are small, low mass, and symmetric, the molecular hydrogen is very hard to detect. The assumption is that by and large CO, which is detectable, will act as a tracer for the hydrogen.

Where the gas is backlit by more distant stars we may detect its presence because it will produce extra absorption lines in the spectra of those stars. The interstellar absorption lines are identifiable if they are for instance narrower or Doppler-shifted differently than the lines due to the background stars. Low-density clouds of neutral hydrogen can emit in the radio, at 21 cm, when the hydrogen's electron flips its spin, dropping from an energy level in which the electron spin is parallel to the proton spin to the slightly lower energy level in which the two spins are anti-parallel. The gas may also emit if it is excited, e.g., by nearby hot stars. An H II region is a region around a hot star within which the hydrogen gas is mostly ionized. It isn't entirely ionized, though, and when a proton and electron recombine to form a neutral hydrogen atom, the electron drops back down to the ground state by way of various energy level transitions that often include the n = 3 to n = 2 H-alpha transition. H II regions are thus bright red emission nebulae (even though the H II nomenclature refers to the ionized state).

Gas and dust tend to be concentrated along a relatively thin plane in the Milky Way, a few hundred to a few thousand parsecs thick, the thickness increasing with increasing distance from the inner galaxy. Molecular gas tends to be concentrated in the galaxy's spiral arms and in an even thinner plane, less than 100 pc thick. In *molecular clouds* the density of both dust and molecular gas is significantly higher than average, with number densities of H_2 ranging from a few 10^1 to $\sim 10^6$ / cm^3; outside a cloud the density might be only 1 particle / cm^3. The dust grains are thought to play a role in the formation of molecules by providing a substrate on which atoms can come into contact and form molecules which may then evaporate back into the gas, and then by protecting those molecules from ultraviolet light that might otherwise dissociate them. Cloud temperatures tend to be less than 100 K.

Molecular clouds come in a range of sizes and average densities and the clouds are themselves often clumpy, containing bubbles and filaments, and may not have well-defined edges. The term *giant molecular cloud* tends to refer to a cloud that's larger than the size of a star cluster, e.g., $10^{3\text{-}7}$ solar masses in a region ranging from a few parsecs to ~200 pc across. Small dense dark clumps, less than a few hundred solar masses, found either within the giant clouds or as isolated small molecular clouds (Bok globules), are the regions within which we find stars actively forming. Stars form in clusters, with the more massive stars forming faster, and sometimes dying explosively before their younger siblings have finished forming. Those massive stars will tend to emit quite a bit of ultraviolet light, meaning that often we find dark molecular clumps, where stars are still forming, in close proximity to H II regions. The densest regions, which will give rise to individual stars or binary systems, are called cores.

The Eagle Nebula, or M16, is a well-known star-formation region, with dark clouds and columns of dust and protostars surrounding a less-dense cavity created by the formation of several massive stars and the blast wave from at least one supernova explosion. M16 is roughly 2,000 pc away, along the plane of the Milky Way in the direction of the constellation Serpens. The following image is infrared, taken by the Spitzer Space Telescope.

Figure 15.1: Eagle nebula / M16
Blue = 4.5 µm;
green = 8.0 µm;
red = 24.0 µm. This image is 28 x 32 arcminutes, or ~16 x 18 $^{1}/_{2}$ parsecs.

http://www.spitzer.caltech.edu/images/1708-ssc2007-01a-Cosmic-Epic-Unfolds-in-Infrared-The-Eagle-Nebula

The “green” pillars of dust on the right-hand side of the cavity have been dubbed the “pillars of creation” because stars are forming within their protective envelope of dust. The following two images show these pillars in greater detail in visible (left) and infrared (right). The dark portion of the central pillar is ~1 pc long.

Figure 15.2: M16 in emission lines of O, S, H, N. NASA / ESA / Hubble Heritage team. https://www.spacetelescope.org/images/heic1501a/

Figure 15.3: M16 @ 1.1 & 1.6 µm. NASA / ESA / Hubble Heritage team. https://www.spacetelescope.org/images/heic1501b/

Gravity is acting within a molecular cloud but so is pressure — the gas molecules, in particular, are moving and for most of the gas in the cloud most of the time that motion is sufficient to prevent the material from starting to collapse to form a star. Clearly some regions do find themselves dense enough to collapse, perhaps aided by a shock wave from a nearby supernova or a collision with another cloud (e.g., when two galaxies collide) or simply the increased pressure in a spiral arm. A current observational project for several groups of astronomers is tracking emission from molecules, e.g., NH_3, or other abundant molecules, that emit in the radio (i.e., where there's lower opacity) in regions within the clouds where proto-star, and potentially proto-planet, formation has progressed to various different extents in an effort to trace the process of star formation.

The physics governing the potential collapse of a core to form a star is similar to that which we will consider in the sections on cosmology and the formation of the first galaxies: as a core starts to collapse its internal pressure will rise; is the core dense enough to continue collapsing or will the increase in pressure halt the collapse?

To get a rough estimate of the timescale for collapse, let's consider the case in which material in the forming star was falling together in the *absence* of increased internal pressure. In other words, let's consider free fall and ask what happens to a small mass m falling under the influence of the gravitational attraction of a core of mass M with initial radius R and average density ρ_0. If we ignore rotation, we could treat m as if it were falling along an "orbit" of semi-major axis $a = {}^1\!/_2\, R$ and eccentricity $e = 1$. Recall that Kepler's third law tells us that the period for such an infalling mass is given by

$$P = \left[\frac{4\pi^2}{GM}a^3\right]^{1/2}.$$

Using the average density and the given relationship between a and R, we can express the mass M as

$$M = \left(4/3\right)\pi R^3\rho_0 = \left(4/3\right)\pi(2a)^3\rho_0 = \left(32/3\right)\pi a^3\rho_0.$$

Also note that what we are actually interested in is $P/2$, the time required for the particle m to fall to the center of the core. Substituting in for M, we have

$$t_{\text{free fall}} = \frac{P}{2} = \frac{1}{2}\left[\frac{4\pi^2 a^3}{GM}\right]^{1/2} = \frac{1}{2}\left[\frac{4\pi^2 a^3}{G\left(32/3\right)\pi a^3\rho_0}\right]^{1/2} = \frac{1}{2}\left[\frac{3\pi}{8G\rho_0}\right]^{1/2}.$$

In a dense core the average number density might be upwards of 10^6 particles / cm^3. For particles that are mostly hydrogen that's on the order of 10^{-15} kg / m^3 and we get a free fall time estimate on the order of $10^{4\text{-}5}$ years.

The timescale on which the pressure in the core will increase depends on the sound speed, which is approximately

$$c_s = \sqrt{kT/m_{H_2}}.$$

For temperatures on the order of a few tens of kelvin the sound speed will be a few hundred meters per second. The question is then whether a pressure wave can cross the core and raise its internal temperature and pressure enough to halt the collapse before enough material has fallen toward the center of the core and raised the internal density enough to counteract the increasing pressure and continue the collapse.

The critical length for collapse vs. stability depends on the ratio of the temperature to the density in the core or clump or cloud. This length is called the Jeans length, λ_J, after British astrophysicist Sir James Jeans (1877 – 1946); smaller overdense regions, which take less time for a pressure wave to cross, will stabilize while larger regions will collapse. In the Milky Way the average mass within a sphere of radius λ_J is equivalent to several thousand solar masses. In other words, regions the size of potential star clusters start to collapse first. As such a region starts to collapse its internal density will rise and the Jeans length will get smaller. The collapsing region is likely to fragment into smaller regions that will proceed to collapse (or not) independently.

Many spiral galaxies are relatively inefficient at producing stars; in the Milky Way today, for example, several solar masses worth of new stars are produced each year, which isn't a lot given the billion-ish solar masses worth of available free gas. The balance of gravity and pressure may not by itself be sufficient to explain the galaxy's star-formation rate. We may need to consider the effect of magnetic fields on gas motions and, more importantly, the degree of internal turbulent motion within a gas cloud. ALMA, the Atacama Large Millimeter/submillimeter Array, is capable of observing the long wavelength spectral lines of molecules within the clouds with sufficient spectral resolution to measure the Doppler broadening. The results? The turbulence is fast, moving the gas around enough to slow gravity's ability to make a cloud collapse. It's also at least sometimes supersonic, which would provide a source of the shock waves which are necessary get the gravitational collapse started to produce a future star.

At some point the center of a core collapsing on its way to becoming a protostar will reach an internal temperature of ~2,000 K, at which point hydrogen molecules can be dissociated by collisions with other atoms or molecules. The opacity in the center of the collapsing core is also increasing and at some point the central regions will become opaque to their own radiation, providing an additional outward pressure that will act to halt the collapse.

The collapsing core is likely going to have to shed material, perhaps 60 - 80% of its original mass, in large part because of conservation of angular momentum. The core was almost certainly rotating and as it became more centrally condensed the equatorial rotation rate increases. Angular momentum is ~$m\ v\ r$. Consider a core collapsing from ~0.1 pc to roughly the radius of the Sun, i.e., changing r from ~ 10^{15} m to ~10^{9} m; if there were no change in mass, the equatorial velocity would have to increase by roughly a million times. A conservative initial rotation rate of ~100 m/s would result in a protostar rotating at close to the speed of light, which is clearly unrealistic. . .mass is going to have to be lost and going to have to carry away angular momentum in the process.

The collapsing core is also likely to have a non-zero initial magnetic field, because material in the ISM has a small field. Just like angular momentum, the magnetic field will be conserved as the protostar collapses. The resulting protostar will have a field, but again as with the angular momentum, some of the magnetic field must be carried away or the final star field will be unrealistically large.

The protostar is thus likely to be surrounded by a rotating disk of somewhat magnetized material aligned with the protostar's equator, a disk within which planets, and possibly a low-mass stellar companion, may form. Some mass is also likely to be lost through jets perpendicular to the disk. Here are two examples of observations of young forming stars. On the left are visible and infrared images of jets and a Herbig-Haro object, the bright emission spots where the jets, from the forming star, are slamming into the surrounding interstellar medium. On the right is a combination visible and near-IR of a region shaped by a relatively massive young star encircled by the disk of dust that cuts vertically across the image. The blue-ish bubbles of hot gas, looking a bit like wings, in this nebula are several light years across.

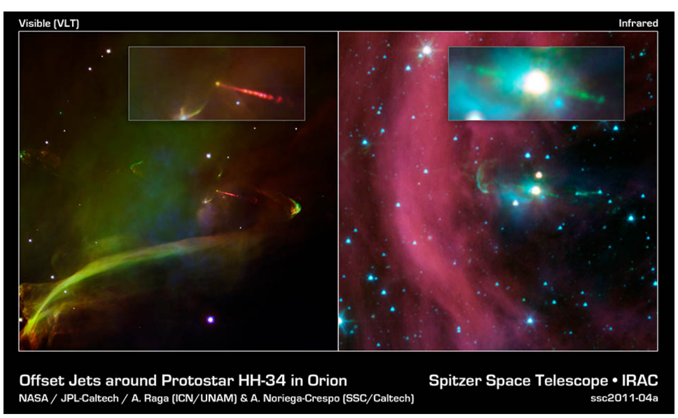

Figure 15.4 - HH-34

http://www.spitzer.caltech.edu/images/3564-ssc2011-04a-Undercover-Jet-Exposed

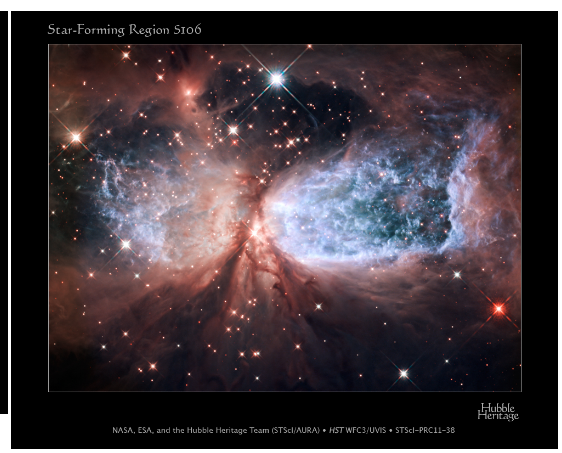

Figure 15.5 — S106

http://hubblesite.org/newscenter/archive/releases/2011/38/image/a/

One of the most famous star-formation regions is the Orion Nebula, shown below in visible light (left) and infrared (right). These images span ~4 parsecs at the ~420 parsec distance to the Orion Nebula. In the visible dust obscures embedded young stars and reflects blue light from hot stars and excited hydrogen gas glows red; in the infrared more stars are apparent because the opacity due to the dust is much lower. The four bright young stars near the center, the most easily apparent members of a small cluster, are often called the “Trapezium”. Several of these stars are more than 10 solar masses and together provide much of the energy to excite and illuminate the surrounding nebulosity. These stars are only, roughly, a few hundred thousand years old. Massive stars tend to form faster than lower-mass stars. Once a star cluster is more than a few million years old we can usually assume that all the cluster member stars are roughly the same age (“co-eval”), but when the stars are still forming that assumption isn’t so great.

Figure 15.6 Orion in visible light

http://hubblesite.org/newscenter/archive/releases/2006/01/image/a/

Figure 15.7: Infrared Orion

NASA / JPL / T.Megeath; Spitzer space Telescope
http://www.spitzer.caltech.edu/images/3628-ssc2011-06a-Stars-Adorn-Orion-s-Sword

The Orion Nebula is just part of a larger star-forming region over 100 parsecs across called the Orion molecular cloud complex. Some stars form in huge regions such as this, where multiple star clusters form over many years, and many more form in much smaller clouds that give rise to only a small cluster. What sort of region gave rise to our solar system? There’s some evidence that the answer to that question is. . .large cloud. The argument is that in a large cloud there would be a higher probability of massive stars forming. Massive stars are ultimately responsible for producing ^{26}Al and ^{60}Fe, short-lived radioactive isotopes. Those isotopes decay into the stable isotopes ^{26}Mg and ^{60}Ni, which we see in meteorites in sufficiently high levels to suggest that the nebula from which the solar system formed had been seeded by material formed in several massive stars.

Pre-main sequence timescales: Supose that we model a forming star as a series of shells falling in from a long ways away to their final location within the star. To get the total gravitational energy of a star requires summing over all the shells of falling material. Successive shells fall harder and faster, as the mass onto which they are falling, and thus the gravitational attraction, increases. This is going to require doing a double integral to get the final gravitational energy (integrating over distance as one shell falls in and then again over all the shells). Here we’ll just state the result, namely that the gravitational energy is proportional to $-G M^2 / R$. The constant of proportionality depends on how the mass is distributed in the star, but 3/5 isn’t a bad estimate.

Example: Estimate the amount of energy released during the pre-main sequence gravitational collapse of the Sun and how long that collapse would take. The virial theorem tells us that the amount of energy radiated away should be half the Sun's gravitational potential energy, or:

$$\Delta E \approx \frac{1}{2}\cdot\frac{3}{5}\cdot\frac{GM_{\odot}^2}{R_{\odot}} = \frac{3}{10}\cdot\frac{(6.67\cdot10^{-11}\,\mathrm{m}^3/\mathrm{kg}\cdot\mathrm{s}^2)\cdot(2\cdot10^{30}\,\mathrm{kg})^2}{7\cdot10^{8}\,\mathrm{m}} \approx 1.1\cdot10^{41}\,\mathrm{J}\,.$$

If the Sun's luminosity were constant over this period (which it isn't, but it's not too bad for purposes of illustration),

$$t \approx \frac{\Delta E}{L_{\odot}} \approx \frac{1.1\cdot10^{41}\,\mathrm{J}}{3.8\cdot10^{26}\,\mathrm{J/s}}\cdot\frac{1\,\mathrm{yr}}{3.16\cdot10^{7}\,\mathrm{s}} \approx 9\cdot10^{6}\,\mathrm{y}.$$

We'd expect the Sun to take something on the order of 10^7 years to collapse to the main sequence. Alternatively, if the Sun were shining by released gravitational energy alone, it would only last about 10^7 years. In the late 19th century, prior to the discovery of nuclear fusion but after the geologic discoveries pointing to an ancient Earth, this short timescale was a bit disconcerting. This gravitational-energy-radiated-away timescale is called the Kelvin-Helmholtz timescale, after two of the physicists who were pondering conservation of energy and how long the Sun had been shining.

Using the equations that describe the conditions inside stars — energy generation rates, the dependence between temperature and pressure, whether or not convection is occurring, etc. — astronomers who study stellar evolution can produce computer models of the visible properties, e.g., T_{eff} and L, of stars of different masses, ages, compositions, and rotation rates. A range of models of of different masses for stars of various ages can be compared with the range of observed properties of stars in clusters. The resulting *evolutionary tracks* can be plotted on an H-R diagram. The following figure traces approximately the pre-main sequence evolutionary tracks for stars of several masses.

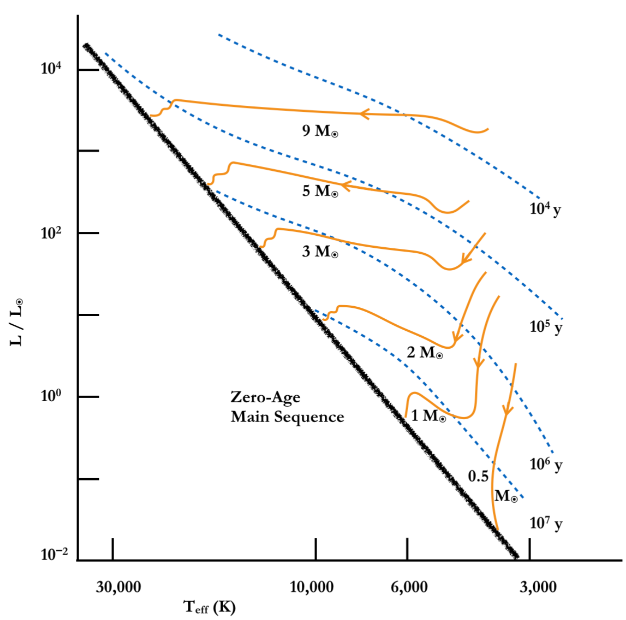

Figure 15.8: Approximate pre-main sequence evolutionary tracks for stars of various masses.

The nearly vertical part of the evolutionary track for stars that aren't more than ~3 solar masses is called the *Hayashi track*, after Japanese astrophysicist Chushiro Hayashi (1920 - 2010), a pioneer in the modeling of pre-main sequence stars. These pre-main sequence stars have become opaque to their own radiation and are shrinking without much change in surface temperature. Energy transport for objects on the Hayashi track is dominated by convection. For stars more than ~0.5 solar masses (smaller stars will remain convective), radiative transport soon becomes dominant and the pre-main sequence star's evolutionary track moves to the left along what's called the *Henyey track*, named for Louis Henyey (1910 - 1970).

Solar-type pre-main sequence stars, those less than roughly 2 solar masses, destined to become main sequence stars of spectral types F and later, are often highly variable in their last stages of contraction to the main sequence. Types of variable stars are often named for the first such object known; in this case, these stars are called T Tauri stars, after a variable in the constellation Taurus. More massive pre-main sequence stars, roughly 2 - 8 solar masses are destined to become spectral type A or B stars. These objects, just prior to hitting the main sequence, are hotter and often show emission lines in their spectra; they are called Herbig Be/Ae stars. The most massive stars tend to be quite rare and to reach the main sequence very rapidly. For both of these reasons we rarely catch these stars in their pre-main sequence phases.

In the diagram above note the terminology *zero-age main sequence* (ZAMS): when sustained hydrogen fusion kicks in and the stars have reached the main sequence their positions in the H-R diagram place them along the under side of the distribution of main sequence stars. (Just to be clear, the ZAMS is not at one end — the ZAMS runs all along the main sequence, it's just on the bottom side of the main sequence). The lowest mass stars are ~0.08 $M_\odot$ — less than that and the object won't get hot enough in the core to sustain hydrogen fusion; the highest mass stars are roughly 100 $M_\odot$ — more than that and the outward thermal and radiation pressure will exceed the inward force of gravity (this is called the *Eddington limit*). Several stars in the Tarantula Nebula in the Large Magellanic Cloud (roughly 170,000 ly away) are larger than 100 $M_\odot$ but that's rare today. Composition matters — the very earliest stars, forming from gas that was almost entirely only hydrogen and helium and thus having lower opacity and experiencing less outward pressure, could be more massive, perhaps several hundred times the mass of the Sun. Stars do evolve and change their visible properties *slightly* while they are on the main sequence but not much.

We expect that relatively more low-mass stars will form, based both on observations of the population of stars near the Sun (for which we have a relatively complete census) and on calculation of the fragmentation of molecular clouds and the collapse of pre-stellar cores. If we look at a star cluster we should see a few massive stars, more solar mass stars, and still more low-mass stars. This is called the *initial mass function* (IMF), an empirical expression for the number density of stars of a given mass range. The IMF falls off exponentially for stars of higher masses. Edwin Salpeter (in 1955) suggested that the IMF may be expressed as

$$\xi(m)\Delta m = \xi_0 (m / M_\odot)^{-2.35} \Delta m / M_\odot,$$

where the calculation returns the expected number of stars with masses in the range m to $m + \Delta m$ per unit volume of space. Later investigators have argued for slightly different values of the exponent and we are only now able to detect very faint stars at sufficiently large distances to begin to answer the question of whether the relation holds down into the mass range of brown dwarfs or whether it flattens out a bit.

Modelling stellar interiors

The virial theorem tells us about the relationship between the amount of gravitational potential energy released as the protostar contracted and the internal temperature of the star once it has finished contracting. Once a star reaches the main sequence it is very nearly in *hydrostatic equilibrium*, meaning that the force due to outward pressure is balance by the inward force due to gravity and most stars are not going to vary much in size while they are on the main sequence. We assume that the material that makes up the star can be treated as an ideal gas during the star's main sequence lifetime. Energy generation takes place in the star's core either by means of the proton-proton chain or the CNO cycle or a combination of the two; stars less massive than the Sun predominantly produce energy by the proton-proton chain because the CNO cycle, involving a higher Coulomb repulsive barrier, requires higher temperatures. Light elements such as Li, Be, and B are easily combined with protons are temperatures of only a few million degrees, meaning that some light-element fusion will also occur in main sequence stars. The most massive stars may have high enough core temperatures to begin 3-α fusion (combining three helium nuclei to produce carbon) before exhausting their supply of hydrogen, but for most stars 3-α fusion does not occur on the main sequence. Energy is transported outward either by radiation or by convection or a combination of the two and the temperature falls (there is a *temperature gradient*) steadily between core and photosphere. This is a very brief summary of the physics involved in determining the conditions in the interior of a star.

Math notes. For those of you interested in seeing more mathematical detail about they physics, here are the relevant equations.

First, hydrostatic equilibrium and mass continuity. Assume that the star is spherically symmetric and consider a shell in the interior at a distance r from the center, of density $\rho(r)$, and having thickness dr. A shell closer to the center experiences a higher pressure to compensate for the larger weight of the outer shells pushing down on it. The equation for the forces being in balance on the shell, i.e., for hydrostatic equilibrium, is

$$\frac{dP}{dr} = \frac{-GM(r)\rho(r)}{r^2},$$

where $M(r)$ is the mass interior to the shell at r. As we move outward from the center the mass increases as we add additional shells:

$$\frac{dM}{dr} = 4\pi r^2 \rho(r).$$

This equation describes mass continuity and, integrated over all shells, provides the total mass of the star:

$$M = 4\pi \int_{r=0}^{r=R} \rho(r) r^2 \, dr.$$

Next, let's consider the equation of state that describes the relationship between pressure, temperature, and density for an ideal gas. There are several ways to express the ideal gas law; one of the most useful is:

$$P(r) = \frac{\rho(r) k T(r)}{\mu(r) m_H},$$

where we are using $\mu(r)$ to represent the mean weight of our particles. (This is usually called the "mean molecular weight" even if we haven't actually got any molecules.) By mass fraction we can express the composition of our star as X (hydrogen), Y (helium), and Z (metals). Given that Z is usually quite small and that hydrogen and helium are mostly ionized, meaning more particles, $\mu(r)$ is likely going to be on the order of $1/2$ or so, i.e., less than one.

Energy generation on the main sequence is going to depend on the density and the relative amount of hydrogen, for the proton-proton chain, or hydrogen and carbon, for the CNO cycle and on the temperature. Usually the energy generation (per unit mass) is given by ε:

$$\varepsilon(r) \propto X^2 \rho T^{\nu} \text{ or } \propto X Z_C \rho \cdot T^{\nu},$$

where we are denoting the mass fraction of carbon alone, not all heavies, with Z_C. For the proton-proton chain, the exponent ν is ~4; for the CNO cycle it's approximately 16-18 — the CNO cycle produces energy much faster than the proton-proton chain. Luminosity will increase in any shell within which energy is being generated; i.e.,

$$\frac{dL}{dr} = 4\pi r^2 \rho(r) \varepsilon(r).$$

Here we've assumed that our energy generation is only due to fusion. A main sequence star is close to being in hydrostatic equilibrium but over the course of its main sequence lifetime its physical conditions will change somewhat, as the relative amount of hydrogen in the core drops. In other words, the energy generation within a shell is likely to include some contribution from the release of gravitational potential energy as well as core fusion.

Finally let's consider the temperature gradient, or, rather *gradients*, since the rate at which energy is transported outward by radiation and by convection are different, resulting in two distinct temperature gradient equations. Conditions in the interior of a star are nearly in thermal equilibrium, meaning that the flux arriving at a shell from below is approximately

$$F(r) = \sigma [T(r)]^4.$$

The temperature is dropping across the shell as *r* increases, so *dT* is going to be negative. Taking the derivative of the flux, we have

$$dF = 4\sigma[T(r)]^3 dT.$$

The energy is not simply passing through the shell, but is interacting with material in the shell at a rate that depends on the flux available, the density of the material, and the opacity of the material, $\kappa(r)$, the latter describing the propensity of the material present to interact with the passing photons. The opacity is a function of wavelength; here $\kappa(r)$ represents a weighted average. We may express the change in flux due to the interactions with the material in the shell as

$$dF = -\kappa(r)\rho(r)F(r)dr.$$

Combining these expressions for *dF* and using $L(r) = 4\pi r^2 F(r)$, as well as adding a factor of 4/3 that our simplified consideration of radiative transport ignored, yields the following:

$$\left.\frac{dT}{dr}\right|_{\text{radiative}} = \frac{-3\kappa(r)\rho(r)}{64\pi\sigma r^2[T(r)]^3}L(r).$$

When the opacity is high enough or the radiative temperature gradient steep enough convection sets in. The convective temperature gradient describes the rate at which temperature falls in a rising, expanding, bubble of hot plasma. We assume that we can consider our plasma to be an ideal gas and that the rising bubble cools only because it is expanding, i.e., not exchanging energy with its surroundings (this is called adiabatic expansion). The convective gradient will depend on the ratio, γ, of the specific heats at constant pressure and volume; $\gamma = c_p / c_v = 5/3$ for an ionized ideal gas. You may have used specific heats before in chemistry or physics; they have units of J / (kg K) and describe the amount of heat needed to raise the temperature of a given mass of material. Without deriving it explicitly, the convective temperature gradient is:

$$\left.\frac{dT}{dr}\right|_{\text{convective}} = (1-1/\gamma)\frac{T(r)}{P(r)}\frac{dP}{dr}.$$

The computer code used to model the interior of a star must check at every step to determine whether energy is more likely to be transported via convection or radiation and use the appropriate temperature gradient in calculating the rate at which the temperature is dropping.

One more note about stellar interiors differential equations and computer modeling: the mass of a star is concentrated toward the center and many of our equations depend on the mass contained within our hypothetical shell. It often makes sense to change variables and compute changes in variables as functions of *dm* rather than *dr*. Regardless, you can probably tell that the process isn't simple because these are coupled differential equations that can't be solved analytically. To do the modeling usually means assuming a set of central conditions, applying the differential equations to calculate values for our variables shell by shell outward toward the surface, and then checking to see whether the predicted surface condition match those of the real star we are trying to model. If not, change the initial assumptions and recompute.

Main sequence and post-main sequence evolution

As noted above, stars arrive on the zero-age main sequence (ZAMS) once sustained core hydrogen fusion has kicked in and the star has achieved hydrostatic equilibrium. The star will remain on the main sequence until it has exhausted almost all of its core hydrogen, which is roughly 10% of it mass. Stars spend 80-ish % of their total energy-producing lifetimes on the main sequence. During this time their structure and surface properties will change modestly as core hydrogen is used. In the core the number of free electrons is dropping as hydrogen gets converted to helium — four hydrogens have 4 electrons but one helium has only 2 electrons, so the number of electrons goes down. The electrons are free because the temperature is high enough, millions of kelvin, that the hydrogen and helium are ionized. Free electrons are a significant source of opacity in the stellar core because they interact so easily with passing electromagnetic radiation. As the opacity drops, photons can escape from the core more rapidly. The number density of particles in the core is also dropping, decreasing every time four protons become one alpha particle, and the mean molecular weight is increasing. The initial effect of these changes is a decrease in outward pressure, which results in an adjustment of hydrostatic equilibrium. The gravity pushing inward hasn't changed, so the core is squeezed a bit and the temperature increases. That will make the rate of fusion reactions increase, since that rate depends on temperature. The fusion rate also depends on the availability of protons, though, which is dropping. As time passes and helium builds up, individual protons, or carbon atoms for the CNO cycle, become less able to find protons with which to fuse, so the overall rate might not increase as much as we would expect from the change in core composition. During its main sequence lifetime, on balance, the star will become a bit more luminous.

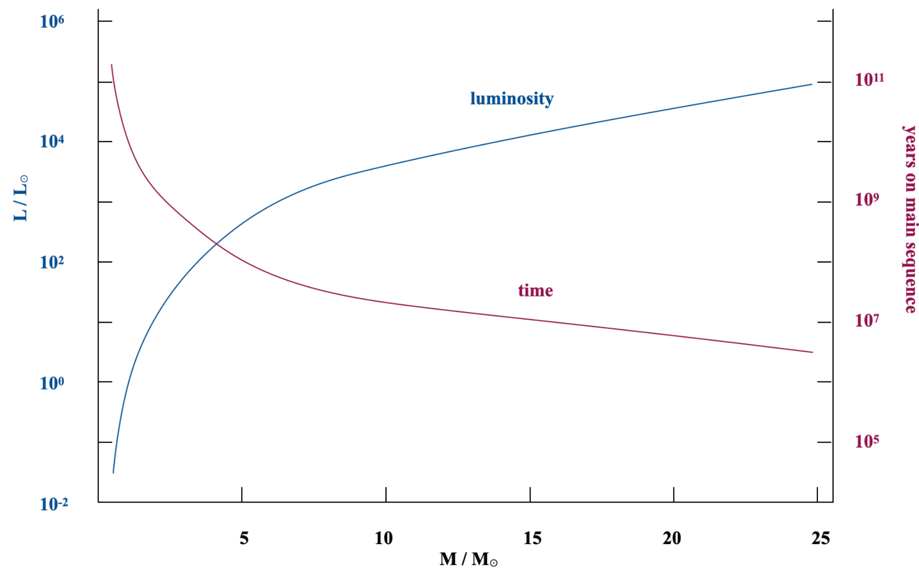

Figure 15.9: Approximate main sequence lifetimes and luminosities for Population I stars of various masses.

What happens as the star reaches the end of its main sequence life depends on its mass. A key property in determining what happens as the star evolves off the main sequence is the core density. Less massive stars have less gravity pushing inward, squeezing the core, but they also produce much less energy, meaning that there is much less outward pressure. The cores of lower main sequence stars are denser than the cores of upper main sequence stars. The following plot shows post-main sequence evolutionary tracks for stars of several masses.

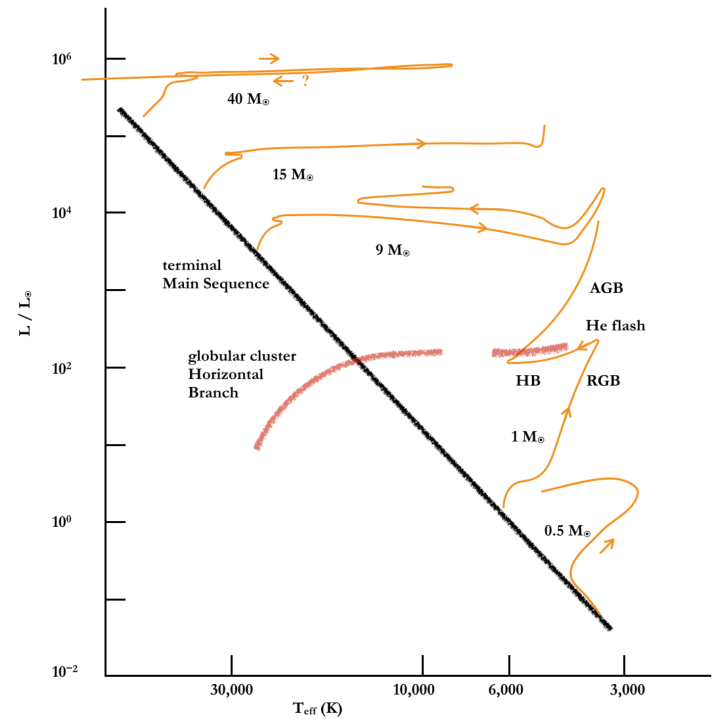

Figure 15.10: Approximate post-main sequence evolutionary tracks for stars of various masses. (Stars of ~3 and ~5 solar masses have been omitted to keep the tracks from getting cluttered.) The terminal main sequence means the main sequence for stars just running out of core hydrogen to fuse. Also shown is the location of the HB (horizontal branch) for a range of stellar masses and compositions; the HB shows up particularly well in color-magnitude diagrams of globular clusters.

The post-main sequence evolution of a solar-type star. Let's look first at the evolution of a star of roughly one solar mass and solar composition. Such a star will take roughly 9-10 billion years to reach the point at which the hydrogen abundance in the core has dropped to such a low level that core hydrogen fusion is no longer occurring at a sufficient rate to maintain hydrostatic equilibrium. We say at that point the star has reached hydrogen exhaustion in the core. That doesn't mean that hydrogen fusion has ceased. Recall that while on the main sequence the core has been slowly shrinking and heating up; this process of releasing gravitational potential energy has also heated the material in the next shell out beyond the core. Even when fusion ceases in the core there will be a shell around that core that's still got plenty of hydrogen and has reached a temperature sufficient to sustain hydrogen fusion. The star readjusts its outer visible characteristics as well, starting to increase in radius and decrease in effective temperature. As the surface temperature drops the opacity increases and that means the depth of the convective zone increases. The star reaches a point at which almost all of its energy is being transported outward by convection. After this, the star climbs up the *red giant branch* (RGB), recapitulating its pre-main sequence descent down the Hayashi track (where its energy transport was also dominated by convection).

During this time the star's core temperature has been creeping upward, but slowly. The core of our red giant contains most of the mass of the star in a region compressed to roughly the size of Earth. The temperature is a few tens of millions of kelvins. This condition of high density and modest temperature is no longer an ideal gas; the core has become electron *degenerate*. When it was an ideal gas the electron's temperature and pressure were related through the ideal gas law and the distribution of electron speeds followed a Maxwell-Boltzmann distribution (described in the introductory chapter). When the electron gas becomes degenerate the temperature and pressure decouple. The low-energy side of the speed distribution is governed by the Pauli Exclusion Principle: in this dense state there are not enough position-momentum combinations for all the electrons we would normally expect to be moving slowly if their speeds were being dictated by the temperature. Some electrons are forced into higher speeds, where there are available position-momentum locations. The high-energy tail of the electron speed distribution is not degenerate and follows the expected Maxwell-Boltzmann distribution. As a result of this decoupling of temperature and pressure for a large fraction of the electrons, squeezing the core more tightly does *not* increase its temperature much. The core also becomes isothermal, the temperature being the same throughout the core rather than decreasing outward from the center as it would for an ideal gas. It will take roughly a billion years for the Sun to move from hydrogen exhaustion to the helium flash, where helium fusion kicks in. The following sketch illustrates this hybrid distribution function, following a Maxwell-Boltzmann speed distribution at the high end and being constrained by too few position-momentum spaces at the low end.

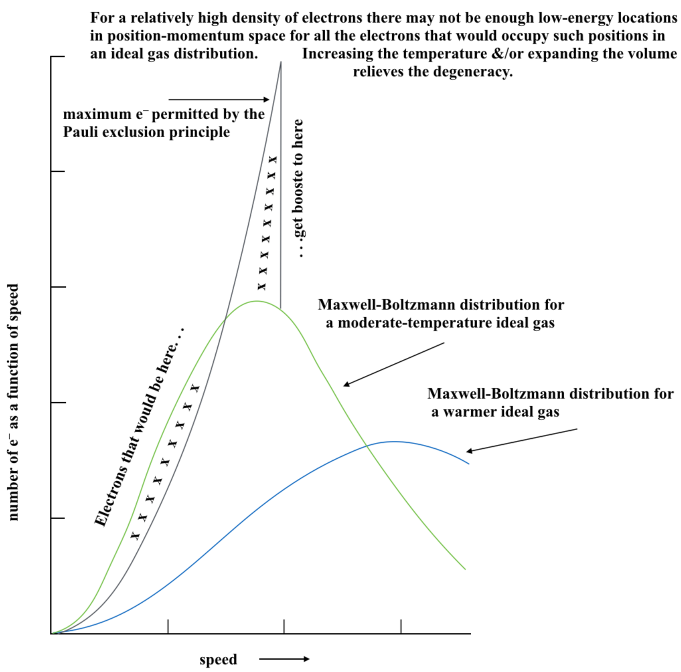

Figure 15.11: Electron speed distribution function for a partly-degenerate electron gas.

Triple-alpha fusion requires temperatures on the order of 10^8 K. Because the degeneracy isn't total the core temperature in a red giant does, slowly, increase and eventually it is sufficiently high for 3-α ignition. Helium fusion starts throughout the core at the same time (seriously — like, within a few minutes!) because the core was isothermal. This event is called the *helium flash*, although it's happening down in the core and we aren't going to notice any visible flash at the surface. The helium fusion rapidly raises the core temperature to ~350 10^6 K, at which point the degeneracy relaxes; that's hot enough that the lower-energy electrons' speeds, which were out-of-synch with the ideal gas speeds for a gas of ~50 10^6 K, can once again be described by a Maxwell-Boltzmann speed distribution. The temperature and pressure are re-connected, the core expands, and the core temperature drops a bit. The surface effects of this internal readjustment soon become apparent. The core is once again carrying its "fair share" of the star's internal energy budget and the outer envelope can shrink. The star will settle onto the *horizontal branch* (HB) where it will fuse helium into carbon in the core and hydrogen into helium in a shell just outside the core. These stars have core masses that are quite similar but they differ in overall mass, due to a range of original masses as well as metallicities and mass-loss rates while on the RGB; stars with more extensive hydrogen envelopes will lie toward the cooler side of the HB. Some carbon in the core will fuse with yet another helium to produce oxygen. Because of the shell fusion prior to the helium flash the star has more helium than it had when it left the main sequence. Triple-alpha fusion is highly dependent on temperature and a horizontal branch star may be 100 times more luminous than it was while on the main sequence. The Sun will spend roughly 10^8 years on the horizontal branch.

Once the star's core helium is exhausted, its evolution will resemble the evolution off the main sequence — the core will be degenerate, shell fusion happens (two possible shells this time), the star expands, this time moving up the *asymptotic giant branch* (AGB). For a one-solar mass star there is no carbon flash — the Sun isn't massive enough to get hot enough in the core for carbon fusion to kick in. Stars on the upper part of the AGB are unstable and variable. Near the end of its life the star is so deeply convective that material from the core may get dredged up to the surface. In particular, it's not abnormal to see AGB stars with high surface carbon abundances (recall that Secchi, in his early spectral type scheme, had a classification for carbon stars). Rapid changes in shell fusion produce *thermal pulses*, the star's magnitude varies noticeably over a few years, and its mass-loss rate skyrockets. The outer layers of the star are ejected into a *planetary nebula*, leaving behind its hot, mostly carbon, degenerate core as a *white dwarf*.

Stars that are initially a few solar masses, up to perhaps 8 $M_\odot$, will follow evolutionary tracks roughly similar to that of the Sun, i.e., MS - RGB - HB - AGB. At greater than about 1 $^1/_2$ solar masses the CNO cycle is the dominant main sequence hydrogen fusion reaction. While on the main sequence, slightly more massive stars will have shallower outer convective zones than the Sun; stars of ~2 solar masses, i.e., those that are early A-type stars on the main sequence, will have no convective zones at all. The more massive stars in this range, the B-type main sequence stars, will start to develop core convective zones. Stars initially more massive than ~2 $^1/_4$ solar masses are expected to reach core temperatures sufficient for helium ignition without having become degenerate.

Before we leave the asymptotic giant branch we should note that AGB stars, with their shell fusion and thermal pulses, are a primary location for the production of *s-process* elements. This is a process for the production of some heavy elements by the slow capture of neutrons. Fusion reactions involving alpha particles provide the main supply of neutrons. For example, both of the following produce neutrons:

$^{13}C + ^4He \rightarrow ^{16}O + n$

$^{22}Ne + ^4He \rightarrow ^{25}Mg + n.$

The neutron captures bump nuclear masses of a given element up from one isotope to another until the chain hits an isotope that beta decays. In the beta decay, a neutron becomes a proton, producing a nucleus one step farther along the periodic table, and an electron and neutrino are emitted. The *s-* is because the neutron captures are slow, as in infrequent, with respect to the beta decay time for those nuclei along the chain that are prone to decay. The s-process starts with elements around iron and can produce heavy elements up to ^{209}Bi. Recent evidence suggests that relatively higher mass, rapidly rotating, Population II stars are also a site for s-process fusion.

Low-mass stars and brown dwarfs. Stars near the 0.08 solar mass lower end of the main sequence are deeply convective. In a star that's totally convective hydrogen from the outer interior will be mixed down into the core, extending the star's main sequence lifetime (which, for a mid-M-type star is more than an order of magnitude longer than the current age of the universe). Stars a bit less than roughly 0.5 solar masses will, like the Sun, have degenerate cores as they leave the main sequence and will become red giants but will not achieve the core temperatures needed to ignite helium fusion. Instead, the outer envelope will be ejected, leaving behind a helium white dwarf. Those less than ~ 0.14 solar masses won't even expand to become red giants, but will simply sink from the main sequence directly down into helium white dwarfs.

Objects that collapse initially as protostars do but with less than ~0.08 $M_\odot$ will not reach high enough core temperatures for sustained hydrogen fusion and will become brown dwarfs. Brown dwarfs range in mass down to the point at which they are hard to distinguish from giant planets. Light elements such as deuterium and lithium are readily destroyed in fusion reactions at lower temperatures than are required for the proton-proton chain. (Li^7 combines with a proton and splits into 2 He^4 nuclei.) Brown dwarfs, particularly those that are more massive and the younger ones, still hot inside from gravitational energy released during their formation, are thus not prevented from doing *any* fusion, they just are not going to establish *sustained* hydrogen fusion. An object less than ~65 times the mass of Jupiter shouldn't get hot enough even to fuse lithium, so the presence of lithium in a cool, low-luminosity object's spectrum is an indication of its sub-stellar nature. In 1995 Gibor Basri used this logic to identify the first young brown dwarf; he subsequently developed a method of determining the ages of young star clusters based on their depletion of lithium. Objects more than ~13 Jupiter masses should be able to do some deuterium fusion, though, and that's useful as a rough, albeit somewhat arbitrary, cut-off for separating low-mass brown dwarfs from high-mass giant planets. Some late M-type "stars" may be younger, hotter brown dwarfs; the spectral types L, T, and Y refer specifically to brown dwarfs. Brown dwarfs are cool enough that their interiors are mostly degenerate. Gravity squeezes more massive brown dwarfs a bit more tightly with the result that regardless of their mass, brown dwarfs are all about the same radius, which is roughly the size of Jupiter. Being faint, low-mass, and very red, brown dwarfs are hard to detect. Like planets, brown dwarfs cool as they age, making older ones even fainter and harder to detect.

High and extremely high mass stars. Single stars of roughly 9 solar masses and higher will evolve to the point of core collapse and will not leave behind a He / C / O white dwarf remnant. These stars are the rare O and early B-type stars while they are on the main sequence. These stars are very luminous and may have relatively strong stellar winds (often Wolf-Rayet stars). These are the first-forming stars whose winds plow into the surrounding dust and gas in molecular clouds, sculpting cavities and pillars and providing some of the shock waves that lead to additional collapse and star formation. During their main sequence lives these stars have convective cores and radiative envelopes. Having a convective core can mean that some additional hydrogen from layers surrounding the core may be carried down in toward the center of the star, providing extra fuel for hydrogen fusion and extending the main sequence life of the star.

With lower-mass stars the kinetic energy of the plasma (gas pressure) is the dominant contribution to the outward pressure; with massive stars, the dominant contribution comes from radiation pressure. Photons carry momentum ($p = E / c$ for individual photons) and a flux of radiation exerts a pressure

$$P_{\text{radiation}} = (a/3)T^4 = \left(4\sigma/3c\right)T^4 .$$

Many of these stars are going to explode as supernovae, producing many of the heavy elements that seed future generations of stars. Let's consider some of the possible fusion reactions that can occur in stars of these higher masses and what happens as the star reaches the end of its life.

We have already alluded to the alpha process, which basically means fusion of a seed nucleus with an alpha particle, i.e., with a helium nucleus, in steps that take us beyond the triple-alpha production of ^{12}C. Dominant steps are those in which the nuclear mass simply increases by 4; e.g., $^{12}C + {}^{4}He \rightarrow {}^{16}O + \gamma$; $^{16}O + {}^{4}He \rightarrow {}^{20}Ne + \gamma$; $^{20}Ne + {}^{4}He \rightarrow {}^{24}Mg + \gamma$; and so on.

Once a massive star has built up a sufficient supply of nuclei such as ^{12}C, ^{16}O, ^{20}Ne, etc., these nuclei can also fuse with each other. There are often several possible paths, with various probabilities of occurring, that such reaction could take. For instance, consider ^{12}C. One obvious possibility is $^{12}C + {}^{12}C \rightarrow {}^{24}Mg + \gamma$, but this isn't the only or even the most likely possibility for the fusion of two carbons. Here are a few more possibilities:

$^{12}C + {}^{12}C \rightarrow {}^{20}Ne + {}^{4}He + \gamma$
$^{12}C + {}^{12}C \rightarrow {}^{23}Na + {}^{1}H + \gamma$
$^{12}C + {}^{12}C \rightarrow {}^{23}Mg + n$ (this one requires energy)
$^{12}C + {}^{12}C \rightarrow {}^{16}O + 2{}^{4}He$ (also requires energy)

We could think about $^{12}C + {}^{12}C \rightarrow {}^{24}Mg$ as the basic reaction with an excited state of ^{24}Mg being an intermediate stage that might de-excite or decay into any of these other four nuclei.

Stars of intermediate mass, 9-11-ish solar mass stars, will ascend the asymptotic giant branch but will be a bit brighter than their lower-mass cousins; at this point they are called super-AGB stars. Stars in this mass range are just massive enough to move beyond helium fusion. They were not degenerate prior to helium ignition but they are thought to be degenerate prior to the ignition of carbon fusion and may undergo a carbon flash. This is analogous to the helium flash but could be considerably more disruptive. These stars may leave a white dwarf remnant made of the O / Ne / Na / Mg that are the principal products of carbon and early alpha-process fusion. They may, or the more massive of them may, go supernova, in a type of core-collapse supernova called an electron-capture supernova. In this model the ^{20}Ne and ^{24}Mg in the core of the star capture electrons, twice, producing ^{20}O and ^{24}Ne, respectively. Each electron capture produces a neutrino. Electron pressure was a significant contributor to the outward pressure holding up the star's core against the inward gravitational force of the overlying layers. The electron-capture process can reduce the electron pressure so drastically that the core of the star collapses. The core winds up as a neutron star and the outer layers bounce off (see below for a few more comments on how an implosion can become an explosion). The electron-capture supernova may be less bright (~10x less bright) than a more massive Fe core-collapse supernova describe below. One candidate for this type of supernova, SN 2018zd, occurred in a spiral galaxy (NGC 2146) that's close enough to us for the progenitor star to be identified in earlier photos. The progenitor star does appear to have been a super-AGB star, as expected for this type of supernova. Researchers in 2021 suggested that the 1054 CE supernova that produced the Crab Nebula might have been an electron-capture supernova.

Higher mass stars are not going to have degenerate cores until the very ends of their lives and are going to have cores that are too massive to be white dwarfs. Electron degeneracy pressure can support an object of roughly 1.4 $M_\odot$ against collapse. This is called the Chandrasekhar limit (after Indian astrophysicist Subrahmanyan Chandrasekhar, 1910 - 1995); it varies a bit by composition, but 1.4 $M_\odot$ is a good estimation for the most massive possible white dwarfs.

In stars massive enough to move beyond carbon fusion, O can fuse; as with carbon, the reaction $^{16}O + ^{16}O$ has several possible outcomes. Oxygen fusion requires temperatures of ~1.5 billion kelvins but also very high densities, on the order of 10^7 g/cm^3. In other words, we are starting to build up a layered star, often described as in some ways like the layers in an onion. In the core, where the density and temperature are highest, the fusion reactions that require the highest temperatures and densities are taking place. The next layer out we have fusion reactions requiring slightly lower temperatures and densities. The star will develop several layers, with the most massive nuclei building up in the core.

At temperatures of several billion degrees the photodisintegration of nuclei starts to become a factor in the fusion of high-mass elements. In other words, it's hot enough that higher-energy photons are capable of splitting nuclei, spitting out protons or alpha particles that are themselves then available to participate in fusion. For example, ^{28}Si might photodisintegrate or, hit by an available alpha particle, might fuse to become ^{32}S. In the last day or so of its life before it goes supernova, this flux of alpha particles contributes to producing elements such as iron and nickel.

A star that is headed for a core-collapse supernova builds up a core of iron (and other nearby elements of mass around 56 a.m.u.), surrounded by shells of less massive elements, e.g., Si and S, O / Ne /Mg, C, He, and finally, for some stars, an outer shell that still contains hydrogen. Some massive stars have such high winds and mass loss rates that they may have lost that outermost layer of hydrogen. Elements such as ^{56}Fe and ^{62}Ni are at the peak of the binding energy curve (see the intro chapter to review this idea) which means that they are the last elements from which fusion reactions *produce* energy. Fusing higher mass elements will require putting energy *in*. Recall the scenario that led to helium fusion: the star's core hydrogen ran out, leading to an imbalance between outward pressure and inward gravity; the core collapses slightly and released gravitational potential energy increases the temperature in the core until eventually a higher-mass fusion reaction can ignite. Because of the increased Coulomb repulsion we expect fusion of higher mass nuclei to require higher temperatures but up until iron we have still, on balance, gotten energy out of each successive set of fusion reactions. Once we reach an iron core, additional collapse will continue to raise the core temperature and make more fusion reactions possible *but* because the fusion of elements beyond the iron group doesn't produce energy there is no concomitant adjustment in outward pressure, no re-establishment of hydrostatic equilibrium, nothing to halt the collapse.

As the star's core starts to collapse, processes of photodisintegration and fusion compete. Disintegration and decay processes increase the flux of available neutrons and many elements are now produced by the *r-process*. The r-process is neutron capture on timescales that are rapid with respect to the beta decay time of any unstable isotopes thus produced. This process is responsible for producing many of the neutron-rich isotopes beyond iron, including uranium. Still higher-mass isotopes, such as isotopes of neptunium or plutonium, almost all have half lives that are so much shorter than the age of the Earth that if the material from which our solar system formed had included these higher-mass isotopes they would by now have decayed. The exception is that there are trace amounts of ^{244}Pu, which seems to be the highest-mass r-process element found naturally on Earth.

Supernovae are distinguished principally by the presence or absence of hydrogen in the spectrum of the explosion. A Type II supernova shows hydrogen. A Type I supernova shows no hydrogen but there are two main reasons for why this might be the case: in a Type Ia supernova the explosion involves a white dwarf, which, as the remnant core of a star, isn't expected to have much hydrogen; Type Ib and Type Ic supernova (distinguished from each other by the presence or absence, respectively, of helium) arise from massive stars that have already lost their outer hydrogen-rich envelopes prior to exploding. In other words, core-collapse supernovae are types Ib/c and II. (There are some additional subdivisions that are not critical to our understanding of what's happening in a core-collapse supernova at this level.)

As its name suggests, a core-collapse *explosion* begins as an *implosion*. In the center of the star high-mass isotopes are being created and disintegrated at a furious pace. In the very center disintegration and decay are winning and a dense core of neutrons is building up. The, granted rather simplistic, picture of what follows is that the layers just outside the neutron core hit it and bounce off, hitting layers farther out that are still falling inward and propelling those outer layers outward in the event we see as the supernova. For an analogy, imaging holding a basketball and on top of that a tennis ball. Very carefully drop the pair. With the air resistance the two will separate as they fall and the basketball will hit the floor first and start back upward. The still-falling tennis ball will hit the upward-moving basketball and the momentum transferred to the tennis ball will send the tennis ball flying. Unlike with our tennis ball analogy, the supernova ejecta are also helped on their way by an enormous flux of neutrinos. Neutrinos, recall, have a very small cross-section for interaction. The production of approximately a solar mass worth of neutrons, though, also produces on the order of 10^{57} neutrinos. That many neutrinos, basically all at once, does make a significant contribution to ejecting the outer layers of the dying star.

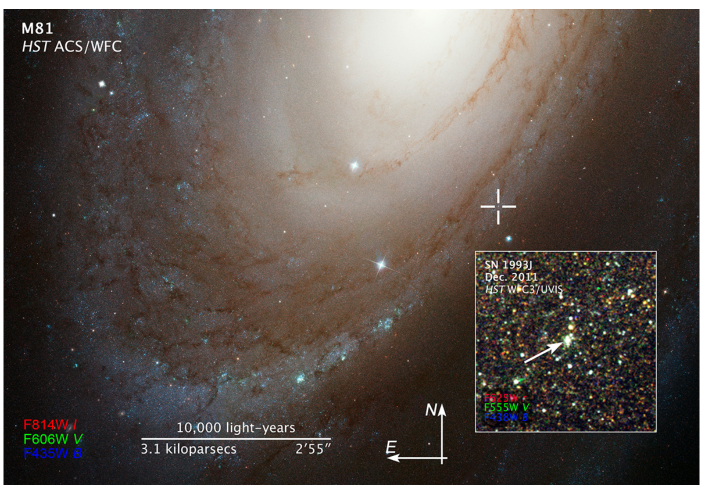

Figure 15.12: Supernova 1993J observed in the galaxy M81. Credit: NASA, ESA, and Z. Levay (STScI).

http://hubblesite.org/image/3412/category/107-illustrations

Supernovae are bright — core-collapse supernovae typically reach absolute magnitudes of −17 - −18, while a Type Ia supernova at its peak may exceed −19. In other words, core-collapse supernovae are typically 1-3 magnitudes less luminous than Type Ia supernovae, at least for stars of roughly solar metallicity. In the image above, the diffraction spikes mark a few foreground stars in the Milky Way; the inset highlights a Type II supernova observed in the galaxy M81 (roughly 11 million light-years away) in 1993.

By the late 1990s it was clear that there are also over-luminous stellar explosions, called *hypernovae,* ~2-3 magnitudes more luminous than Type Ia supernovae. There are several possible reasons for such over-luminous supernovae, generally less well-modeled than normal supernovae. One type of hypernova could be the explosion of a very massive Population III star, possibly by way of a "pair-instability" supernova (PISN). In this sort of event, conditions are energetic enough for the creation of electron-positron pairs, which leads to a reduction in core pressure, which leads to a runaway explosion leaving no remnant (in this case no remnant black hole, since this type of event should happen in stars too massive to leave a neutron star). The massive, early, pre-cursor stars to PISN should be long gone but it's possible that small stars formed from the slightly enriched material they left behind might still be with us. A report in 2023 points to the discovery of a metal-poor star (LAMOST J1010+2358) whose composition matches the expected chemical footprints of a first-generation star that died in a pair-instability supernova. The composition of this star supports the model of high-mass (greater than ~140 solar masses) first-generation stars.

The core collapse of a rapidly rotating high-mass star, often but not always a low-metallicity object, might produce relativistic jets in the ejected material, possibly linked to some longer-duration gamma ray bursts (GRBs), which are very energetic flashes of gamma rays lasting from a few milliseconds to a few hours. These "collapsars" are a possible model for some hypernovae. They are also a possible source of a huge flux of neutrons and thus one possible source of heavy r-process elements (e.g., gold, silver, platinum, etc.).

If the weight of the infalling material is too great to be stopped by a neutron core, the star may instead collapse into a black hole. Most short-duration GRBs are thought to arise from the merger of two neutron stars or a neutron star and a black hole; these events don't seem to be associated with supernovae. We will look in more detail at stellar remnants, including merger events, in the next chapter, but first let's consider a few variations on our evolution theme, in particular composition differences, rotation and binarity.

Composition differences. The lower main sequence in an H-R diagram has width in part because it is populated by stars of various ages, many of which have drifted away from the ZAMS. It also has width, though, because of variation in the metallicity of stars. Metals often have absorption lines in the blue portion of the stellar spectrum. In low-metallicity stars — for example the Population II stars in globular clusters — blue light that would otherwise be absorbed is instead free to escape from the star's photosphere. As noted in Chapter 14, the lower main sequence for a globular cluster (which no longer has any stars on the upper main sequence) will be bluer than the lower main sequence for a solar-metallicity cluster.

The largest of the earliest (Population III) stars, composed almost exclusively of hydrogen and helium, are expected to have been more massive than the most massive stars today, perhaps reaching several hundred solar masses. Without metals to provide extra opacity (both by line absoprtion and by virtue of having low first ionization energies), i.e., without as much for the radiation to exert pressure on, the Eddington limit for these stars should be higher than for today's stars. With metallicity and opacity and radiation pressure comes mass loss; a massive star today may shed enough mass over the course of its lifetime that the likelihood of leaving behind a black hole is significantly lower than it would have been for a massive low-metallicity star.

Detection of Population III stars is a current research project for several groups of astronomers. The most obvious, and most difficult, place to look is in the most distant, i.e., youngest, galaxies. A paper in 2015 reports strong evidence for the presence of Pop. III stars in two galaxies observed at a time when the universe was ~800 million years old (i.e., about 13 billion years ago; for those of you already familiar with redshift, the galaxies have $z = 6.6$). A 2018 report of a low-mass ultra-low metallicity binary star (2MASS J18082002-5104378, [Fe/H] ~ −4, component masses of ~ 0.76 and ~ 0.14 solar masses) in the Milky Way offers a tantalizing possiblity that low-mass Pop. III stars, able to survive until today, might have been able to form in a protostellar disk around a more massive primary star. Many more detections of Population III stars are needed before we can be confident that we understand the properties of these first stars.

Rotation and binarity. Stars rotate. In a star of roughly solar mass, a dynamo at the base of the outer convective zone maintains and regenerates the star's magnetic field. For most of their lives such stars exhibit *magnetic braking*, in which the charged particles in the stellar wind are forced by the magnetic field to co-rotate out to large distances, rather than falling into Keplerian orbits. These particles thus carry away angular momentum, effectively applying a brake to the star and slowing its rotation slightly. As the rotation rate decreases, so does the effectiveness of the magnetic dynamo and the strength of the magnetic field. Solar-mass stars slow rapidly at first and then more slowly as time passes. Stars on the left-hand side of the H-R diagram have core convective zones and don't experience magnetic braking. A massive main sequence star's surface equatorial speed may be may be more than 100 km/sec. If the star's core is rotating at a fairly high rate as well then as the star evolves and its core shrinks, and the core density increases, the star is going to have to shed angular momentum or else its core will wind up rotating at an unphysically high rate. We saw a similar problem with conservation of angular momentum in the pre-stellar collapse phase of stars' lives. The interiors of massive stars may experience rotationally-induced mixing, and there are some observations to suggest that at least some massive stars show enhanced surface heavy element abundances, which could be a signature of material mixed upward from the interior. High rates of rotation may also act to reduce the inward pressure on a star's core, resulting in a lower core temperature than we might have otherwise expected, and lengthening the star's life.

Many massive stars are a part of binary systems in which the two stars are close enough to each other to experience mass transfer between the two at one or more points in their lives, or even possibly merge. Adding mass may also add angular momentum, increasing a star's rotation rate. Adding mass to a main sequence star, in addition to confusing our understanding of its interior structure, should result in a star with a longer main sequence life. In other words, if we observe a massive star that is not as far along in its evolution as we expect for its age, it might be because that star didn't start life as a massive star but rather as a lower-mass star that was evolving more slowly. We do see a class of objects for which acquisition of mass, possibly including mergers, is a reasonable explanation. They are called *blue stragglers*. Recall that by the time all the stars in a cluster have formed we can justifiably use the approximation that the cluster stars are all the same age. The IMF (initial mass function) tells us that we should expect a cluster to have a few massive stars and many more low mass stars. As a cluster ages, its stars evolve off the main sequence. The mass at which stars are just evolving off the main sequence is called the cluster's *turn-off point*. Stars more massive than the turn-off point have left the main sequence; stars less massive are still on the main sequence. In color-magnitude diagrams for many clusters we see stars that lie along the main sequence blueward of the turn-off point. For single stars this would make no sense. For stars that have in, say, middle-age suddenly gained some extra mass, being blueward of their cluster turn-off point is plausible. The schematic CMD, below, for two old clusters, roughly representing NGC 188 and the slightly younger M67, illustrates where blue stragglers lie and the change in turn-off point as a cluster ages. The line indicating where all the stars of the same age should fall in a CMD or H-R diagram is called an *isochrone*. A note of caution: an isochrone — plotting stars of lots of different masses but all one age — can look a lot like the evolutionary track for a single star — plotting lots of different times for a single mass.

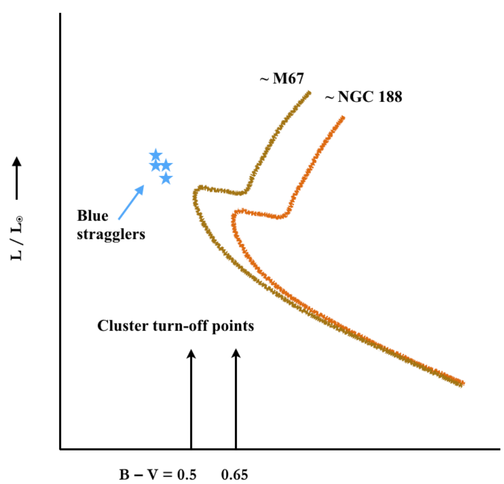

Figure 15.13: Approximate locations for the stars in M67 and NGC 188, indicating the locations of the cluster turn-off points and blue stragglers. NGC 188 is older than M67.

Mass transfer also happens for evolved stars, e.g., if in the process of expanding as a red giant the originally more massive in a binary fills its Roche lobe. Its atmosphere can then spill through the L1 Lagrange point and onto its originally less massive companion. A binary consisting of a lower-mass red giant and a higher-mass blue main sequence star strongly suggests that mass transfer has happened. If the pair formed together, there's no reasonable way the now more massive star could be less evolved unless it was originally the less massive of the two and only became the more massive of the pair when its companion evolved, expanded, and transferred mass onto it.

Type Ia supernovae involve mass transfer from a companion onto a white dwarf, pushing it over the Chandrasekhar limit, i.e., over the amount of mass that the pressure of the degenerate material in the white dwarf can support against gravity.

The Instability strip. There are many types of variable stars and many evolved stars vary in some fashion or other, but there's a particular location in the H-R diagram that is especially important: the *instability strip*. This is the regions where Cepheid variables live, named for prototype star δ Cephei. Stars whose properties fall in this region pulse, expanding and contracting, with associated changes in temperature and luminosity, with periods that are directly related to their masses. Because this region is nearly vertical, that translates into a relationship between period and luminosity. And because the brightest Cepheids are among the most luminous stars known, they are visible in other galaxies. Observing their periods permits estimations of their average absolute magnitudes which, when compared with their average apparent magnitudes, permits us to determine their distances and, thus, the distances to the clusters or galaxies within which they lie. We'll discuss them in more detail in the chapters on galaxies and cosmology; for now, the following figure is a sketch of where the instability strip lies. Post-main sequence evolutionary tracks for stars of 5 and 9 solar masses are included; note that some stars may evolve through the instability strip multiple times in their evolution. Cepheids have periods ranging from a few days to a few tens of days; RR Lyrae stars, found in globular clusters, are less massive and vary over a few hours. (Note: there are other types of variable stars that vary for different reasons and that also have associate instability strips; when it's used without any other qualifying terms, instability strip refers to Cepheids.)

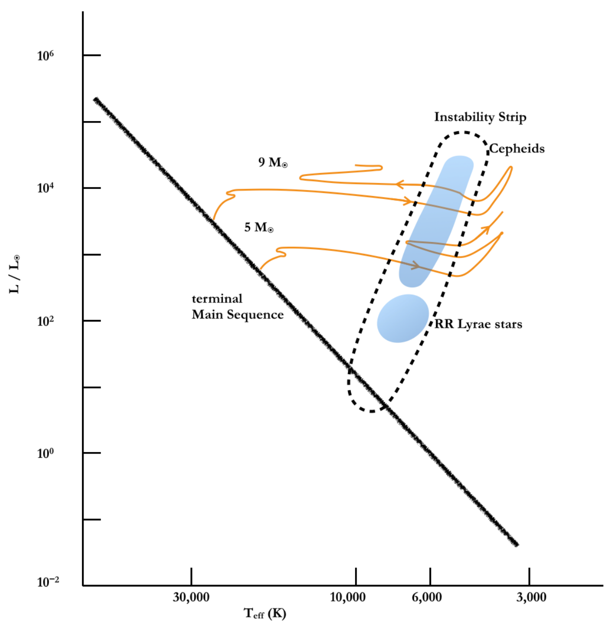

Figure 15.14: The location of the Instability Strip.

A few comments about other types of variable stars. Mira, Algol, UV Ceti, RS CVn, BY Dra, P Cygni, in addition to δ Ceph, RR Lyr, and T Tauri, mentioned already — lots of individual stars have their names attached to various patterns of stellar variability. Here, briefly, are a few notes about some types of variable stars.

Mira, or Omicron Ceti, is an AGB star, very red, very luminous, with a very extended atmosphere. Mira variables pulse with periods of over 100 days; sometimes a star's period will change, perhaps due to one of those thermal pulses mentioned above. These stars are losing mass, some of which may form dust around the star, and are on their way toward ejecting their outer layers into planetary nebulae.

Algol, or β Persei, is a prototype for an eclipsing, semi-detached binary star system. It's an example of stars so close together that they've experienced mass transfer, in which the initially more massive star has expanded and lost mass onto the initially less massive star. The now-secondary star is still a cool giant, with a magnetically active and occasionally flaring chromosphere. Algol itself is actually in a triple system; the two main stars are only 0.062 AU apart, and a third orbits that pair at a distance of only 2.69 AU.

Chromospherically magnetically active stars often show emission lines in their spectra and may be flare stars, exhibiting rapid unpredictable increases in brightness which may be extreme analogues to solar flares. Gliese 65 is a binary with two red dwarf stars, spectral types M5.5 V and M6 V, both of which are variable. The slightly smaller of the two is also known as UV Ceti and is the prototype for low-mass stars, and possibly brown dwarfs, that flare excessively. RS Canum Venaticorum (RS CVn) stars, which also often flare, are close, often eclipsing, binary systems. Normally an RS CVn is a system of two cool, chromospherically active stars, which show high levels of magnetic activity. The prototype, RS CVn, is a pair of subgiants, with spectral types F4 V-IV and K0 IV. Among the most chromospherically active stars is the binary BY Draconis. BY Dra variables are K or M main sequence stars that rotate quite rapidly, either because they are young or because they are in such a close binary pair that they are tidally locked, both stars rotating at the same rapid pace that they orbit each other. Some chromospherically active stars are contact binaries or possibly single stars that have merged. W UMa is an example of the former; it's a pair of stars sharing a common envelope, with the more massive star probably on its way to totally engulfing its less massive companion. FK Com is a rapidly rotating, very active, single star which may represent the post-fusion state of the W UMa stars. As you may be able to tell from these descriptions, there's considerable overlap between these types of variability!

As mentioned above, massive stars often have high mass-loss rates, and they may or may not be variable in the process. One prime example is P Cygni, a supergiant Luminous Blue Variable (LBV) of roughly 30 solar masses. The mass loss is indicated by the characteristic shape of the spectral line profiles (dubbed P-Cygni profiles) which show blue-shifted absorption and red-shifted emission components. Hot excited gases emit and cool gases absorb. But remember that what counts as hot or cool depends on the background. Recall that on the Sun we have prominences and filaments, which are the same physical structure; an arch of gas above the limb, seen against the cold background of space, is seen in emission, while the same arch of gas seen against the disk of the Sun shows up as a dark feature. Some of the material being lost from P Cygni is in front of the star and coming towards us; that material will show up in absorption, and blue-shifted because it is moving towards us. The majority of the material being lost won't be in front of the star and will show as an emission feature and with a broad range of Doppler shifts. Some of the emitting material will be moving slightly towards us, some sideways, and most relatively away from us. The line profile will thus have a bit of an absorption dip to the blue side of line center and then quite a bit of emission, most of which will be somewhat red-shifted. Various types of stars with mass loss will show spectral lines with P-Cygni profiles, so to avoid confusion LBVs may be referred to as S Doradus stars, after a very bright variable star in the Large Magellanic Cloud.

Cataclysmic variable stars, including classical novae, are binary systems involving stellar remnants, usually white dwarfs, onto which material is being transferred from a companion star via an accretion disk. When the accreted material ignites, the system increases in brightness rapidly. We'll look at novae in more detail in the next chapter, stellar remnants.

It's not unusual for evolved cool stars to be variable, in a manner that is definitely not chaotic but also not strictly periodic. Semi-regular variables vary in brightness from a fraction of a magnitude up to a few magnitudes; to the extent that the variation is roughly periodic, the periodicity ranges from a few weeks to a few years. Pulsation may be involved in the variability of some of the semi-regular variables. Examples of semi-regular variables include Mu Cephei, also called the "Garnet Star", an M2 supergiant with a radius roughly 1,000 times the radius of the Sun. Another is Betelgeuse, the bright red supergiant located at one of the shoulders of the constellation Orion. In addition to its semi-regular variability, Betelgeuse created a bit of excitement in late 2019 by dropping by a bit more than a magnitude in brightness, in the visual, over the course of about four months. By March 2020 it was on its way back up. Observations in the infrared and submillimeter wavelengths didn't show anything out of the ordinary, though, ruling out an imminent supernova explosion as well as the more mundane passing dust cloud as possible explanations. As of mid-2020, a wave of extra-large starspots seemed to be the leading culprit for the episode. In mid-2021 a group of researchers using the European Southern Observatory's Very Large Telescope proposed that an erupted bubble of gas from Betelegeuse cooled and condensed into dust, partially obscuring our view of the star's surface. These explanations aren't mutually exclusive: cool dark spots in the photosphere may be what permits the dust to condense.

There are a few stars that are decidedly idiosyncratic. Eta Carinae is a LBV; it's a pair (at least) of massive stars, one of roughly 100 solar masses and the second about half that, with a 5.5-year orbital period. The more massive of the two seems to be on the verge of going supernova — although, since it's ~7,500 light years away, it might already have done so — and has experienced several huge eruptions over the past eight hundred or so years. In the mid-1800s, for example, η Car shed roughly 20 solar masses of material, brightening erratically over about twenty years' time from 4th magnitude to ~ −1, rivaling Sirius in the night sky, before fading back into obscurity over a couple of decades, heavily obscured by dust condensed from the ejected material. One suggestion for this odd behavior is that the system might originally have been a triple. With different initial masses, the components would evolve at different rates. The initially most massive star (1) could have expanded and shed mass onto its closest companion (2). The mass shift could have resulted in such a change in their mutual orbits that the least massive of the three stars (3) could have gotten disrupted, and partially ejected, by the now more massive of the remaining pair (2). Maybe.

Light from that Great Eruption is still reaching us, in the form of light echoing off dust clouds in the extended Carina nebula, as seen in the figure at the left.

Hot outflowing material from η Car, including iron and nickel shed in the 19th century, produces spectral emission lines, shown in the figure below.

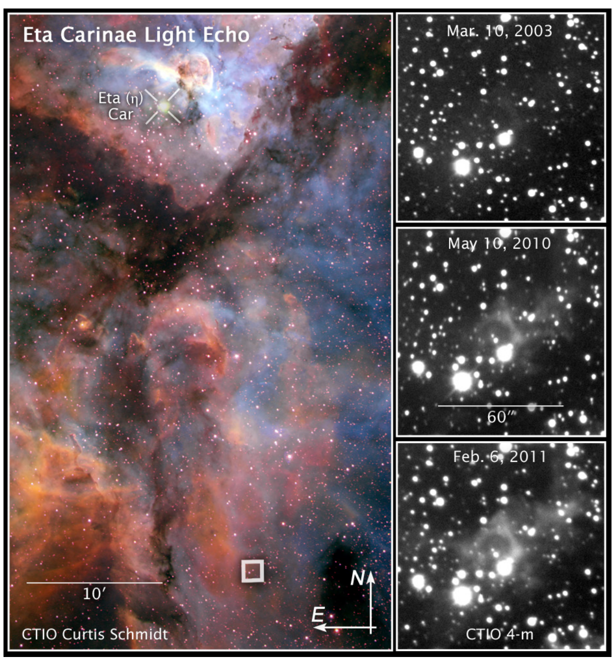

Figure 15.15: Light echoes from Eta Carinae
Credit: NASA, NOAO, ESA, and Z. Levay and A. Rest (STScI).
http://hubblesite.org/image/2998/news_release/2012-12

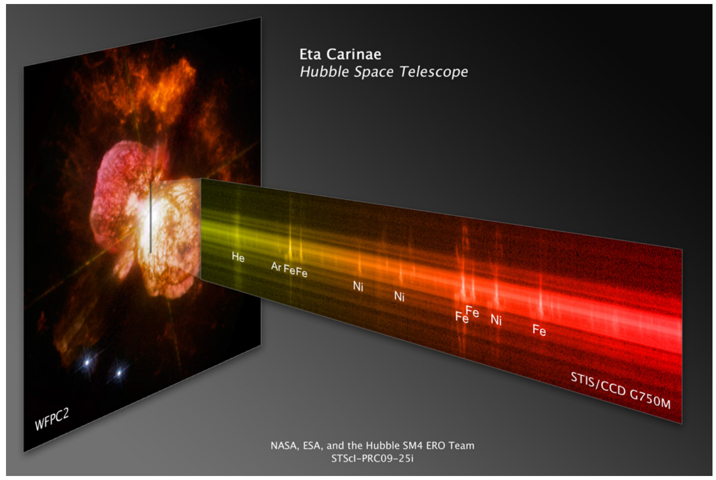

Figure 15.16: Visible image and visible + near IR emission spectrum of Eta Carinae; image scale: 0.6 arcmin or 0.4 pc across.

Credit: NASA, ESA, and the Hubble SM4 ERO team

http://hubblesite.org/image/2612/category/63-massive-stars

Sample questions

1. A few question about concepts related to stellar nucleosynthesis processes:

a) ^{56}Fe and ^{62}Ni are at the peak of the binding energy curve; what does that mean?

b) Both the CNO cycle and the triple-α process involve carbon; what's the difference?

c) How does Coulomb repulsion play a role in the core temperatures needed for various stellar nucleosynthesis reactions?

d) How do the r-process and s-process differ?

e) Give examples of a couple of α-process elements.

2. Some questions about CMD or H-R diagrams and the evolution of stars of different masses:

a) What are the mass limits for stars at the top and bottom of the main sequence? and why does each of these limits exist?

b) Sketch an H-R diagram and add an evolutionary track for the Sun; label the Hayashi track, red giant branch, where the helium flash occurs, and the horizontal branch.

c) Sketch a CMD diagram and add an isochrone for a cluster of stars of roughly 10 billion years age; label the turn-off point and indicate where we might find blue stragglers. Hint: that's about the age at which a star such as the Sun would be leaving the main sequence.

d) Sketch an H-R diagram (or add to one you've already drawn) indicating the location of the instability strip, where we would find brown dwarfs, and the region where we find white dwarfs.

e) Explain why the lower main sequence for stars of low metallicity could be bluer than the lower main sequence for Pop. I stars.

f) What is the ZAMS?

3. A few more about star formation:

a) What is the IMF (*not* the International Monetary Fund or the interplanetary magnetic field or a video game file format. . .)?

b) Describe the Jeans' length.

c) Define hydrostatic equilibrium.

d) What is a Population III star?