Chapter 16: **Stellar remnants.**

- white dwarfs and planetary nebulae
- neutron stars and supernova remnants
- stellar-mass black holes
- sample questions

White dwarfs and planetary nebulae

At the ends of their lives stars leave behind white dwarfs, neutron stars, or black holes as well as the ejecta from what were the stars' outer envelopes. White dwarfs are the electron-degenerate remnant cores left behind by all but the most massive stars. Determined masses for white dwarfs lie between ~0.2 and 1.3 $M_\odot$ with observed radii of roughly 0.008 - 0.02 $R_\odot$. That's roughly one solar mass in a volume that's roughly the size of the Earth. . .meaning that white dwarf densities are roughly 10^6 times the density of the Sun. As mentioned in the stellar evolution chapter, white dwarfs are expected to have masses below ~1.4 $M_\odot$, the Chandrasekhar limit, which is the limit that degeneracy pressure can support against further gravitational collapse. One consequence of degeneracy is that *more* massive white dwarfs have *smaller* radii.

Math note: for non-relativistic material, pressure and density are related by

$$P \propto \rho^{5/3}\text{, or, in terms of mass, } P \propto M^{5/3} / R^5$$

(unlike in an ideal gas where the equation of state gives $P \propto \rho T$).

If we have hydrostatic equilibrium, $P \propto M^2 / R^4$. Setting our two expressions for pressure equal,

$$M^2 / R^4 \propto M^{5/3} / R^5 \Rightarrow R \propto 1/M^{1/3}.$$

More mass means a degenerate object is smaller.

White dwarfs are initially very hot, since they were the cores of red giants. Their surfaces cool quite rapidly to a few tens of thousands of kelvin (into the B and A spectral range) but thereafter white dwarfs cool very slowly even though they are no longer producing any energy themselves. Because they are degenerate the interiors of white dwarfs can only cool by conduction, which is not very efficient compared with radiation or convection. When the interior has cooled enough, down to roughly 10^6 K, observations (consistent with a slow-down in the cooling rate over a large sample of white dwarfs) suggest that the interior carbon and oxygen may start to crystallize, giving the white dwarf a metallic core.

William Herschel detected a white dwarf in the triple star system 40 Eridani in 1783; the white dwarf companions to Sirius and Procyon were observed in the 1800s. The odd nature of these tiny hot faint objects was not obvious until the early 1900s when it became clear that they fell ~10 magnitudes below the main sequence in the then-new H-R diagram. Today, when we have many examples of white dwarfs still surrounded by expanding planetary nebulae, the nature of these oddballs is a bit more clear.

Here are three images of planetary nebulae acquired by the Hubble Space Telescope. They clearly are not planets. . .the name dates from the time when these faint fuzzy objects looked vaguely like planets in small, low-resolution telescopes. The Ring Nebula is located in the constellation Lyra, at a distance of ~700 pc. The image below is ~0.4 pc vertically. NGC 6543 is ~1,000 pc away, in the constellation Draco; the field of view of this image is similar to that of the Ring image. The distance to NGC 5189, which is in the southern hemisphere sky, is a bit more uncertain, between ~ 540 – 900 pc; at the center of this nebula is a binary, a white dwarf + a Wolf-Rayet star (hot surface, emission lines, high winds). The gases in a planetary nebula are excited by the hot star at the center. Eventually, over perhaps 20,000 years, the gases dissipate and the central white dwarf is left to cool off on its own.

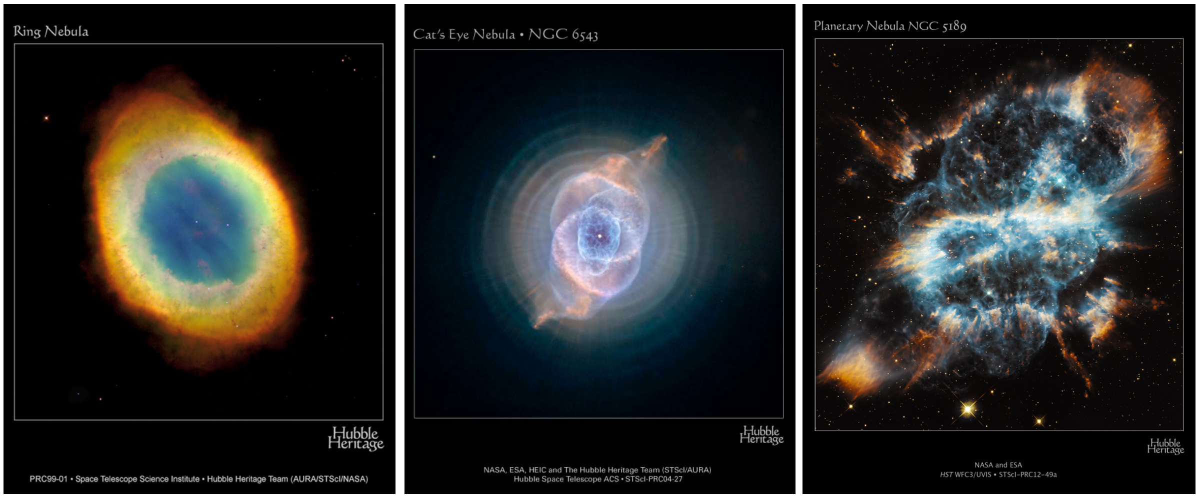

Figure 16.1 a: Ring Nebula; b: NGC 6543; c: NGC 5189

http://hubblesite.org/newscenter/archive/releases/1999/01/image/a/ http://hubblesite.org/image/1679/category/34-planetary-nebulae http://hubblesite.org/newscenter/archive/releases/2012/49/image/a/

Planetary nebulae may be, or at least appear to be, roughly spherical, but the majority are not. The gases in the nebulae were the envelope of an AGB star. . .which may have had a significant rotation rate, a strong magnetic field, a binary companion, all of which could contribute to the non-symmetric ejection of the star's envelope.

One possible consequence for a white dwarf in a binary system is that mass transfer may occur as its companion fills its Roche lobe, spilling at least some of its envelope, likely to be mostly hydrogen, onto the white dwarf. The companion could be an evolving star or a very close main sequence star. Consider first a C/O white dwarf safely below the Chandrasekhar limit. As compact as it is, the gravity at the surface of the white dwarf is relatively high, so the accreting material will hit hard, its temperature will go up, and it won't have much chance to expand. Temperatures in this new hydrogen layer can reach the point, approximately 20 million K, at which CNO fusion ignites. Once ignited, ~5% of the accreted material fuses, producing a large amount of energy very fast which makes the accreted layer expand and cool, and the fusion ceases. Some of the accreted material will be ejected at high speeds, often over a thousand km/sec. The white dwarf has become a *nova*, so-named because the increase in brightness can make an otherwise very faint object appear to be a "new" star. The nova may increase by as much as 10 magnitudes over the course of a few days and then, as the material dissipates, the brightness decreases over many months. The following figure is an example, showing roughly what the light curve for a naked-eye visible 1975 nova in the constellation Cygnus looked like.

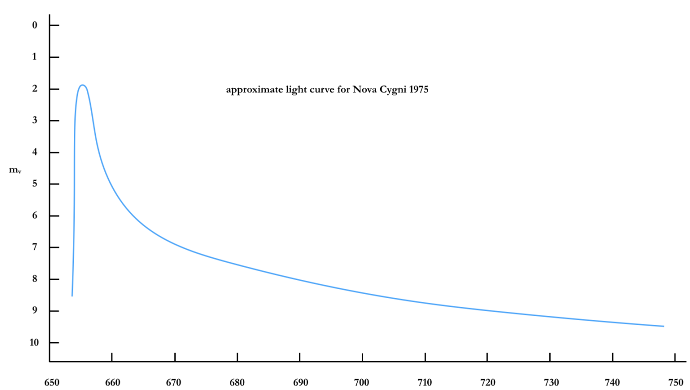

Figure 16.2: Nova Cygni 1975 light curve.

Based on Young, Corwin, Bryan, & de Vaucouleurs 1976, *ApJ* **209** 882.

In 2019 a group of astronomers using ESO's Very Large Telescope (VLT) reported the discovery in the globular cluster M 22 (NGC 6656) of a small emission nebula, with a total mass of the order of 10^{-5} $M_\odot$, that appears to be the remnant bubble of ejecta from a nova. Given the distance to M 22 and the expected magnitude of a nova, the explosion should have been visible to the unaided eye; interestingly, this one is at about the right size and location to correspond to a 'guest star' recorded by Chinese observers in the year 48 BCE.

Several white dwarfs are considered recurrent novae, i.e., they are known to have flared more than once. Many others may produce repeat novae but on timescales longer than the few hundred years we've been observing the sky with telescopes. Because a nova explosively brings together helium and a means of ejecting material rapidly into the cooler interstellar medium they are strong candidates for sources of ^{7}Li enrichment of the ISM.

A Type Ia supernova results if the additional mass, usually hydrogen, from a companion star pushes the white dwarf over the Chandrasekhar stability limit or if two white dwarfs merge, with a resulting combined mass that's over 1.4 $M_\odot$. And just to make things more complicated, there are some suggestions that a shell of helium added from a companion white dwarf could ignite sufficiently forcefully to cause the receiving white dwarf to explode even if the mass is under the Chandrasekhar limit; hopefully a wiggle in the brightening light curve allows such 'double-detonation' events to be identified. A Type Ia supernova can reach an absolute magnitude of −19 - −20, as the entire white dwarf blows itself apart. The light curve for a Type Ia supernova declines at a rate that permits it to be distinguished from the various possible core-collapse supernovae. Type Ia supernovae are identifiable in moderately distant galaxies; they thus serve as a way to probe conditions when the universe was younger and we'll consider them in that role in the chapter on cosmology.

Neutron stars and supernova remnants

As mentioned in the section on evolution of massive stars, a core-collapse supernova may leave behind a neutron star. Photodistintegration in the very center of the dying star splits nuclei, and the flux of very energetic electrons and protons merge to form neutrons that join with those squeezed directly out of the disintegrating nuclei. Neutron stars are made largely of degenerate neutrons and, like the white dwarfs, have an upper limit to the amount of mass that can be supported against gravity by the degeneracy pressure, known as the Tolman-Oppenheimer-Volkoff limit. It's less clear exactly what that mass limit — calculations suggest somewhere around 3 $M_\odot$, which agrees with observations that most neutron stars with determined masses are under ~2 $M_\odot$. Analysis of the detection of gravitational waves from the merger of two neutron stars (event GW170817) suggests a limit of ~ 2.2 $M_\odot$. In 2019 astronomers at the Green Bank radio observatory reported a rapidly rotating pulsar / neutron star with a mass of 2.17 $M_\odot$. This object is in a binary system with a white dwarf, conveniently oriented so that radiation from the pulsar will periodically pass near the white dwarf, which induces a slight gravitational effect due to the warping of spacetime by the mass of the white dwarf. That effect allows the mass of the white dwarf to be calculated, after which it is a straight-forward binary star orbit calculation to determine the mass of the neutron star. Very cool. A rapidly rotating neutron star can be ~20% more massive than a non-rotating neutron star because the centrifugal acceleration can help balance the inward gravitational acceleration; rotate too fast, though, and the neutron star will break up.

If a white dwarf is roughly the density of an atom, a neutron star is roughly the density of a nucleus, on the order of 10^{14} g/cm^3, packing a few solar masses into a ball with a radius of only 10-15 km. Neutron stars are hotter than white dwarfs, cooling over a few thousand years from ~10^8 -ish K at formation to surface temperatures of $10^{5\text{-}6}$ K and, like white dwarfs, cooling much more slowly thereafter. Don't expect to see neutron stars included on an H-R diagram — they are tiny, with spectra that peak in the far UV, and, as their progenitor stars are rare, we don't expect to have any of them close by enough to observe easily.

Neutron stars are the result of the collapse of the core of a supergiant and, as with other collapsing objects, we expect angular momentum and magnetic field strength to be conserved as the core collapses. Thus it is not surprising that neutron stars rotate rapidly and have strong magnetic fields. It is surprising how fast and how strong: there are neutron stars that rotate several *hundred* times per second and some that have magnetic field strengths over

10^{10} T (or $\sim 10^{7\text{-}8}$ times stronger than Earth's field). At the extreme end are those dubbed *magnetars*, which rotate very rapidly and seem to be the source for bursts of high-energy radiation (called Soft Gamma Repeaters and Anomalous X-ray Pulsars). It has been suggested that these may be young neutron stars and/or possibly relatively massive neutron stars formed from the merger of a pair of neutron stars.

As we see with many planets, the axis of the magnetic field doesn't have to line up with the rotation axis of the neutron star. After the supernova that created it, the neutron star is likely to be surrounded by some remaining plasma. Electrons, in particular, can get trapped by the strong magnetic field and forced to spiral around the field lines. Since they are being accelerated, the electrons produce synchrotron radiation; usually this is over a broad range of frequencies that are most obvious to us in the radio regime (where they are often relatively brighter than other sources). If one magnetic pole sweeps past us with every rotation of the neutron star we will get a blip, a pulse, of the synchrotron radiation every rotation period, like the flashing beam from a very rapidly rotating light house. We call these neutron stars from which we receive pulses of radiation *pulsars*. Terminology note: the neutron star is not pulsat*ing*; the name comes from the repeated *pulses* of radiation.

The techniques of radio astronomy were developed in large part in the decades following the second world war and it was through radio astronomy that the first pulsars were identified. S. Jocelyn Bell Burnell was a graduate student at Cambridge University in the mid-late 1960s; her work included participating in constructing a radio antenna designed to observe the recently identified quasars (now known to be the very bright nuclei of distant galaxies). In 1967 she discovered the first pulsar, showing that what on first appearance looked like noise in the data was in fact a rapid periodic signal from a stellar source. In what many consider to be a classic outrage, Bell Burnell's adviser shared in the 1974 Nobel Prize for Physics, which noted his having played a role in the discovery of pulsars, while Bell Burnell herself was excluded from the honor.

Pulsars are remarkably stable clocks, although they do tend, overall, to slow as they age. A pulsar with a companion may, for a time, accrete mass, along with its associated angular momentum, as that companion expands. This is likely to speed up the pulsar's rotation rate (or slow it, depending on the direction from which the infalling material is accreted) and produce x-rays from the hot infalling accreted material. With or without added mass, occasionally the crust of the neutron star readjusts its shape and the rotation rate spins up again, so we can't simply use rotation rate alone as a measure of the age of the neutron star.

Because pulsars emit over a broad range of wavelengths they are useful for probing the properties of the interstellar medium: the ISM is not totally empty and thus has an index of refraction and thus the speed at which the radiation from the pulsar travels toward us does not equal c, the vacuum speed of light. The deviation from c depends on frequency, with the lower frequencies traveling a bit more slowly than the higher frequencies. The more material the radiation passes through, the greater the dispersion in arrival times for different frequencies that make up one individual pulse packet. If we have an independent estimation of the distance to the pulsar, the dispersion provides information about the density of the ISM. If, on the other hand, we have reason to think that we know the density of the ISM along the line of sight toward the pulsar, we could use the dispersion to calculate the distance to the pulsar.

The shock wave that powers a supernova is likely to eject the majority, and in some cases all, of the mass of the progenitor star into a *supernova remnant* (SNR), on its way back into the interstellar medium where it may seed another generation of star formation. Arguably the most famous SNR is the Crab nebula, the remains of a supernova recorded by Chinese astronomers in 1054 C.E. Here is a Hubble Space Telescope image of the Crab, or M1.

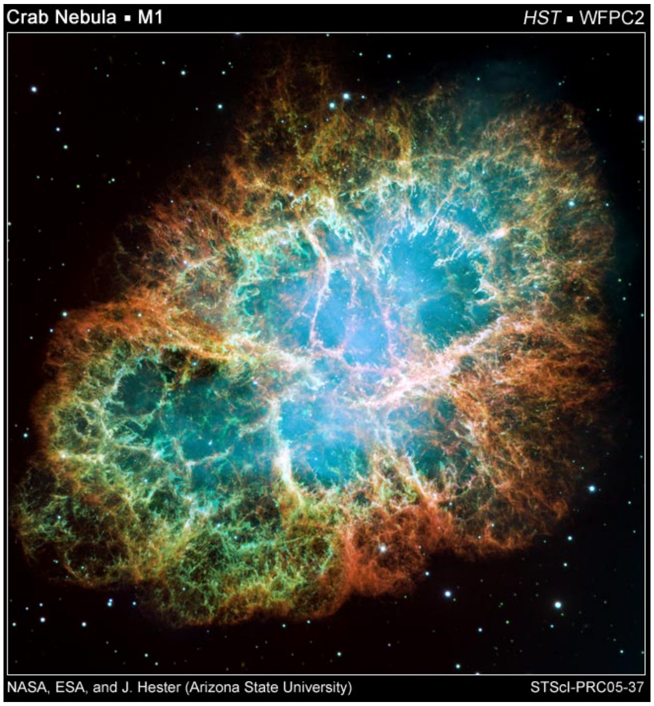

Figure 16.3: The Crab Nebula is ~ 2 kpc away, in the direction of the constellation Taurus. This image is 6 arcminutes horizontally, or ~3.7 pc at the distance to the Crab. This image emphasizes emission from oxygen and sulfur isotopes. The Crab is expanding at ~1,500 km/sec.

http://hubblesite.org/image/1823/category/2-stars

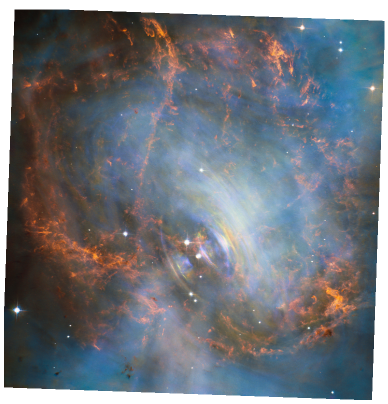

Figure 16.4: The pulsar at the center of the Crab is quite fast, spinning ~30 times / sec. In this image of the core of the Crab Nebula, spanning about 1 parsec, the pulsar is the lower of the two bright stars just below the center.

Credit: NASA and ESA; J. Hester (ASU) and M. Weisskopf (NASA/MSFC) — Hubble Space Telescope

http://hubblesite.org/image/3760/news_release/2016-26

Tycho and Kepler are both associated with observations of Milky Way supernovae, in 1572 and 1604, respectively, being among the better-known of those astronomers who observed these events. Tycho's supernova was a Type Ia, estimated to be ~3 kpc away and to have reached a peak apparent visual magnitude of −4. In the image below left, there's a small blue arc just below left center that Chandra astronomers think may indicate additional material that was blown off the companion star by the shock wave from the supernova. The blue in this image is mostly synchrotron emission from very high energy electrons in the outward-moving shock wave and the red and green are expanding debris from the explosion that's been heated to millions of degrees by a reverse shock wave that bounced back inwards. Kepler's supernova, shown in the image below right, was also a Type Ia, and bright enough to be visible in the daytime for several weeks. The bubble of iron-rich dust and gas is still expanding outward at ~ 2,000 km/sec. In a Type Ia supernova a white dwarf, or perhaps two, is totally destroyed by the explosion and no stellar remnant is left behind.

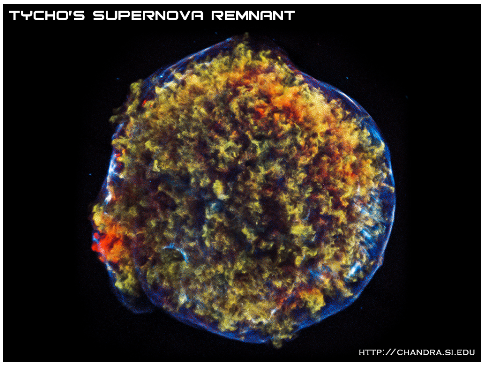

Figure 16.5: Chandra X-ray telescope observation of Tycho's SNR; this image is 9.5 arcminutes across. Red = 1.6 - 2.0 keV; green = 2.2 - 2.6 keV; blue = 4 - 6 keV. Credit: NASA/CXC/SAO.
http://chandra.harvard.edu/photo/2014/15year/

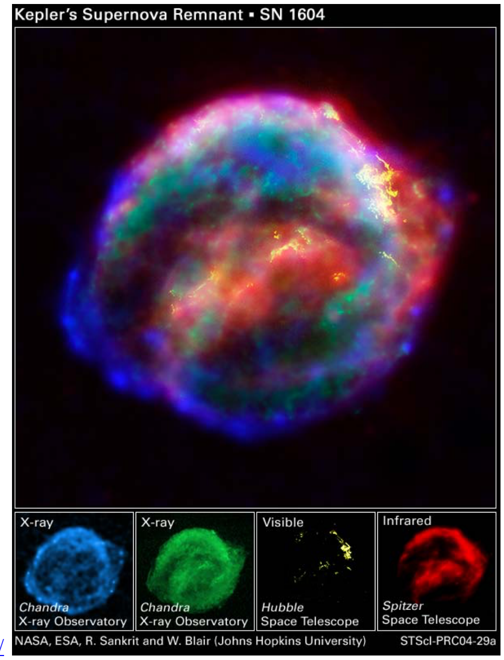

Figure 16.6: Kepler's SNR in a montage of visible, IR, and X-ray. The image is 5 arcminutes wide.
http://chandra.harvard.edu/photo/2004/kepler/

We haven't had a naked-eye visible supernova in the Milky Way since those two, and, statistically, we're overdue; of course Milky Way supernovae may have occurred more recently but been impossible to see because of dust along the plane of our galaxy. In the more distant past it's likely that supernovae occurred closer to Earth — in 2016 two independent teams of researchers reported an excess abundance of ^{60}Fe, half life ~2.6 million years, in samples of sediments from several deep ocean locations. A supernova or two within ~100 parsecs or so, erupting within the last few million years, is a plausible explanation as a mechanism for delivering ^{60}Fe to Earth.

SN1987A

The most recent naked-eye visible supernova was SN 1987A, observed in the Large Magellanic Cloud (LMC; ~168,000 light years away) in February 1987 (the "A" means it was the first supernova detected in 1987). Ian Shelton, an astronomer working at the Las Campanas Observatory in Chile, was the first to catch the supernova. He had fortuitously photographed the same region in the LMC the previous night. Using various instruments astronomers have been watching it ever since. The star that exploded was identified as a blue supergiant, ~20 $M_\odot$, which is a bit perplexing because models prior to that point had suggested that red supergiants should be the ones to explode; this star seems to have been on a loop back across to the blue side of the H-R diagram.

The assertion that this was a core-collapse supernova was bolstered by the detection of neutrinos 2-3 hours before the first light arrived from the event. Three neutrino detectors, in Japan, the United States, and Russia, observed the arrival of a burst of antineutrinos produced in the neutron creation as the core collapsed. Neutrinos are notoriously hard to detect and in this case a "burst" means about 25 neutrinos total, above background levels, by the three detectors. The neutrinos arrived a bit ahead of the light — neutrinos travel at nearly the speed of light and get out of the supernova with minimal interference, leaving the star's core pretty much as it is collapsing; light, on the other hand, is impeded by the opacity of the explosion debris and the visible signal of the explosion only starts on its way to us once the shock wave from the core collapse reaches the surface of the star.

The supernova took about three months to peak, reaching maximum brightness in late May 1987. The light curve dropped off sharply in the month following peak brightness and then, July - November, declined exponentially with a half-life of 77-78 days. That's what we would expect based on the half lives of ^{56}Ni, 6 days, and ^{56}Co, 77 days. ^{56}Ni is one of the dominant isotopes expected to be produced in a core-collapse supernova; it rapidly decays to ^{56}Co, which then decays to ^{56}Fe. We observed the light from the expanding debris cloud heated by the γ rays produced by these radioactive decays. By the end of 1987 the material from the explosion had expanded enough that the opacity dropped to the point where γ rays could simply escape, and the light curve decreased more rapidly thereafter. The following sketch shows roughly what the light curve looked like in the months following the supernova.

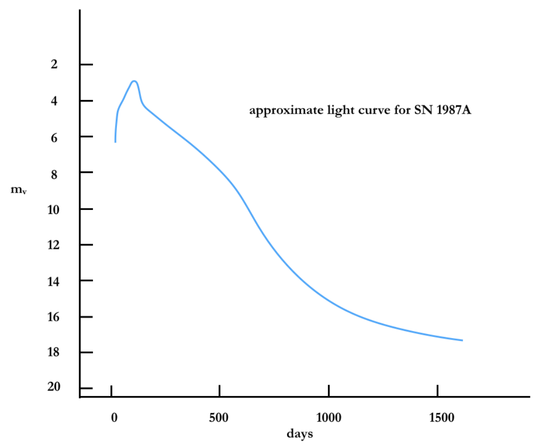

Figure 16.7: Supernova 1987A light curve

The following images show SN 1987A three-plus decades on.

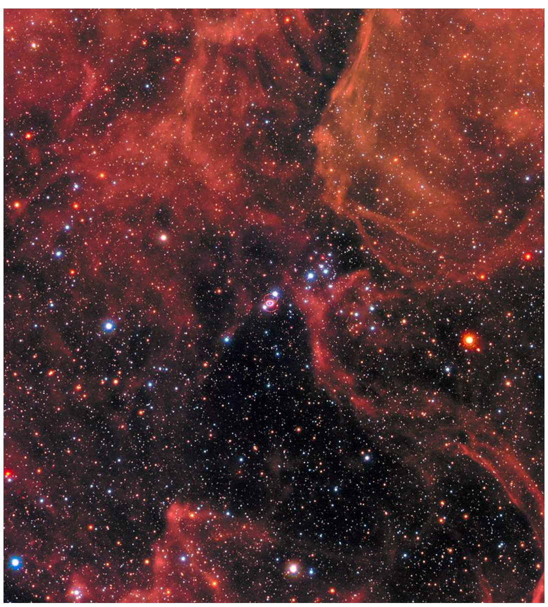

Figure 16.8: January 2017 image of the expanding remnant of SN1987A. Credit: NASA, ESA, R. Kirshner (CfA), M Mutchler, R. Avila (StScI). http://hubblesite.org/image/3987/news_release/2017-08

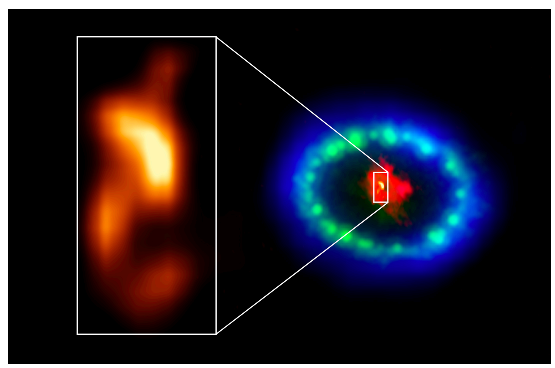

Figure 16.9: 2020 release of multi-wavelength image of SN1987A. Red: millimeter ALMA (ESO/NAOJ/NRAO) showing dust and cold gas; green: visible HST (NASA/ESA) and blue: x-ray Chandra (NASA) show the expanding shock wave and hot gas. North is up; ring ~3 arcsec, or ~ $^3/_4$ pc across. https://public.nrao.edu/news/alma-finds-possible-sign-of-neutron-star-in-supernova-1987a/

The progenitor star would have had a strong stellar wind, particularly in its supergiant phases. After the supernova, high-energy photons and then, later, material from the explosion caught up with bubbles of previously ejected material (and perhaps with a companion star). Several months after the supernova the photons caught up with and excited emission from the earlier material; it took a few years for the shock wave of supernova ejecta to arrive. That resulting collision generated x-rays, observed starting in ~2001. Observations in 2015 suggest that clumps of material are being broken up by the passing shock and will likely dissipate over the next few decades.

One of the oddities of SN 1987A is that even as the debris has expanded we had not yet been able to detect the neutron star expected to have been formed by the supernova. Millimeter-wavelength observations indicate the presence of newly formed dust at the blast site, which may inhibit our ability to see a neutron star. It's also possible that the neutron star didn't last, but that enough material fell back onto it to push it into becoming a black hole. On the other hand, it may just be that we hadn't waited long enough: Observations reported in July 2020 using the Atacama Large Millimeter/submillimeter Array (ALMA) hint at a hot "blob" starting to reveal itself at roughly the location expected for the remnant neutron star. In 2021 X-ray space telescopes reported the detection of hard X-ray emission from that site, of the sort that we would expect from the material close to a pulsar. We may still need to wait another decade or two for the dust to clear enough for us to get decent visible-wavelength observations of what might be the long-sought-after neutron star.

GW170817

On August 17, 2017, gravitational wave detectors — the two Laser Interferometer Gravitational-wave Observatory (LIGO) sites in the U.S. working with the European-based Virgo detector — observed for the first time a signal from the inspiral and merger of a pair of neutron stars. Neutron stars spiral in and merge a bit more slowly than black holes would and the gravitational wave signal in this case was detectable for about 100 seconds. Unlike the merger of a pair of black holes, the merger of a pair of neutron stars was expected to create not just gravitational waves but also electromagnetic radiation. 1.74 seconds after the gravitational waves signaled the merger of the two neutron stars the first gamma rays were detected by the Gamma-ray Burst (GRB) Monitor on the Fermi space telescope. Dozens of ground- and space-based observatories were quickly alerted and telescopes slewed nearly in unison toward a relatively close galaxy (only about 130 Mly away) in the constellation Hydra. Those first few seconds confirmed two theoretical predictions: that gravitational radiation travels at the speed of light and that neutron star collisions can produce short GRBs.

Observations of this event continued at various wavelengths long after the initial detections of the explosion, yielding both new insights and new questions. A neutron star merger had been predicted to produce a *kilonova*, observed as a short GRB followed by a long lower-energy afterglow powered predominantly by the decay of the radioactive r-process isotopes produced in the explosion. A kilonova should be roughly 1,000 times brighter than a classical nova, although still less luminous than a typical supernova. In the case of GW170817, observations are consistent with roughly 0.05 solar masses worth of heavy elements having been produced, including about ten Earth masses of gold and platinum. This supports the hypothesis that events such as this are responsible for producing approximately half of the isotopes heavier than iron, individual massive stars alone not being capable of producing enough heavy elements to match the measured cosmic abundances. That mergers such as these aren't the only source of these elements is indicated by the fact that spectra show the presence of r-process elements way too early in history of the universe, well before a pair of neutrons stars could lose enough energy to spiral in close enough to merge (see also 'collapsars' in the previous chapter).

The short GRB, lasting approximately 2 seconds, wasn't as bright as expected, perhaps having been produced by a jet of material not quite directed toward us or perhaps having been dimmed and spread out by hitting a fog or cocoon of previously ejected surrounding material. The latter model is supported by radio observations: at radio frequencies the afterglow of the explosion continued to brighten for several months, consistent with a jet whose emission diffused across an extended cocoon of material.

At least two prior events are proposed to have been kilonovae but that's hard to verify without the gravitational wave signature. That signature in the case of GW170817 is consistent with a total combined mass of approximately 2.8 solar masses for the stellar remnants involved in this merger. The lack of emissions that would indicate a surviving neutron star suggest that while the result of the merger could initially have produced a massive neutron star, that object must have collapsed into a black hole within milliseconds.

Stellar-mass black holes

A black hole is a region from within which the escape speed exceeds the speed of light. The possibility that light might be affected by gravity was first suggested in 1783 by John Michell, an English cleric and scientist. His thinking was that light would be slowed down by an object of sufficient gravity, creating a "dark star". Today we would say rather that the frequency of light changes as it climbs out of a gravitational potential, but Michell definitely deserves recognition for his insights. The size of a black hole is usually indicated by the distance at which the escape speed equals the speed of light:

$$v_{\text{escape}} = c = \sqrt{2GM/r} \Rightarrow r_{v_{\text{esc}}=c} = 2GM/c^2 .$$

For one solar mass, this is about 3 km. This distance is called the *Schwarzschild radius,* named for Karl Schwarzschild, who early on found an exact solution to Einstein's equations of general relativity for the case of a single, non-rotating point mass. The spherical surface with radius = $r_{\text{Schwarzschild}}$ is called the *event horizon,* since if light can't escape, we can't obtain information about any event inside this region. If the mass within the event horizon is not rotating, we know of no force that would support it against gravitational collapse; the mass at the center of a non-rotating black hole is said to be a *singularity.* It's not clear whether the singularity literally means a point, zero size; it's more reasonable to say that at this point our classical theory breaks down and we are in need of a quantum theory of gravity. The following sketch illustrates the warping in the fabric of spacetime due to the presence of a black hole mass.

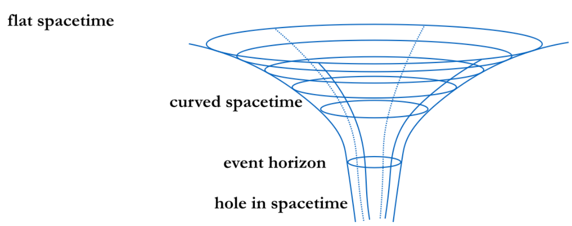

Figure 16.10:
Warping the fabric of spacetime.

Stars rotate, which suggests that most stellar-mass black holes should have angular momentum. Space is dragged around by a rotating black hole, meaning that outside a rotating black hole there's an "around" curvature, not just an inward warping to the fabric of spacetime. This effect is called *frame dragging* (it's also known as the Lense-Thirring effect) and in principle makes it possible to measure the angular momentum of a black hole. There is an oblate region around a rotating black hole, outside the event horizon, called the *ergosphere,* within which no particle could stand still because the space is being dragged around so fast. Frame dragging is more extreme for a black hole, but it exists for any rotating mass and has been measured for the Earth: Gravity Probe B, a spacecraft in low-Earth orbit in 2004-5, carried four high-precision gyroscopes and measured a terrestrial frame-dragging rate of $\sim -37 \cdot 10^{-3}$ ″/yr. Inside a rotating black hole the mass is not compressed to a point singularity but rather to a torus.

It is theoretically possible for a black hole also to have electric charge, but since stars are likely overall to be electrically neutral we don't expect to have charged black holes in actuality. So that's it: mass and angular momentum. Other quantities that might describe the matter that got incorporated into the black hole — e.g., entropy, the numbers of baryons and leptons — don't seem to get preserved by the black hole. In an otherwise quite conservative universe this loss is confusing, and has been dubbed the black hole information paradox.

Light climbing out of a gravitational potential well experiences a gravitational redshift. Imagine dropping a probe toward a black hole from a safely orbiting spacecraft. The probe is equipped with a strobe light and sends a flash back your direction every second at a specified frequency. As the probe drops toward the black hole, i.e., deeper and deeper into the hole's gravitational field, you observe that the pulses from the probe arrive less frequently and at longer and longer wavelengths. If you could "ask" the probe, it would respond that all systems are fine and that it continues to emit the specified frequency of light at the specified time intervals. Due to the increasing gravitational field, the clock on the probe is running more slowly than your clock and the light emitted by the probe is losing energy. Once the probe reaches the event horizon of the black hole the redshift goes to infinity. If you could see it, the last pulse the probe emits seems to hang there in space, traveling outward at the speed of light in a region of spacetime that's falling inward at the speed of light toward the singularity. Any mass is going to produce gravitational redshifts and clocks that run more slowly than they would in space, well away from any star or planet. (Practical application?: we have to compensate for the fact that clocks on GPS satellites run at different rates than clocks in GPS receivers on the ground.)

Math note: the gravitational redshift observed by a distant observer is given by: $z = \frac{\Delta\lambda}{\lambda_{emitted}} = \frac{\lambda_{obs}}{\lambda_{emit}} - 1 = \frac{1}{\sqrt{1 - \frac{r_{Sch.}}{r_{emit}}}} - 1 = \frac{1}{\sqrt{1 - \frac{2GM}{c^2 \cdot r_{emit}}}}$ where $r_{Sch.}$ is the Schwarschild radius. If $r_{emitted}$ is much larger than $r_{Sch.}$ then we have an expression of the form $\sqrt{1-x}$ for which x is small and may approximate it as $1 - x/2$. Following some algebra we may say, if $r_{emitted}$ is small, $z \approx \frac{GM}{c^2 r_{emitted}}$.

Stars with initial masses of more than ~20 $M_\odot$ don't seem to be able to shed enough mass in the course of their lives to leave behind remnants of less than ~5 $M_\odot$. In other words, the most massive stars seem bound to leave remnants too massive to be neutron stars or white dwarfs. A stellar mass black hole may result directly from the collapse of the core of such a star or an intermediate neutron star may form but be pushed by additional infalling material over the neutron star stability limit and on to infinite collapse. There is little doubt that stellar-mass black holes exist. We have considered how to determine the masses of objects in binary systems; there are several in which a dark companion unambiguously has a mass greater than 5 $M_\odot$. Black holes also make their presence known by the behavior of material falling toward them. Particles don't just fall straight across an event horizon. Material spirals inward, losing energy, collecting for a time in an accretion disk. Accretion disks are energetic, turbulent places, and an accretion disk around a black hole can get hot enough to emit x-rays. Some material will also be lost in jets perpendicular to the accretion disk (we will consider much more extreme jets in the sections on active galaxies). V404 Cygni is a binary system 2.39 kpc away containing a solar-ish star and a black hole of ~12 solar masses. Emission from this system is variable (hence the "V"); 2015 saw an outburst of activity, in visible light as well as x-rays, after ~26 years of relative quiet. A more well-known neighbor in Cygnus, Cygnus X-1, is an x-ray emitting binary system with a black hole of ~21 $M_\odot$ in orbit with a blue supergiant star (new research published in 2021 found Cygnus X-1 to be farther away than previously thought — 1,900 pc — and thus more massive than previously thought). Cygnus X-1 was the subject of a famous bet between Stephen Hawking and Kip Thorne in 1975. Hawking bet that the object wasn't a black hole, not because he didn't think Cyg X-1 was a black hole but because if it were shown not to be a black hole he'd at least have the consolation of having won the bet. Hawking conceded in 1990 that the weight of the evidence was overwhelmingly in favor of Cyg X-1 being a black hole. As of mid 2023, the closest known black hole, at ~ 478 pc, is an ~10 $M_\odot$ black hole in a binary system with a solar-mass main sequence star. Identified in observations by the Gaia satellite, it's known as Gaia BH1.

Once a black hole forms, it can grow by accreting more infalling material or perhaps colliding and merging with another remnant. In principle the black hole can, eventually, shrink: the warped spacetime that is the black hole event horizon should produce virtual particle - anti-particle pairs that normally promptly annihilate; should one of these fall into the black hole, the other would become real, carrying away mass-energy from the black hole. The black hole radiates in this way ("Hawking radiation") as if it were a black body. Most black holes' event horizons are curved too gently to radiate much; i.e., the black holes are not very "hot", but are actually cooler at the present epoch than the cosmic background radiation. A stellar-mass black hole is going to have a lifetime many orders of magnitude longer than the current age of the universe.

Einstein's equations of general relativity permit interesting permutations on the black hole theme that are not necessarily physically realistic. A time-reversed black hole would be a white hole, a point from which energy would be emitted. The event horizon of a white hole would be a region nothing could enter. Calculations suggest that white holes would not be stable. A worm hole is a tunnel through spacetime, e.g., if spacetime were folded back on itself so that two holes were connected. Like white holes, wormholes seem to be inherently unstable. Practically speaking wormholes would seem to permit one to travel back in time, leading to the classic paradox of, hypothetically of course, killing off one's grandparents before one's parents had been born, leading to the paradox of the time traveller not being present to prevent themselves from being present. . .Stephen Hawking has quipped that there must be a principle he calls the "chronology protection conjecture" to keep the universe safe for historians.

Sample questions

1. Check some of the densities for stellar remnants compared to stars and atomic nuclei. First, assume a mass of 1.4 solar masses, or $2.8 \cdot 10^{30}$ kg. That would be roughly an F5 main sequence star, right at the high end for a white dwarf, and just a bit low for a neutron star. Calculate some densities; express your answers in g/cm^3:
   - a) for an F5V star of radius ~ 1.3 times $R_\odot$.
   - b) for a white dwarf of radius ~ 0.005 $R_\odot$.
   - c) for a neutron star of radius ~ 12 km.
   - d) for the nucleus of ^{12}C, which has a radius of ~$2.7 \cdot 10^{-15}$ m.

2. Look at the math note about gravitational redshifts. Let the emitted wavelength be H-β, 486.13615 nm. Calculate the observed wavelength for light emitted from the surface of
   - a) that 1.4 solar mass, 12 km radius neutron star
   - b) that 1.4 solar mass white dwarf, radius ~ 0.005 $R_\odot$.
   - c) Can you use the approximate form for z for one or both of these objects?

3. Calculate the radius of the event horizon for an 8 solar-mass black hole.

4. Define / describe / explain:
   - a) AGB star
   - b) planetary nebula
   - c) Chandrasekhar limit
   - d) the Pauli Exclusion Principle & degeneracy
   - e) Type Ia supernova
   - f) Type II supernova
   - g) neutron star
   - h) pulsar
   - i) GW170817
   - j) event horizon or Schwarzschild radius
   - k) singularity
   - l) black hole information paradox
   - m) frame dragging
   - n) gravitational redshift
   - o) Hawking radiation

Answers to selected problems are on the next page

1 a) ~ 0.9 g/cm^3
b) ~$16 \cdot 10^6$
c) ~ $3.9 \cdot 10^{14}$
d) ~ $2.4 \cdot 10^{14}$

2. a) either 654.3 or 570.2 nm, depending on whether you use the more exact for or the approximate form, respectively
b) 486.426 in either case

3. ~23.7 km (i.e., 8 times larger than for one solar mass)