

Chapter 3: **Coordinates & time**; much of this chapter is based on earlier work by Katherine Bracher

- celestial sphere and celestial coordinates
- time; connecting celestial and terrestrial coordinates
- sample problems

## The Celestial Sphere

Imagine yourself on the Earth, surrounded by a huge sphere that is the sky. On this sphere appear all the stars, the planets, the Sun and Moon, and they appear to move relative to you. Some of these motions are real and some are of course illusions caused by the motions of the Earth. The objects are not all at the same distance from us, but they *look* as though they are and this is a useful way to describe the appearance of the sky.

This imaginary sphere is called the **celestial sphere**; the Earth is a tiny sphere, concentric with it, at its center. As you look straight up above you, the overhead point is called the **zenith**. If you imagine a circle,  $90^\circ$  away from the zenith, you have your **horizon**. See figure 1. Ideally, you could observe the half of the celestial sphere that is above the horizon; the other half is hidden from you by the Earth. Most of the time mountains and trees and houses will block some of your view. The point directly opposite the zenith (i.e., directly under you) is called the **nadir**.

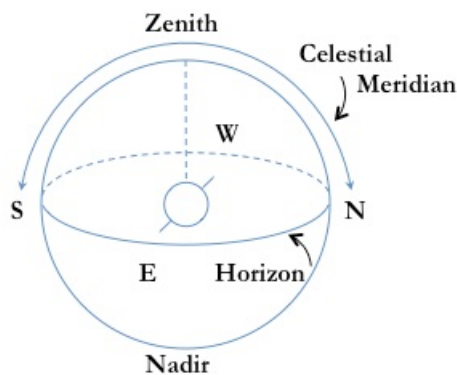


Figure 3.1 — Celestial meridian, horizon, zenith.

You could also draw an imaginary circle down from the zenith to the north point on the horizon and to the south point on the horizon. This circle (or semi-circle, since you can only see half of it) is called your **celestial meridian**, and it cuts the celestial sphere into an eastern and a western side.

The locations of the zenith and the horizon relative to the stars on the celestial sphere depend on where you are on Earth and what time of day or night it is. Your **latitude** is how many degrees you are north or south of the Earth's equator; your **longitude** is how many degrees you are east or west of the Greenwich (England) meridian. What is up in your sky at any given time depends on where on Earth you are.

To understand how your position is related to what's up in the sky, we need a few more definitions. You are located on a planet that turns on an axis; the ends of that axis are the north and south poles of the Earth. If you extend that axis on out to the celestial sphere, you locate the **north celestial pole (NCP)** and the **south celestial pole (SCP)**. The great circle that you can draw around the celestial sphere that is halfway between the poles is the **celestial equator**. You could also get the celestial equator by extending the Earth's equator out to intersect the celestial sphere. See fig. 2.

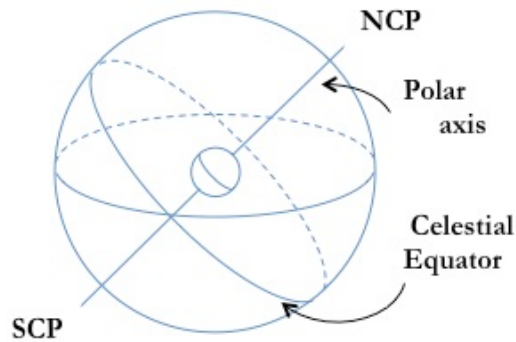


Figure 3.2 — Celestial equator, poles.

The celestial equator divides the celestial sphere in half, into a northern and a southern hemisphere. Parts of both of these may be above your horizon, depending on your location on Earth.

Let us now combine these two drawings (Figure 3); the celestial equator and the horizon intersect at the east and west points on the horizon. The celestial equator and the celestial meridian intersect at the point labeled Q.

As the Earth rotates daily, the celestial sphere appears to turn from east to west around the celestial poles, and so the stars, Sun, Moon, and planets move across the sky from east to west; this is called their **diurnal motion**. When an object reaches its highest point in the sky and crosses the celestial meridian, it is said to **transit**. The celestial sphere requires approximately 24 hours – actually  $23^{\text{h}}56^{\text{m}}$  – for one complete turn.

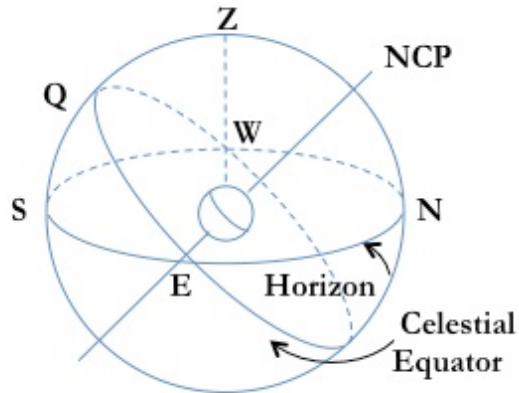


Figure 3.3 — figures 1 and 2, combined.

Why not 24 hours? As the Earth rotates once on its axis it also revolves about one degree around the Sun. With respect to the distant stars, the Earth rotates in  $23^{\text{h}}56^{\text{m}}$ ; this is the Earth's *sidereal* period. The extra four minutes is the time required to rotate just a bit further so that we return to the same orientation with respect to the Sun; 24 hours is the Earth's *synodic* period.

### Celestial Coordinate Systems

Any coordinate system on a sphere has to have two things to define it: a great circle (a circle whose plane passes through the center of the sphere) and an arbitrary starting point along that circle. The horizon, celestial equator, and celestial meridian are all great circles. There are several astronomical coordinate systems; the most useful for us will be the horizon system and the equatorial system.

## I. Horizon System

In this system the fundamental circle is the horizon and the arbitrary starting point is the north point on the horizon. We define two coordinates: **Altitude** is measured in degrees above (+) or below (–) the horizon; the zenith has an altitude of  $90^\circ$ . **Azimuth** is measured around the horizon from north toward the east and ranges from  $0^\circ$  to  $360^\circ$ ; west has an azimuth of  $270^\circ$ .

In figure 4, the arc RX is the altitude of object X; the arc NER is the azimuth of object X. In this case, it looks like its altitude is about  $45^\circ$  and its azimuth about  $140^\circ$ . The advantage of this system is that it's easy to estimate the coordinates of a star or other celestial object since you can tell where the horizon and north are. The disadvantage is that because of the rotation of the Earth, the altitude and azimuth of your object change continually.

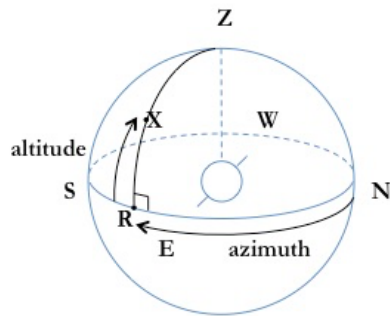


Figure 3.4 — altitude and azimuth coordinates.

## II. Equatorial System

In this system the fundamental circle is the celestial equator and the arbitrary starting point is the vernal equinox. The **vernal equinox** is the point on the sky where the Sun crosses the celestial equator on the first day of spring in the northern hemisphere, about March 21. For now, you can consider it to be basically a fixed point on the celestial equator. The two coordinates in this system are defined as: **Declination**, which is the angular distance north or south from the celestial equator to your object. It is measured in degrees along an **hour circle**, a great circle crossing the celestial poles and perpendicular to the equator. The south celestial pole has a declination of  $-90^\circ$ .

**Right Ascension** is measured along the celestial equator eastward from the vernal equinox to the object's hour circle. It is always positive and may be expressed either in degrees or in hours and minutes (the latter is more common, if a bit more confusing; the  $360^\circ$  corresponds to 24 hours, so  $1^h = 15^\circ$  or  $1^\circ = 4^m$ ). The autumnal equinox (where the Sun is in September) has an RA of  $12^h$ .

In figure 5 the arc RX is the declination of object X and the arc  $\gamma$ R is its right ascension. The arc from the NCP through X to R is part of the object's hour circle. This object appears to have a declination of about  $70^\circ$  and an RA of about  $1^h$ .

The advantage to this system is that it is more or less permanent, not dependent on your location or the time of day or season. Because of this it is very useful for making catalogues and atlases of the sky. The disadvantage is that it's hard to relate it to the sky you're looking at: where is the celestial equator, anyway? And how about the vernal equinox? This latter question is especially tricky, because, like everything else in the sky, the vernal equinox moves diurnally around the sky as the Earth rotates. So locating it in the sky at any given moment is not real obvious. We do it by specifying where the vernal equinox is relative to the celestial meridian, which means we need to define a supplementary coordinate to connect the equatorial system with the sky as you are observing it:

**Hour Angle** is the arc along the celestial equator from the celestial meridian to an object's hour circle. Note that the starting point, the celestial meridian, is tied to you. Hour angle is negative (–) for an object that is east of the celestial meridian (rising) and positive (+) for an object after it transits the meridian. Like RA, HA is usually expressed in hours and minutes. Due to the rotation of the Earth, the hour angle of an object changes uniformly at a rate of  $15^\circ$  (i.e., 1 hour) per hour of time. An object on the celestial meridian has  $HA = 0^h$ ; this increases to  $12^h$  and

then for the remaining half of the rotation of the celestial sphere it is counted as negative, from  $-12^h$  back to  $0^h$  (although you could continue with positive values, from  $12^h$  to  $24^h$ ).

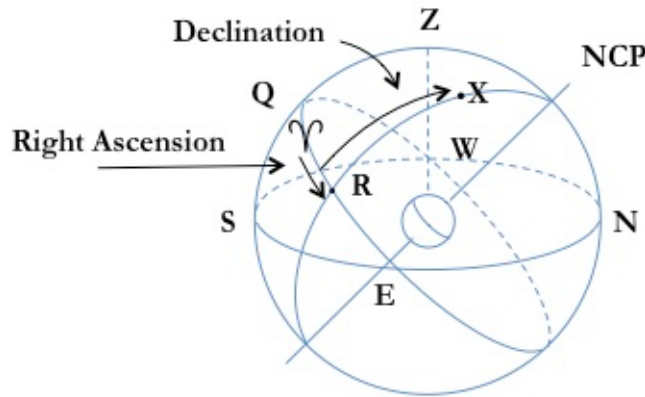


Figure 3.5 — RA and Dec

In figure 5, the hour angle of the vernal equinox is the arc  $Q\gamma$  (about  $-45^m$ ) and the hour angle of object X is the arc  $Q\gamma R$  (about  $-1^h45^m$ ). Both cases in this drawing are negative, since both objects are east of the celestial meridian.

As the Earth rotates, then, the vernal equinox moves around the sky like everything else. We can specify where it is relative to your celestial meridian by giving its hour angle, and we call this the **sidereal time**. Sidereal time is the hour angle of the vernal equinox; it is  $0^h$  when the vernal equinox transits,  $6^h$  when it sets,  $18^h$  when it rises; ST doesn't use a.m./p.m. (or  $+/-$ ) designations, it just increases steadily from  $0^h$  to  $24^h$ . The ST describes a positioning of the celestial sphere and its permanent coordinate system relative to you, the observer, and will allow you to use the RA coordinates to find objects in the sky.

Now, let's put ST together with RA: RA is measured eastward from the vernal equinox and  $ST = HA$  of the vernal equinox. This implies that ST is also the same as the RA of whatever object is on the celestial meridian. And after that object transits, say, its HA increases at the same rate as the ST. So more generally, we can say that

$$ST = HA + RA$$

(as long as we're talking about the HA and RA of the same object!). Object X, back in figure 5, has an  $HA = -1^h45^m$  and its RA is  $1^h$ . So the ST is  $-1^h45^m + 1^h0^m = -45^m$  or  $23^h15^m$ . Note that this is what the HA of the vernal equinox appears to be in the drawing.

Example: Suppose that the star Vega,  $RA = 18^h35^m$ , is on the celestial meridian: What is the sidereal time?  $ST = HA + RA = 0^h$  (Vega's on the meridian) +  $18^h35^m$ , or  $18^h35^m$ . At that same moment, what is the hour angle of the star Deneb,  $RA = 20^h40^m$ ? Using Deneb's RA, solve for HA:  $18^h35^m = HA + 20^h40^m$ , whence  $HA_{Deneb} = -2^h05^m$ . Since this is negative, Deneb is east of the celestial meridian, still rising, and will transit in  $2^h05^m$ .

Another example: What is the sidereal time when Arcturus,  $RA = 14^h13^m$ , is  $3^h$  west of the meridian?  $ST = +3^h00^m + 14^h13^m = 17^h13^m$ . In other words, the vernal equinox has an hour angle of  $17^h13^m$  or  $-6^h47^m$  and is therefore just below the eastern horizon.

We don't have to use the HA and RA of a star to calculate the sidereal time, we can use the Sun, too, although this is trickier since the Sun's RA changes throughout the year as we revolve in our orbit around it. The Sun's HA changes through the day as the Earth rotates and is thus indicated pretty closely by the time of day, as long as you don't forget that at 12 noon the Sun has an HA of  $0^h$ .

First the right ascension: The Sun's RA is  $0^h$  on March 21 (recall that we define the vernal equinox as the point where the Sun's path crosses the equator, so the Sun is at the vernal equinox—a point in the sky,  $RA = 0$ —on

the *day* of the vernal equinox.) Thereafter it increases by about  $2^h$  per month as we revolve around the Sun. That's about  $1^h$  every two weeks or  $4^m$  per day. A table of date vs. the Sun's RA would give:

Date	RA <sub>Sun</sub>	Date	RA <sub>Sun</sub>	Date	RA <sub>Sun</sub>
21 March	0	21 July	8	21 November	16
21 April	2	21 August	10	21 December	18
21 May	4	21 September	12	21 January	20
21 June	6	21 October	14	21 February	22

If you could see the stars on the celestial sphere behind the Sun, you would see the Sun steadily moving through the constellations of the zodiac. In March, at the vernal equinox, the Sun is in Pisces. By mid-April it's in Aries, by mid-May, Taurus, etc.

You can of course interpolate between the dates in the table above; thus on April 7 the Sun's RA is about  $1^h$ , on July 7 it's about  $7^h$ , on Oct. 28 it's about  $14^h30^m$  and so forth. If the RA of the Sun were  $8^h40^m$ , what would the date be? At  $4^m$  per day, that's 10 days after July 21, so it would be July 31.

Now for relationship between hour angle and time of day: the Sun's hour angle is given by the time of day *provided* you use **local solar time**; i.e.,

$$t = HA_{\odot} + 12^h.$$

This gives us solar time on a  $24^h$  clock. For example, at 9 a.m.,  $HA_{\odot} = -3^h$ , and  $t = -3 + 12 = 9:00$  a.m.; at 5 p.m.,  $HA_{\odot} = +5^h$ , and  $t = +5 + 12 = 17:00$  (5:00 p.m.).

Example: If the  $HA_{\odot}$  is  $-4^h20^m$ , what is the local solar time?  $t = -4:20 + 12:00 = 7:40$  a.m. Check that this makes sense: yes, according to the hour angle the Sun is still 4+ hours from reaching the meridian, so it should be early morning.

If you are not at the central longitude of your time zone you may have to correct for the difference between your local time and your time zone time. More on this below, but note here that each degree of longitude corresponds to 4 minutes difference in time. Walla Walla, WA, is at  $118^{\circ}$  W longitude,  $2^{\circ}$  east of the center of our time zone. Our local time is thus 8 minutes *later* than our time zone time.

Remember that what we are after is getting the sidereal time using the Sun. If we combine our equations we can get the sidereal time from the date and the time of day:  $ST = HA_{\odot} + RA_{\odot}$ . This should look ok: ST is the HA + RA of any object, including the Sun.

Example, what's the sidereal time at 8 p.m. on June 21? On June 21,  $RA_{\odot} = 6^h$ . At 8 p.m., the  $HA_{\odot} = +8^h$ . Therefore the  $ST = 8 + 6 = 14^h$ . So the vernal equinox has an HA of  $-10 (= 14)$  and is well below the eastern horizon.

### Time and Your Latitude and Longitude

Let's go back now to our combined drawing of the horizon and equator; figure 6 is similar to figure 3, with added emphasis. Note that in the concentric spheres of the Earth and sky, the celestial arc QZ is the same as your **latitude**, i.e., the number of degrees you are away from the terrestrial equator is the same as the number of degrees your zenith is from the celestial equator. Thus the arc Z-NCP must be  $(90^{\circ} - \text{lat.})$ , and since  $ZN = 90^{\circ}$ , this means that the arc NCP-N is the same as your latitude also. Thus it is always true that your latitude is the same as the

altitude of the pole, or, in the northern hemisphere, roughly the altitude of Polaris, the North Star. So an estimate of the altitude of Polaris will give you an estimate of your latitude. In the days pre-GPS, this was wildly important for navigation. It's a bit harder in the southern hemisphere, where there is no bright pole star.

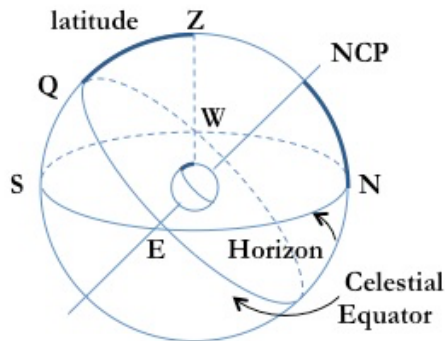


Figure 3.6 — latitude projected onto the sky.

You can even do this latitude calculation if you don't have Polaris readily visible. For objects that happen to be transiting the celestial meridian, we can easily relate the observer's latitude, the object's declination, and its altitude (or its zenith distance, which is  $[90^\circ - \text{lat.}]$ ). Given any two of these quantities, you can figure out the third, by drawing a picture of the meridian (essentially a simplified celestial sphere drawing).

Example. First, suppose that Altair (dec  $+9^\circ$ ) is on the celestial meridian  $15^\circ$  south of the zenith. What is your latitude? Draw the picture (Figure 7) and put in what you know:

First, you know that the star is  $15^\circ$  south of the zenith; so draw that in. Then you know that it's  $9^\circ$  north of the celestial equator, so you can put in the equator, which locates the point Q. And we've seen that the arc QZ is the same as your latitude; so here QZ must be  $15^\circ + 9^\circ = 24^\circ$ . Note that since the celestial equator is to the south of your zenith, that must mean that the NCP is above your northern horizon, so you are in the northern hemisphere. If you wanted, you could now put in the NCP,  $24^\circ$  above the northern horizon.

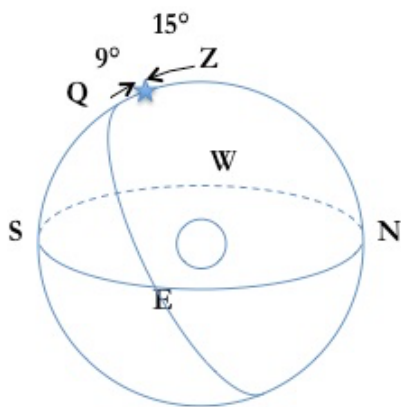


Fig. 3.7a

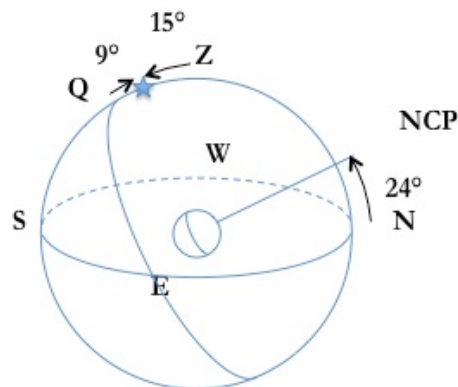


Fig. 3.7b

Another example. What if Altair were crossing your meridian  $20^\circ$  north of your zenith? In that case, where are you?

Draw another picture and put in what you know: the star is  $20^\circ$  north of the zenith and the celestial equator is  $9^\circ$  south of the star. That locates Q, and you can see that QZ is  $11^\circ$ . But this time the celestial equator is north of your zenith, so you are in the southern hemisphere, at a latitude of  $11^\circ$  S (or  $-11^\circ$ ) and the SCP would be  $11^\circ$  above your southern horizon. The NCP will be below the northern horizon. See figure 8.

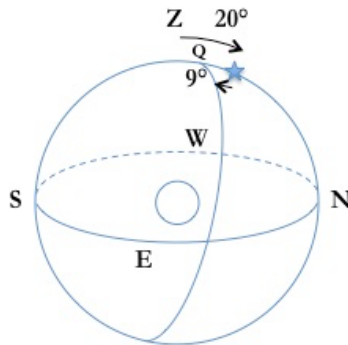


Fig. 3.8a

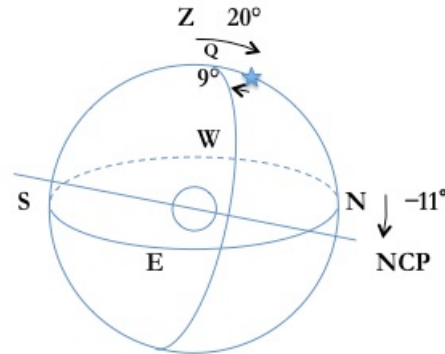


Fig. 3.8b

If a star is **circumpolar**, that means that it never goes below the horizon during its diurnal motion (or it could refer to stars that are so close to the other pole that they never come above the horizon). How can you tell whether a particular star is circumpolar for you or not? It means, for us in the northern hemisphere, that the star must be close enough to the NCP (have a large enough declination) that even at its lowest it never drops out of sight. In other words, the NCP must be higher above the horizon than the distance of the star from the NCP. This latter is called the star's **north polar distance**, or NPD, equal to  $(90^\circ - \text{dec.})$ . The altitude of the NCP, recall, is the same as your latitude. So the criterion for a star being circumpolar is that its NPD must be less than your latitude.

Figure 9 shows a circumpolar star X at both its highest position,  $X_1$ , and at its lowest position,  $X_2$ , along its diurnal path. Even at  $X_2$  it is still above the horizon. Its NPD (the arc NCP - X) is less than the altitude of the NCP (the arc NCP - N). So this star is circumpolar. But you can see that a star on the celestial equator would not be circumpolar, since half its circle would be below the horizon

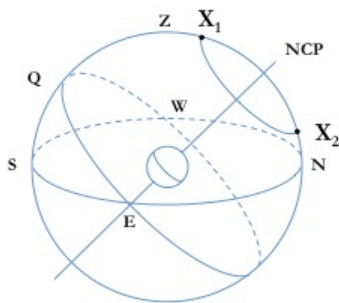


Figure 3.9 — circumpolar star.

Example: would Altair be circumpolar if you were at  $24^\circ$  N? No, because Altair's  $\text{NPD} = 90^\circ - 9^\circ = 81^\circ$ , and the altitude of the NCP is only  $24^\circ$ . For Altair to be circumpolar you'd have to be at a latitude of  $81^\circ$  N or higher, way up in the Arctic. You could draw yourself a picture to convince yourself of this.

So now you can find latitudes. To find your **longitude**, east or west of the Greenwich meridian, you need to look at local times. For those pre-GPS navigators, finding longitude is not nearly as easy as finding latitude! You have to have a good clock, because the difference between your longitude and Greenwich (whose longitude is  $0^\circ$ ) is the same as the difference between your local solar time (LST) and Greenwich Mean Time (GMT), the local solar time at Greenwich. Thus if you determine your LST ( $\text{HA}_\odot + 12^{\text{h}}$ , recall) and compare that to a GMT clock, the

difference is your longitude;  $\Delta t = \Delta \text{longitude}$ . If you are west of Greenwich, your local time is earlier than GMT; if you are east of Greenwich, your local time is later than GMT. (You could think about watching some live event on TV; if you are in the Pacific time zone and the event is in the Eastern time zone, it's later there than it is where you are.)

Examples:

If your GMT clock reads 21:20 (i.e., 9:20 p.m. in Greenwich), and your own local time is noon, what is your longitude? The time difference is  $9^{\text{h}}20^{\text{m}}$  (21:20–12:00), so your longitude is  $140^{\circ}$  (recall that  $1^{\text{h}} = 15^{\circ}$  and  $4^{\text{m}} = 1^{\circ}$ ) and it's west since your time is earlier than GMT.

Walla Walla is  $118^{\circ}$  west of Greenwich; if the LST here is 8:00 a.m., what time is it in Greenwich? Since our longitude is  $118^{\circ}$  or  $7^{\text{h}}52^{\text{m}}$  west of Greenwich, our time is 7:52 earlier than theirs; so the GMT is  $8:00 + 7:52 = 15:52$  or 3:52 p.m.

When it's 1 p.m. local time in Walla Walla, what time is it in Tokyo, longitude  $140^{\circ}$  E? The longitude difference, around to the east, is  $118^{\circ} + 140^{\circ} = 258^{\circ}$ , putting them  $17^{\text{h}}12^{\text{m}}$  later than here. So their time is  $13:00 + 17:12 = 30:12 = 6:12$  a.m. tomorrow. Note that we could also get this by saying that Japan is equivalently  $6^{\text{h}}48^{\text{m}}$  west of us, so its time would be 6:48 earlier than ours, or  $13:00 - 6:48 = 6:12$  a.m. again. Here, though, you have to remember that going west you cross the International Date Line, so you jump ahead a day to get 6:12 a.m., consistent with the eastward result.

When it's 9:03 a.m. in Walla Walla, what is the LST in Seattle (longitude  $123^{\circ}$  W)? The longitude difference is  $5^{\circ}$  or 20 minutes, so it's 20 minutes earlier in Seattle, or 8:43 a.m.

**Time Zones** exist to avoid things like having to change your watch by 20 minutes every time you go from Walla Walla to Seattle. You change your time zone time only every  $15^{\circ}$ , and then by 1 hour. Everybody in one time zone keeps the same time. In Seattle or Walla Walla, that is Pacific Standard Time (PST), and is the local time on the  $120^{\circ}$  W meridian. Thus Walla Walla's LST is  $8^{\text{m}}$  later than PST and Seattle's is  $12^{\text{m}}$  earlier, but our watches all read the same (more or less!) and we all keep standard time rather than local time.

When we are on Daylight Savings Time (April to October), we set our clocks *ahead* one hour: Pacific Daylight Time (PDT) = PST +  $1^{\text{h}}$ . Most states in the U.S. do this, except Arizona, Hawaii, and parts of Indiana. Many European countries do also; they often call it Summer Time.

Some time zones of interest:

$150^{\circ}\text{W}$ - Hawaiian Standard Time	= $10^{\text{h}}$ earlier than GMT	Honolulu
$135^{\circ}\text{W}$ - Alaskan Standard Time	= $9^{\text{h}}$ earlier than GMT	Anchorage
$120^{\circ}\text{W}$ - Pacific Standard Time	= $8^{\text{h}}$ earlier than GMT	Seattle, SF, LA
$105^{\circ}\text{W}$ - Mountain Standard Time	= $7^{\text{h}}$ earlier than GMT	Denver, Albuquerque
$90^{\circ}\text{W}$ - Central Standard Time	= $6^{\text{h}}$ earlier than GMT	Chicago, St. Louis
$75^{\circ}\text{W}$ - Eastern Standard Time	= $5^{\text{h}}$ earlier than GMT	NY, Boston, D.C.
$0^{\circ}$ - Greenwich Mean Time	= GMT	London
$15^{\circ}\text{E}$ western Europe	= $1^{\text{h}}$ later than GMT	Paris, Bonn
$30^{\circ}\text{E}$ eastern Europe	= $2^{\text{h}}$ later than GMT	Athens, Istanbul
$45^{\circ}\text{E}$ Arabia	= $3^{\text{h}}$ later than GMT	Baghdad
$135^{\circ}\text{E}$ Japan	= $9^{\text{h}}$ later than GMT	Tokyo



**“Real” solar time**

Due to the eccentricity of the Earth’s orbit the speed of the Earth along its orbit varies through the year. This means that the rate at which the Sun moves around the celestial sphere varies; it isn’t in the same place in the sky as it was 24 hours ago. The variation in solar declination introduced by the  $23.5^\circ$  tilt of the Earth’s axis also means that a “day”, meaning the length of time from sunrise to sunset, varies considerably from 12 hours over the course of the year (how much it varies depends on your latitude, as well.) The local solar time that we’ve been discussing is the time kept by a fictitious object known as the **mean Sun**, which keeps perfectly regular time along the celestial equator.

These effects (and other very minor perturbations in the Earth’s motion) combine so that over the year the true Sun is sometimes fast, sometimes slow, with respect to the mean Sun. The **equation of time** is the difference between **apparent solar time** (what your sundial gives you) and local mean solar time. The maximum differences occur about the middle of February and the beginning of November, when the apparent solar time is about 15 minutes behind or ahead of, respectively, the mean time. Unless you always wanted to know what that figure-8-ish line was on your world globe (it’s called the *analemma* and it gives the equation of time), this isn’t of much practical importance! On the other hand, in the dark couple of weeks after the winter solstice, when you are wondering why the Sun has yet to start rising earlier in the morning even though you know that the days are getting longer, you can now blame the equation of time.

Math note: Let’s consider that latest sunrise concept a little further. The shortest day of the year for the northern hemisphere is the winter solstice, about Dec. 21<sup>st</sup>. The date of the Earth’s perihelion, when it is closest in its orbit to the Sun and moving the fastest, is about Jan. 3<sup>rd</sup>. In early December the fact that the Sun’s declination is still getting farther south from day to day means that the Sun will rise later from day to day. How much later depends on your latitude and the angle the Sun’s path makes with your horizon. Regardless of the declination, the fact that we are near perihelion means that over the course of a day the Sun will move a bit farther to the east than usual along its path among the stars. That fact means, also, that it will take a bit longer to get from one sunrise to the next. After we pass the winter solstice the Sun’s declination starts moving north again but we have a few more weeks until perihelion, and this means that we have to wait a while for the Sun to start rising earlier in the morning. As mentioned above, the dates of earliest sunset and latest sunrise depend on your latitude – think about being above the Arctic circle, where the sun rises on ~ March 21<sup>st</sup> and sets on ~ September 21<sup>st</sup>. For those of us living at ~46 degrees north latitude, the earliest sunset occurs on about December 8<sup>th</sup> and the latest sunrise on about January 8<sup>th</sup>.

**Sample problems:**

1. If the sidereal time is 14:30, what is the hour angle of the vernal equinox? Is it up?
2. If Rigel (RA =  $5^{\text{h}}12^{\text{m}}$ ) is crossing the celestial meridian, what is the sidereal time?
3. If Deneb (RA =  $20^{\text{h}}40^{\text{m}}$ ) is observed  $5^{\text{h}}37^{\text{m}}$  west of the meridian, what is the sidereal time?
4. If the sidereal time is 16:40, what is the hour angle of Fomalhaut (RA =  $22^{\text{h}}55^{\text{m}}$ )?
5. What is the Sun’s approximate RA on the following dates?

June 7

Jan. 22

Sept. 28

6. What is the Sun's hour angle at the following local times?  

8:15 p.m.
8:15 a.m.
noon
18:30
7. What is the sidereal time on  

Dec. 28 at 3:45 a.m.
July 7 at 6 p.m.
8. If Deneb (Dec.  $+45^\circ$ ) is on the celestial meridian  $9^\circ$  south of the zenith, what is your latitude?
9. If Deneb is on the celestial meridian  $54^\circ$  north of the zenith, what is your latitude?
10. From Walla Walla (lat.  $+46^\circ$ ), what is the altitude of Deneb when it transits?
11. From Walla Walla, what is the altitude of the south celestial pole? What is the declination of a star at the zenith? What is the farthest south dec. you can see?
12. If you are in Los Angeles (lat.  $34^\circ\text{N}$ ) and observe a star crossing the celestial meridian  $22^\circ$  above the southern horizon, what is that star's declination?
13. From Walla Walla, is the star at the end of the handle of the Big Dipper (its dec. is  $+50^\circ$ ) circumpolar?
14. From Hawaii (lat.  $+20^\circ$ ) can you see the star  $\alpha$  Centauri (dec.  $-61^\circ$ )? Can you see it from Walla Walla? (If you're interested, that sort of notation means that we are referring to the brightest— $\alpha$ —star in the constellation Centaurus.)
15. When it's noon in Walla Walla, the local time in Louisville is 2:12 p.m. What is the longitude of Louisville?
16. When it's 3:45 p.m. in London, what is the PST? What is the standard time in Baghdad?

Sample problem solutions are on the next page:

1. The sidereal time = HA of the vernal equinox, given as 14:30. Therefore if the ST = 14:30, so does the HA of the VE = 14:30. Is it up? No – recall that 0<sup>h</sup> is transiting the celestial meridian, 6<sup>h</sup> is setting, 12<sup>h</sup> is down, 18<sup>h</sup> is rising, so 14:30 is not up.
2. Recall that sidereal time more generally is  $ST = HA + RA$  (of the same object). Rigel is on the celestial meridian so its HA = 0. Thus  $ST = 0 + 5^h12^m$ . This is an example where we use the fact that the  $ST = RA$  of whatever is transiting the celestial meridian.
3. Deneb, with  $RA = 20^h40^m$ , is  $5^h37^m$  west of the celestial meridian. West means that Deneb's HA is positive.  $ST = HA + RA = 25:77$ . . . which is larger than 24! Fix this by subtracting 24 and adjusting that 77 minutes:  $25:77 = 26:17 = 2:17$ .
4. What's the HA of Fomalhaut, with  $RA = 22^h55^m$ , if the  $ST = 16:40$ ?  $ST = HA + RA$  becomes  $16:40 = HA + 22:55$ , or  $HA = 16:40 - 22:55$ . Negative hours are hard to deal with; you might turn hours:minutes into decimal hours, which are easier to subtract.  
 $HA = 16:40 - 22:55 = 16.67 - 22.92 = -6.25 = -6:15$ .

5. The Mean Sun's RA on various dates; recall that on March 21<sup>st</sup> the Sun's RA = 0<sup>h</sup> and that it increases by 2 hours per month.

date	June 7	Jan. 22	Sept. 28
RA	5 hours	20 hours	12 <sup>h</sup> 30 <sup>m</sup>

6. The Mean Sun's HA at various times of day; recall that 0<sup>h</sup> HA is at 12:00 (i.e., noon) and that HA increases as the day goes on, with the Mean Sun setting at 18:00 hours (6 p.m.) with an HA of +6.

time	8:15 p.m.	8:15 a.m.	noon	18:30
HA	8 <sup>h</sup> 15 <sup>m</sup>	20 <sup>h</sup> 15 <sup>m</sup> or -3 <sup>h</sup> 45 <sup>m</sup>	0	6 <sup>h</sup> 30 <sup>m</sup> , i.e., it's 6:30 pm

7. Combinations of date and time.

date and time	Dec. 28 at 3:45 a.m.	July 7 at 6 p.m.
RA	$RA_{\text{sun}} = 18:30$	$RA_{\text{sun}} = 7:00$
+ HA	$HA_{\text{sun}} = 15:45$	$HA_{\text{sun}} = 6$
= ST	$ST = 33:75 = 34:15 = 10:15$	$ST = 13^h$

8. Deneb, with declination = +45°, is on the celestial meridian 9° south of the zenith. Recall that the latitude = declination of the zenith. Thus latitude = 45 + 9 = 54°.
9. Suppose Deneb instead is 54° north of the zenith. From Deneb it's 45 degrees south to get to the equator and another 9 degrees to get to the zenith. Latitude is degrees from the equator, so latitude = 9° south.
10. Deneb is transiting the meridian for latitude = 46°. At 46 degrees latitude, the equator crosses the celestial meridian at an altitude of 44 degrees. Deneb is 45 degrees north of the equator. Its altitude when it transits is 44 + 45 = 89°.
11. From 46°, what is . . .
  - a) altitude of SCP? 46° below southern horizon, i.e., directly opposite the NCP;
  - b) declination of a star at the zenith? 46°, because latitude = declination of zenith;
  - c) lowest declination we can see? 44° south, because the equator crosses our CM at 44° altitude, so a star 44° below the equator would just reach our southern horizon.

12. At latitude  $34^\circ$  north, a star transits CM  $22^\circ$  above the southern horizon. At  $34^\circ$  north the equator crosses the meridian at  $56^\circ$  altitude. That's 34 degrees higher than the star, so the star must be at  $34^\circ$  south declination.
13. From  $46^\circ$  latitude is declination  $+50^\circ$  circumpolar? Yes; anything closer to the NCP than your latitude is circumpolar, so anything north of  $44^\circ$  declination is circumpolar.
14. Can you see  $\alpha$  Centauri, dec.  $-61^\circ$  from. . .
  - a) Walla Walla? no – see #11; nothing further south than  $-44^\circ$  is visible;
  - b)  $20^\circ$  north? yes – at  $20^\circ$  the CM crosses at an altitude of  $70^\circ$ , so a star with a declination of  $-61^\circ$  would cross at an altitude of  $9^\circ$ .
15. When it is noon at  $118^\circ$  west longitude it is 2:12 local time in Louisville. That's 2.2 hours to the east, and each hour of time corresponds to  $15^\circ$  of longitude. Thus Louisville is at  $2.2^h \cdot 15^\circ/h = 33$  degrees east of Walla Walla, or  $118 - 33 = 85^\circ$  W longitude.
16. At  $15^h45^m$  GMT (3:45 p.m.), the standard . . .
  - a) Pacific time is 8 hours earlier, or 7:45 a.m.;
  - b) Baghdad time is 3 hours later, or 18:45 (6:45 p.m.).