Section 2.6

1. Find the derivative of $f(x) = \int_0^{3x} \ln(1+t^2)\,dt$.

Using the FTC and the chain rule, we find that

$$f'(x) = \ln(1+(3x)^2)\cdot 3 = 3\ln(1+9x^2).$$

2. Find $F''(\pi/4)$ given $F(x) = \int_x^1 f(t)\,dt$ and $f(x) = \int_1^{2x} \frac{\sin t}{t}\,dt$.

Note that $F(x) = -\int_1^x f(t)\,dt$. By the FTC

$$F'(x) = -f(x)$$

$$F''(x) = -f'(x) = -\frac{\sin 2x}{2x}\cdot 2 = -\frac{\sin 2x}{x}.$$

We also used the chain rule for $f'(x)$

It follows that $F''(\pi/4) = -\frac{\sin(\pi/2)}{\pi/4} = -\frac{4}{\pi}$.

3. Evaluate $\lim_{x\to 0} \frac{1}{x^4}\int_0^x \sin(2t^3)\,dt$.

$$\lim_{x\to 0} \frac{1}{x^4}\int_0^x \sin(2t^3)\,dt = \lim_{x\to 0} \frac{\int_0^x \sin(2t^3)\,dt}{x^4} \qquad \frac{0}{0} \text{ form}$$

$$= \lim_{x\to 0} \frac{\sin(2x^3)}{4x^3} \qquad \text{L'Hopital's Rule FTC}$$

$$= \lim_{x\to 0} \frac{6x^2\cos(2x^3)}{12x^2} \qquad \text{L'Hopital's Rule again}$$

$$= \lim_{x\to 0} \frac{\cos(2x^3)}{2} \qquad \text{simplify}$$

$$= \frac{1}{2}.$$