Math 456 Assignments for Spring 2016 (due on given date)

for Tuesday, January 19

1. Read the introduction to Chapter 5 and Section 5.1. Since this material should be review (as well as a preparation for orals), it should not take too long to read. Do think carefully about the ideas and pay close attention to the notation.
2. Do exercises 2, 3, 6, 8, 11, 12, 17, and 18 in Section 5.1.
3. Turn in a carefully written solution for Exercise 5.1.11b, where the (b) refers to part (b) of Theorem 5.5.

for Wednesday, January 20

1. Review Section 5.1 if necessary.
2. Do exercises 9, 10, 19, 20, and 21 in Section 5.1. Be prepared to discuss the solutions to these exercises in class.

for Friday, January 22

1. Read Section 5.2 very carefully. Note that this reading may take 90 minutes or so.
2. Do exercises 2, 4, 10, and 13 in Section 5.2. Note that Exercise 2 appears before Exercise 4 (duh), which asks for a proof of the converse of the Cauchy criterion. This means (see the preface) that you are to do Exercise 2 without using the Cauchy criterion.
3. Turn in solutions for exercises 4 and 10.

for Tuesday, January 26

1. You should consider rereading Section 5.2 to make certain the ideas are clear.
2. Do exercises 7, 14, 15, 19, and 20 in Section 5.2.
3. Turn in solutions for exercises 7 and 19 in Section 5.2.

for Wednesday, January 27

1. Read Section 5.3 through the statement of Corollary 5.16.
2. Do exercise 18 in Section in 5.2 (note the need for Corollary 5.16) and exercises 6, 7, and 8 in Section 5.3.

for Friday, January 29

1. Finish reading Section 5.3.
2. Do exercises 10, 11, 12 (you should NOT use the Fundamental Theorem of Calculus for any of these exercises), 13, 17, 18, 26, and 28 in Section 5.3.
3. Turn in solutions for exercises 10 and 17 in Section 5.3.

for Tuesday, February 2

1. Read Section 5.4 through the proof of Theorem 5.24.
2. Do exercises 1, 2, 3, 4, 5, 6, 7, and 8 in Section 5.4.
3. Turn in solutions for exercises 5 and 6. For exercise 5, look carefully over previous results on integration as well as the version of the Bolzano-Weierstrass Theorem presented in Chapter 2 of our textbook. For exercise 6, you may use the Fundamental Theorem of Algebra if you find it helpful. This is the first of our no help assignments. To be specific (perhaps overly so given some past difficulties) concerning what it means to work alone, you may not seek help from classmates, you may not seek help from other professors or students, you may not seek help from me, you may not seek help from any books other than the textbook (and your notes), you may not seek help from the Internet, and you may not seek help from Maple, Wolfram Alpha, or any electronic devices. Following these guidelines falls under the category of the plagiarism form that you signed when you first arrived at Whitman. Even talking briefly to someone about the problems or sharing notes is a violation of the policy. I want to see what you can do on your own with these problems and I want you to learn to rely on your own mental resources to solve a problem.

for Wednesday, February 3

1. Finish reading Section 5.4.
2. Do exercises 16, 17, 25, and 27 in Section 5.4. There is a mathematical error in exercise 25; see if you can determine and explain what it is.

for Friday, February 5

1. Each of you will present a solution to an exercise at the board. The presentations will be “chalk and talk” rather than powerpoint, etc. This means that you do not present the solution using complete sentences, but provide an explanation in a way to get across the main points. In addition to your presentation, turn in a carefully written solution to your exercise. The exercises are 5.6.1 (Kevin, use two different methods), 5.6.5 (Marissa), 5.6.13 (Tate), and 5.6.18 (Elliot). Be aware that these exercises are nontrivial.
2. As time permits, spend some time working on exercises 2, 3, 4, and 7 in Section 5.6.

for Tuesday, February 9

1. Read the remark preceding Exercise 5.6.45.
2. Do exercises 45–55 in Section 5.6. Note that there is some redundancy in the first few of these exercises.
3. Turn in solutions for exercises 53 and 55 in Section 5.6.
4. Ponder the following definition and think about how to finish the proof of the theorem that follows it. These ideas are needed in proving the converse of Exercise 5.6.55.

   Definition: Let δ be a positive function defined on the interval $[a, b]$. A δ-fine tagged partition of $[a, b]$ is a tagged partition ${}^tP = \{(t_i, [x_{i-1}, x_i]) : 1 \le i \le n\}$ of $[a, b]$ such that $[x_{i-1}, x_i] \subseteq \big(t_i - \delta(t_i), t_i + \delta(t_i)\big)$ for each $1 \le i \le n$.

   Theorem: If δ is a positive function defined on $[a, b]$, then there exists a δ-fine tagged partition of $[a, b]$.

   Proof: Let E be the set of all points $x \in (a, b]$ for which there exists a δ-fine tagged partition of $[a, x]$. The set E is clearly bounded. It is also nonempty since it contains the interval $(a, a + \delta(a))$—the one element set $\{(a, [a, x])\}$ is a δ-fine tagged partition of $[a, x]$ for each $x \in (a, a + \delta(a))$. Let $z = \sup E$ and note that $z \in [a, b]$. To complete the proof, it is sufficient to prove that z belongs to E and that $z = b$.

for Wednesday, February 10

1. Read Section 5.5 carefully.
2. Do exercises 8, 11, and 12 in Section 5.5.

for Friday, February 12

1. Read Section 7.1. Study the examples very carefully and do your best to understand this new abstract concept.
2. Do exercises 1, 2, 3, 5, 6, 8, 9, 13, 15b, 15c, and 16 in Section 7.1.
3. Turn in a solution for exercise 14 in Section 7.1.

for Tuesday, February 16

1. Read Section 7.2. Uniform convergence is a very important topic in real analysis.
2. Do exercises 3, 4, 5, 6, 7, and 9 in Section 7.2.
3. Turn in solutions for exercises 4c and 5 in Section 7.2.

for Wednesday, February 17

1. Read Section 7.3.
2. Do exercises 1, 2, 3, 4, and 5 in Section 7.3.
3. Turn in a solution for exercise 5b in Section 7.3.

for Friday, February 19

1. Review the first three sections of Chapter 7 if necessary.
2. Do exercises 6, 7, 9, 11, 12, 15, and 18 in Section 7.3.
3. Turn in a solution for exercise 6 in Section 7.3.

for Tuesday, February 23

1. Read Section 7.4.
2. Do exercises 1, 2, 4, 5, 16, and 23 in Section 7.4.
3. Turn in solutions for the following two exercises. This is the second of our no help assignments; see the guidelines for these assignments from the February 2 assignment.

i. Use the concept of δ-fine tagged partitions to prove that a function f that is continuous on $[a, b]$ is uniformly continuous on $[a, b]$.

ii. Present a solution to Exercise 7.7.33. If desired, you can streamline the steps outlined in the problem.

for Wednesday, February 24

1. Read Section 7.5 carefully.
2. Do exercises 1, 3a, 3b, 3e, 3f, 3g (find $f^{(82)}(0)$ also), 6, 7, and 12b in Section 7.5. These exercises are all computational in nature. Be prepared to present your solutions on the board.

for Friday, February 26

1. Read Section 7.6 through the statement of Theorem 7.29.
2. Do exercises 1, 3, 4, 5a, 6, and 7 in Section 7.6.
3. Turn in a solution for exercise 3 in Section 7.6.

for Tuesday, March 1

1. Finish reading Section 7.6. Do your best to make sense of the proofs for these two important results in analysis.
2. Spend some time reviewing the material we have covered thus far during the semester in preparation for the exam on March 2.

for Wednesday, March 2

1. We have an exam on the material that we have covered in Chapters 5 and 7. You should do the following to prepare for the exam.

i. Be familiar with the definitions of the basic concepts and the statements of the important theorems.

ii. Be able to write proofs for the results in Theorems 5.12, 5.17, 5.23, and 7.8. This is not intended to be an exercise in memorization but rather an opportunity for you to understand the main idea of each proof and then be able to reproduce some correct variation of the proof.

iii. Be familiar with the types of problems we have been doing so that you can solve similar ones or use related ideas to solve novel problems.

iv. The take-home portion of the exam is due at the beginning of class.

for Friday, March 4

1. Read the introduction to Chapter 8 and Section 8.1.
2. Do exercises 1 through 10 in Section 8.1. Given your backgrounds, I believe that this reading and these exercises will go quickly. However, if not, we will slow down and go over the concepts carefully.
3. Turn in solutions for exercises 5 and 8 in Section 8.1. These two exercises are intended to have short and simple solutions.

for Tuesday, March 8

1. Read Section 8.2 carefully.
2. Work on exercises 1 through 12 in Section 8.2. Be prepared to present your solutions at the board.
3. Turn in solutions to exercises 9 and 12 in Section 8.2.
4. Begin working on the following two exercises. These two exercises represent the third of our no help assignments; see the guidelines for these assignments from the February 2 assignment. This assignment is due at the beginning of class on Friday, March 11.

i. Solve Exercise 8.1.38.

ii. Prove that the Cauchy product of the alternating harmonic series with itself converges. You may find that the sequence $\{\gamma_n\}$ defined in Exercise 5.2.18 is helpful.

for Wednesday, March 9

1. Review Sections 8.1 and 8.2 as necessary.
2. Work on exercises 25, 26, 27, 35, 36, 40, and 41 in Section 8.1, and exercises 18 and 21 in Section 8.2. Note that the function f in Exercise 8.2.21 is defined on $[a, b]$, has a right-hand limit at each point of $[a, b)$, and has a left-hand limit at each point of $(a, b]$.

for Friday, March 11

1. Read Section 8.3 through the proof of Theorem 8.24.
2. Work on exercises 1, 2, 4, 5, 12, 14, and 18 in Section 8.3. Be prepared to present your solutions at the board.
3. Turn in a solution to exercise 13 in Section 8.3.
4. The third special assignment is due at the beginning of class.

for Tuesday, March 29

1. Read Section 8.4 through the proof of Theorem 8.28.
2. Work on the following list of exercises: 28, 29, and 39 in Section 8.1; 18, 19, and 20 in Section 8.2; 23 and 34 in Section 8.3; and 1–7 in Section 8.4.
3. Turn in solutions for exercise 19 in Section 8.2 and exercise 5 in Section 8.4.
4. As we have done once before, each of you will present a solution to an exercise at the board. The exercises are 8.1.28/29 (Marissa), 8.1.39 (Tate), 8.2.18 (Elliot), and 8.3.23 (Kevin).
5. Begin working on exercises 7.7.6 and 8.4.27. These two exercises represent the fourth of our no help assignments; see the guidelines for these assignments from the February 2 assignment. This assignment is due at the beginning of class on Tuesday, April 5.

for Wednesday, March 30

1. Read Section 8.4 through the discussion of the Cantor set.
2. Work on exercises 8–13 in Section 8.4. Be prepared to discuss these exercises in class.

for Friday, April 1

1. Continue reading Section 8.4 through the proof of Theorem 8.37.
2. Work on exercises 18–24 in Section 8.4.
3. Turn in solutions for exercises 23 and 24 in Section 8.4. You may, of course, use previous parts of Theorem 8.34.

for Tuesday, April 5

1. Finish reading Section 8.4.
2. Work on exercises 30–37 in Section 8.4. Many of these exercises should go quickly.
3. Turn in a solution for exercise 34 in Section 8.4.
4. The fourth special assignment (see 3/29) is also due on this date.

for Wednesday, April 6

1. Read Section 8.5 through the 12 examples of metric spaces. Convince yourself that the triangle inequality holds for each of these metric spaces.

for Friday, April 8

1. Read Section 8.5 through the statement of Theorem 8.46.
2. Work on exercises 1–16 in Section 8.5. Some of these should go very quickly but others may give you pause. Begin by working on the exercises you find most interesting. Be prepared to discuss any solutions or questions that you have.
3. Turn in solutions for exercises 14 and 15 in Section 8.5.

for Tuesday, April 12

1. No class today due to the undergraduate conference; attend a few talks by your peers.

for Wednesday, April 13

1. Read Section 8.5 through the statement of Theorem 8.53.
2. Work on exercises 17–35 in Section 8.5. Keep track of any questions or difficulties that arise.
3. Turn in solutions for exercises 18e and 21ab. For 18e, you may use earlier parts of the exercise without proof. These two exercises represent the fifth of our no help assignments; see the guidelines for these assignments from the February 2 assignment. This assignment is due at the beginning of class on Wednesday, April 13.

for Friday, April 15

1. Work on exercises 36–43 in Section 8.5. Keep track of any questions or difficulties that arise.
2. Turn in solutions for exercises 36 and 43.

for Tuesday, April 19

1. Read Section 8.5 through the statement of Theorem 8.60.
2. Work on exercises 46–50 in Section 8.5. Keep track of any questions or difficulties that arise.
3. Turn in a solution for exercise 46.

for Wednesday, April 20

1. We have an exam on the portions of Chapter 8 that we have covered thus far. Look over the topics and exercises that we have discussed and focus on the larger picture.
2. Recall that there is a take-home portion of the exam due at the beginning of class. It consists of Exercise 8.2.20bc (you may assume part (a)), Exercise 8.4.39bcd, and Exercise 8.4.53 (note that this exercise is not quite as obvious as it first appears).

   The guidelines are the same as they were for the other take-home exam, namely,

   Write neat and careful solutions to each of the following problems. When appropriate, write your solutions using complete sentences and avoid the use of abbreviations and symbols. Each of these problems (I will count them as six problems) is worth 6 points. The ground rules for this portion of the exam are the same as those for special assignments (that is, you must work alone with no help from classmates, professors, books other than our textbook, or the Internet). Do not let the questions monopolize all of your time. This portion of the exam is due at the beginning of class on Wednesday, April 20.

for Friday, April 22

1. Look over the exercises that have been assigned thus far in Section 8.5 and make note of any that you would like to discuss.
2. Work on exercises 51–56 in Section 8.5; these should go fairly quickly.
3. Turn in a solution for exercise 54.

for Tuesday, April 26

1. Work on exercises 58–63e in Section 8.5. Give careful values for the computational parts of exercise 63.
3. Turn in solutions for exercises 58b and 63b.

for Wednesday, April 27

1. Work on parts f, g, and h of Exercise 8.5.63.

for Friday, April 29

1. Finish reading Section 8.5 to get a sense of the remaining topics.
2. Be prepared to discuss exercises 64–70 in Section 8.5.
3. Turn in a solution for exercise 8.5.77.
4. Note that a special assignment is due next Tuesday.

for Tuesday, May 3

1. Read the proof (sent as a pdf file) for Exercise 8.5.63i.
2. Read the rest of the exercises in Section 8.5 and determine which ones you might like to consider.
3. Turn in solutions for exercises 99ab and 103 in Section 8.5. These three exercises represent the sixth of our no help assignments; see the guidelines for these assignments from the February 2 assignment. This assignment is due at the beginning of class on Tuesday, May 3.

for Wednesday, May 4

1. Read the paper on the existence of bounded derivatives that are not Riemann integrable.
2. Read the CMJ article.
3. Ponder exercises 85, 91f, and 91g in Section 8.5.

for Friday, May 6

1. Add exercises 91e and 104 in Section 8.5 to your list of ponderings.

for reading days and beyond

1. Look over the topics that we have covered this semester.
2. Our final exam is scheduled for 2–5 pm on Monday, May 16. It will focus on ideas and concepts that you have learned over the semester as well as problems involving those ideas. In addition to knowing and being familiar with standard definitions, you should know how to prove the following:

   i. continuous functions and monotone functions are integrable

   ii. a proof of the Fundamental Theorem of Calculus

   iii. a proof of the standard Mean Value Theorem for integrals

   iv. a proof showing how uniform convergence preserves continuity

   v. the general form for power series and how to work with them

   vi. standard definitions, concepts, and results for metric spaces