Name: due October 7

Math 299 **Assignment 5** **Fall 2025**

Use your own paper to work on the problems listed below, then choose one of the problems to write a careful (rewritten and polished) solution on this page.

1. Prove that $\sum_{i=1}^{n}(-1)^{i+1}i^2 = \frac{1}{2}(-1)^{n+1}n(n+1)$ for each positive integer n.

2. Prove that $f_1f_2 + f_2f_3 + f_3f_4 + \cdots + f_{2n-1}f_{2n} = f_{2n}^2$ for each positive integer n.

3. Prove that $f_{n+1}f_{n-1} = f_n^2 + (-1)^n$ for each positive integer $n > 1$.

4. Find (by collecting data) and prove (using induction) a formula for $f_1 + f_4 + f_7 + \cdots + f_{3n-2}$ that is valid for all positive integers n.

5. Let $a_1 = 1$ and $a_{n+1} = 3 - (1/a_n)$ for each integer $n \geq 1$. Find (by collecting data) and prove (using induction) a formula for a_n involving Fibonacci numbers that is valid for all positive integers n.

Name: due September 30

Math 299 **Assignment 4** **Fall 2025**

Use your own paper to work on the problems listed below, then choose one of the problems to write a careful (rewritten and polished) solution on this page.

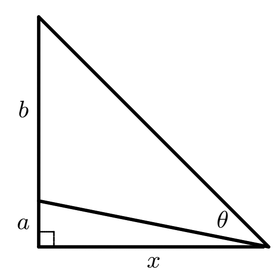
1. Prove that $2\cos(3\pi/7)$ is a root of the polynomial $z^3 - z^2 - 2z + 1$.
2. Prove that $\displaystyle\sum_{k=1}^{n-1} \cot^2(k\pi/n) = \frac{(n-1)(n-2)}{3}$ for all integers $n \geq 2$.
3. Use the result of the previous problem to prove that $\displaystyle\sum_{k=1}^{n-1} \csc^2(k\pi/n) = \frac{n^2-1}{3}$ for all $n \geq 2$.
4. Prove that the equation $\displaystyle\sum_{k=1}^{n-1} \frac{2^m}{\big(1 + i\cot(k\pi/n)\big)^m} = n$ is valid for all positive integers $n \geq m+1$.
5. Prove that $\displaystyle\sum_{k=0}^{n} \sin^2(k\pi/n) = \frac{n}{2}$ for all integers $n \geq 2$.
6. Use the result of the previous problem to find a formula for $\displaystyle\sum_{k=0}^{n} \cos^2(k\pi/n)$ for all $n \geq 2$.
7. Prove that $\displaystyle\sum_{k=0}^{n} \sin^4(k\pi/n) = \frac{3n}{8}$ for all integers $n \geq 2$.

Name: due September 23

Math 299 **Assignment 3** **Fall 2025**

Use your own paper to work on the problems listed below, then choose one of the problems to write a careful (rewritten and polished) solution on this page.

1. Prove that the area of the square on the side of a regular pentagon inscribed in a circle is equal to the sum of the areas of the squares on the sides of the regular hexagon and the regular decagon inscribed in the same circle.
2. Prove that $\csc(\pi/10) - \csc(3\pi/10) = 2$.
3. For a positive integer n, let $f_n(x) = \cos x \cos(2x) \cos(3x) \cdots \cos(nx)$. Find the smallest n such that $|f_n''(0)| > 2023$.
4. For the figure below, find the value of $x \in (0, \infty)$ that will maximize θ. Treat a and b as constants.

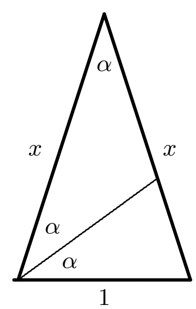


Name: due September 16

Math 299 **Assignment 2** **Fall 2025**

Use your own paper to work on the problems listed below, then choose one of the problems to write a careful (rewritten and polished) solution on this page.

1. Referring to the last figure in problem 5 from Class Period 2, use similar triangles to prove the Pythagorean Theorem.
2. Prove the law of sines (see problem 6 from Class Period 2).
3. Find and prove identities for $\sin(3x)$ and $\cos(3x)$.
4. Find the values of $\sin(\pi/5)$ and $\cos(\pi/5)$. One option is to note that $\sin(3\pi/5) = \sin(2\pi/5)$ and the figure below provides another option (using $\alpha = \pi/5$).



Name: due September 9

Math 299 **Assignment 1** **Fall 2025**

Use your own paper to work on the problems listed below, then choose one of the problems to write a careful (rewritten and polished) solution on this page.

1. Find the area of the region in the first quadrant bounded by the graph of $y = x^3$, the x-axis, and the tangent line to $y = x^3$ at the point (a, a^3), where a is a positive constant.
2. Evaluate $\displaystyle \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n+i}$.
3. Find the minimum distance from a point on the curve $y = 4/\sqrt{x}$ to the origin.
4. Let a and b be positive constants. Find an equation for the line that passes through the point (a, b) and cuts off the least area in the first quadrant.