Math 455: Real Analysis **Fall 2013**

for Tuesday, September 3

1. Read the preface and Section 1.1. Except for the last part of Section 1.1, this material should read rather quickly. (If you have not had abstract algebra, then just skim the material on fields.) Start now to really learn what it means to read and understand a math textbook as you will be doing quite a bit of this.
2. Do exercises 2, 4b, 7, 10, 11, 12, and 14 in Section 1.1. Many of the solutions to these exercises involve proof by contradiction. You may use the Fundamental Theorem of Arithmetic for exercises 11 and 14.
3. Turn in a carefully written solution to exercise 1.1.15; it should not be all that long. Be certain to include a (perhaps abbreviated) statement of the exercise and to use words and complete sentences in your proof. I ask that you work alone on this exercise (getting no help from any source other than the textbook) so that I can get a sense of your level of preparation for writing proofs.

for Wednesday, September 4

1. Read Section 1.2 through the discussion of geometric sums. Once again, most of this material should be familiar to you.
2. Do exercises 1, 2, 6, 7, 12, 17, and 19 in Section 1.2. For exercise 7, the solution should basically consist of one multi-step equation and just a few words. You should use the result of exercise 12 in your solution for exercise 17. The fact that previous results can often be used to simplify the proofs of later results is an important observation; keep this in mind as you do later assignments.

for Thursday, September 5

1. Finish reading Section 1.2. The last part of the section will be less familiar to you so be aware that this reading will go more slowly. Hopefully our discussion in class will have smoothed things out.
2. Do exercises 25c, 27, 30, and 34 in Section 1.2.
3. Turn in a carefully written solution for exercise 1.2.37. A proof of this result (one that does not involve calculus) uses mathematical induction. As you should have learned about these types of proofs, be very careful with your wording. Pay attention to where the assumptions $x \neq 0$ and $x > -1$ are used.

for Friday, September 6

1. Read Section 1.3 through the proof of Theorem 1.17. The Completeness Axiom and the Archimedean Property are extremely important so be certain to ask questions if you find anything confusing about them.
2. Do exercises 1, 2, 5, 12, 13, and 15 in Section 1.3.

for Tuesday, September 10

1. Finish reading Section 1.3, noting that the two remaining proofs make for slow reading. Go over any assigned exercises that you have not yet determined how to do.
2. Do exercises 19, 20, 21, and 30 in Section 1.3.
3. Turn in a solution for exercise 1.3.31. This is the first of our no help assignments; see the syllabus for the guidelines on these types of assignments. For the record, this is not an easy exercise so allot sufficient time to take on the challenge. There are several possible solutions for this particular exercise; one of them is fairly short and clean. Do not panic if you have difficulty with this exercise. I am giving one of these problems early on so that you can learn to deal with the frustration of problems that do not have clear cut solutions.

for Wednesday, September 11

1. Read Section 1.4. Hopefully, much of this material is familiar to you from Math 260 or previous math courses. Study carefully the proof that the set of real numbers is uncountable and note the use of the Completeness Axiom since this approach is probably different than the one you have seen.
2. Do exercises 2, 6, 8, and 9 in Section 1.4. Note that exercise 2 is useful in a solution for exercise 6.

for Thursday, September 12

1. Read Section 1.5. Much of this material should be familiar to you but be certain that you have the vocabulary down; this includes being able to state the definitions if requested or needed.
2. Do exercises 11, 23, 24, 25, 26, 42, and 45 in Section 1.5. Some of these exercises are relatively easy but a few involve a little more thought. The somewhat unusual functions that appear in these problems are important to learn. Be certain you can visualize the graphs of the functions that appear in exercise 11 as they will be the basis for important examples later in the semester. The functions in exercises 42 and 45 may seem rather bizarre, but it is important to realize that functions can behave in strange ways so definitions of properties (such as continuity) must be stated very carefully.
3. Turn in a solution for exercise 1.4.20. (Note Section 1.4 is intended.)

for Friday, September 13

1. Read Section 2.1 through the proof of Theorem 2.4. Become very familiar with the adjectives for sequences, being able to state their definitions and giving examples of sequences with or without a given property. This will most likely be your first introduction to proofs that involve "Let $\epsilon > 0$ be given" so study the example carefully.
2. Do exercises 4, 5, 7, 9, 10, and 11 in Section 2.1. For exercises that ask for examples, try to find several different sequences with each property.

for Tuesday, September 17

1. Continue reading Section 2.1, making it through the proof of Theorem 2.7.
2. Do exercises 14, 15, 16, 17, and 19 in Section 2.1.
3. Turn in solutions for exercises 2.1.9d and 2.1.26, where exercise 2.1.9d involves the sequence $\{13n/(5n+8)\}$. This is the second of our no help assignments; see the syllabus for the guidelines on these types of assignments. A solution for exercise 2.1.9d is very similar to the example in the text. For exercise 2.1.26, you should be extremely careful with the wording of your proof.

for Wednesday, September 18

1. Finish reading Section 2.1, reviewing any previous material that was not clear.
2. Do exercises 32, 33, 34, 35, 36, 41, 42, and 43 in Section 2.1.

for Thursday, September 19

1. Read Section 2.2 through the proof of Theorem 2.13. The two main results presented here are very important so read this material carefully and note any questions you have.
2. Do exercises 5, 8, 10, 12, 14, 16, and 20 in Section 2.2.
3. Turn in solutions for exercises 2 and 15 in Section 2.2.

for Friday, September 20

1. Finish reading Section 2.2.
2. Do exercises 21, 25, 26, 31, 32, 36, and 37 in Section 2.2. For the second part of exercise 25, you might want to consider the special case $a_0 = 0$ and $a_1 = 1$ first.

for Tuesday, September 24

1. Read Section 2.3 through the statement of the Bolzano-Weierstrass Theorem; we will not consider any of the rest of this section at this time.
2. Do exercises 2, 3, 10, and 11 in Section 2.3 and exercises 9 and 25 in Section 2.4. Since we have a test coming up on Thursday, you might also want to spend some time reviewing Chapters 1 and 2.
3. Turn in solutions for exercises 2.4.2 and 2.4.17. This is the third of our no help assignments; see the syllabus for the guidelines on these types of assignments. For exercise 2.4.2a, you may find it helpful to first consider the case in which $x = 0$, then show how the general case reduces to this special case.

for Wednesday, September 25

1. Continue reviewing Chapters 1 and 2 in preparation for the test on Thursday.
2. Do exercises 5, 16, and 19 in Section 2.4.

for Thursday, September 26

1. We have a test on Chapters 1 and 2. You need to be familiar with the concepts and theorems that are included in these chapters. You should be able to state definitions of commonly used terms (such as supremum and Cauchy sequence) and provide examples to illustrate these terms. By flipping through the pages of these two chapters, you should be able to identify the main results we have considered. You should be able to prove the Archimedean property of the real numbers, the linearity properties of sequences, that convergent sequences are bounded, and that bounded monotone sequences converge.
2. On the test, you will be asked to state some definitions or results, give some examples, solve some familiar problems, and solve one or two problems that you have not seen. The best preparation for the test is to have been keeping up with the material during these first few weeks of the semester.

for Friday, September 27

1. There is no assignment for this day. We will be starting Chapter 3.

for Tuesday, October 1

1. Read Section 3.1 through page 85.
2. Do exercises 3, 4, 5, 6, 9, and 10 in Section 3.1.
3. Turn in solutions for exercises 3.1.4c and 3.1.7. This is the fourth of our no help assignments; see the syllabus for the guidelines on these types of assignments.

for Wednesday, October 2

1. Read Section 3.1 through the proof of Theorem 3.5.
2. Do exercises 15, 16, 18, 22, 23, 27, and 28 in Section 3.1.

for Thursday, October 3

1. Finish reading Section 3.1.
2. Do exercises 32, 33, 34bce, 35, 36, and 38 in Section 3.1.
3. Turn in solutions for exercises 35b and 37 in Section 3.1.

for Friday, October 4

1. Read Section 3.2 through Corollary 3.12.
2. Do exercises 1, 2, 4, 9, 10, and 15 in Section 3.2.

for Tuesday, October 8

1. Finish reading Section 3.2, perhaps reviewing some of the previous material in Chapter 3.
2. Do exercises 23, 24, 25, 28, 29, and 30 in Section 3.2.
3. Turn in solutions for exercises 3.2.22 and 3.2.31. This is the fifth of our no help assignments; see the syllabus for the guidelines on these types of assignments. For exercise 3.2.31, note the following fact. If S is a finite set and $a \notin S$, then the number $d = \min\{|x - a| : x \in S\}$ is positive and $(a - d, a + d) \cap S = \emptyset$.

for Wednesday, October 9

1. Read Section 3.3 through page 102. Pay particular attention to the proofs of the IVT and the EVT.
2. Do exercises 5, 6, 9, 11, 12, 13, 17, and 18 in Section 3.3.

for Thursday, October 10

1. Continue reading Section 3.3 through page 105.
2. Do exercises 22, 27, 30, 31, and 36 in Section 3.3.
3. Turn in solutions for exercises 7 and 14 in Section 3.3. For problem 7, you can (and should) use the result of exercise 3.1.40.

for Friday, October 11

1. Read Section 3.4 through page 112. Pay particular attention to the proof of Theorem 3.28.
2. Do exercises 1, 2, 3, and 4 in Section 3.4.

for Tuesday, October 15

1. As you are no doubt aware, there is no class on Tuesday due to the October break.

for Wednesday, October 16

1. Finish reading Section 3.4. Spend at least 20 minutes on the proof of Theorem 3.29.
2. Do exercises 6, 7, 9, and 11 in Section 3.4.
3. Turn in solutions for exercises 3.4.8 and 3.4.10. This is the sixth of our no help assignments; see the syllabus for the guidelines on these types of assignments. The main idea for exercise 3.4.10 should be clear but you need to be rather careful with the details.

for Thursday, October 17

1. We have a test over the material we have covered since the first exam but be aware that you do need to know much of the content from earlier chapters. If you have been keeping up (doing the reading, working on the problems, thinking about the concepts), you should be in good shape for the exam. You should be able to state and prove the IVT, the EVT, and the uniform continuity theorem. As usual, you should be prepared to state definitions, give examples, and solve problems.

for Friday, October 18

1. Read Section 3.5 through page 117; we will not be discussing any more of this section.
2. Do exercises 6, 7, 8, and 9 in Section 3.5. The reading and exercises are not intended to take too much time.

for Tuesday, October 22

1. Do exercises 9, 20, and 22 in Section 3.6. For exercise 22, change the $\mathbb{R}$ at the end to $[0, \infty)$.
2. Turn in solutions for exercises 3.6.17 and 3.6.21. These problems will be worth 10 points each so the total will be out of 20. You will probably find part (b) of exercise 17 to be rather challenging; give it your best shot. There are several ways to proceed for exercise 21. A direct argument is possible but most students find an indirect proof involving the continuous function $|f|$ to be easier. This is the seventh of our no help assignments; see the syllabus for the guidelines on these types of assignments.

for Wednesday, October 23

1. Do exercises 4, 7, 25, and 27 in Section 3.6.

for Thursday, October 24

1. Read the introduction to Chapter 4 as well as Section 4.1 through the middle of page 135.
2. Do exercises 1, 2, 4, 5, 6, 7, 9, and 16 in Section 4.1. Note that exercises 1, 2, 6, and 16 are Calculus I problems so you should be able to do them easily at this point in your mathematical career.
3. Turn in solutions for exercises 8 and 13 in Section 4.1.

for Friday, October 25

1. Finish reading Section 4.1.
2. Do exercises 12, 18, 19, 28, and 31 in Section 4.1.

for Tuesday, October 29

1. Read Section 4.2 up to the paragraph prior to Theorem 4.13.
2. Do exercises 5, 7, 8, 10, 14 and 15 in Section 4.2. Note that most of these problems require working with a parameter rather than a number.
3. Turn in solutions for exercises 11 and 18 in Section 4.2. This is the eighth of our no help assignments; see the syllabus for the guidelines on these types of assignments.

for Wednesday, October 30

1. Read Section 4.2 through the statement of Theorem 4.16.
2. Do exercises 20, 21, 26, 27, 28, and 33 in Section 4.2.

for Thursday, October 31

1. Finish reading Section 4.2.
2. Do exercises 35, 36, 37, and 39 in Section 4.2.

for Friday, November 1

1. Read Section 4.3 through the proof of Theorem 4.24.
2. Do exercises 3, 4, 5, 6, 7, 11, and 13 in Section 4.3.
3. Turn in solutions for exercises 8 and 12 in Section 4.3.

for Tuesday, November 5

1. Do exercises 4, 5, 6, 7, 8, 9, 11 and 15 in Section 4.4. For exercise 9, replace $(y-x)^r$ with $|y-x|^r$.
2. Turn in solutions for exercises 13 and 42 in Section 4.4. This is the ninth of our no help assignments; see the syllabus for the guidelines on these types of assignments.

for Wednesday, November 6

1. Do exercises 12, 20, 24, and 40 in Section 4.4. For exercise 20, your example should satisfy $\lim_{x\to\infty} f(x) = \infty$.

for Thursday, November 7

1. We have a test over the material we have covered since the second exam but be aware that you do need to know much of the content from earlier chapters. If you have been keeping up (doing the reading, working on the problems, thinking about the concepts), you should be in good shape for the exam. You should be able to state and prove Rolle's Theorem, the Mean Value Theorem, the intermediate value property for derivatives, the various rules for differentiation, and the fact that differentiability implies continuity. As usual, you should be prepared to state definitions, give examples, and solve problems.

for Friday, November 8

1. Read pages 163–166 of the text. This should only take about 20 minutes.

for Tuesday, November 12

1. Read Section 5.1.
2. Do exercises 2, 8, 9, 10, 11, 12, 17, and 18 in Section 5.1.
3. Turn in solutions for exercises 15, 20, and 19b (using exercise 20) in Section 5.1. These problems will be worth 5 points each so the total will be out of 15. This is the tenth of our no help assignments; see the syllabus for the guidelines on these types of assignments.

for Wednesday, November 13

1. Read Section 5.2 through the proof of Theorem 5.10; this reading may take 90 minutes or so.
2. Do exercises 2 and 4 in Section 5.2.

for Thursday, November 14

1. Finish reading Section 5.2.
2. Do exercises 7, 11, 12, and 14 in Section 5.2.
3. Turn in solutions for exercises 10 and 14a in Section 5.2.

for Friday, November 15

1. Read Section 5.3 through the proof of the Fundamental Theorem of Calculus.
2. Do exercises 6, 8, 10, 11, 12, (you should NOT use the Fundamental Theorem of Calculus for any of these exercises) 15, and 16 in Section 5.3.

for Tuesday, November 19

1. Finish reading Section 5.3.
2. Do exercises 13, 17, 18, 26, and 29 in Section 5.3.
3. Turn in solutions for exercises 18 and 20 in Section 5.2 (yes, the section is correct). You may use Theorem 5.15 for exercise 18. These problems will be worth 10 points each so the total will be out of 20. This is the eleventh of our no help assignments; see the syllabus for the guidelines on these types of assignments.

for Wednesday, November 20

1. Read Section 5.4 through the proof of Theorem 5.22.
2. Do exercises 1, 2, 3, 4, 5, 6, 7, and 8 in Section 5.4; most of these exercises should not take too long.

for Thursday, November 21

1. Finish reading Section 5.4.
2. Do exercises 16, 17, 18, 19, 20, and 25 in Section 5.4. There is a mathematical error in exercise 25; see if you can determine and explain what it is.
3. Turn in solutions for exercises 5.4.12 and 5.6.4, thus pretending that you are in a calculus class again.

for Friday, November 22

1. Review Sections 5.1 through 5.4 as necessary, identifying any gaps in your understanding.
2. Do exercises 1, 2, 6, 10, and 12 in Section 5.6.

for Tuesday, December 3

1. Read the remark prior to exercise 37, then read (and think about but not solve) exercises 37 through 44. This should give you a sense for the Darboux approach to integration.
2. Turn in solutions for exercises 3 and 18 in Section 5.6. For exercise 3, note that F' may not exist at the points where $F'(x) \neq f(x)$. Exercise 18 is not too difficult once you recognize what needs to be done. The key issue is that the points s_i and t_i (which technically also depend on n but the notation then becomes very awkward) are different points (tags) in the subinterval. These problems will be worth 10 points each so the total will be out of 20. This is the twelfth of our no help assignments; see the syllabus for the guidelines on these types of assignments.

for Wednesday, December 4

1. Review the portions of Chapter 5 that we have discussed and look over the exercises that have been assigned. Bring any questions you have to class so that we can discuss them.

for Thursday, December 5

1. We have a test on the first four sections of Chapter 5 but, as usual, you do need to know some of the content from earlier chapters. If you have been keeping up (doing the reading, working on the problems, thinking about the concepts and notation), you should be in good shape for the exam. You should be able to state the definitions of the common terms we have been using and give examples of functions exhibiting various properties. As on the quiz, if you are asked to state the definition of the Riemann integral, you may assume that tagged partitions and Riemann sums are already defined. You should be prepared to prove the linearity properties of the integral, that Riemann integrable functions are bounded, that continuity implies Riemann integrability, that monotonicity implies Riemann integrability, both parts of the Fundamental Theorem of Calculus (we did the second part in class), and the Mean Value Theorem for integrals. Working on problems in Section 5.6 is good practice for learning how to solve problems you have not yet seen.

for Friday, December 6

1. Spend about 30 minutes reading Section 6.1, including all of the exercises. This material should look familiar from Calculus II but you should find that you have a much a deeper understanding of it now.

for Tuesday, December 10

1. Read Section 6.2 carefully.
2. Be prepared to discuss problems 1, 2, 3, 4, 5, 15, 16, and 21a in Section 6.2.
3. Turn in solutions for problems 3 and 5ab in Section 6.5. These problems will be worth 10 points each so the total will be out of 20. This is the thirteenth (and final!) of our no help assignments; see the syllabus for the guidelines on these types of assignments.

for Wednesday, December 11

1. Read Section 6.3. You can ignore the $\liminf$ and $\limsup$ symbols for now and treat them as ordinary limits. Be certain you understand the definitions, theorems, and proofs in this section. Since the general ideas should be somewhat familiar to you from calculus, focus on the proofs and concepts rather than just the computational details.
2. Be prepared to discuss exercises 3, 5, 6, 8, 9, and 12 in Section 6.3.

for Thursday, December 12

1. Review Sections 6.1, 6.2, and 6.3 as necessary, reminding yourself how the proofs of the main results go and looking over some of the exercises.
2. Be prepared to discuss exercises 6.1.23 (which we hinted at in class today), 6.2.13 (for practice with the Cauchy condensation idea), 6.2.21, 6.3.4 and 6.3.21 (since these two are related), 6.3.27, and 6.5.22.

for Friday, December 13

1. Spend 30 to 60 minutes reviewing the portions of the textbook that we have covered this semester.
2. You should be able to give the definition of any concept that we have used regularly during the semester, be able to state any major result we have considered, and be able to generate examples that illustrate these ideas and concepts. You should also be prepared to prove the following results.
    1. Archimedean property of the real numbers
    2. there exists a rational number in any interval
    3. convergent sequences are bounded
    4. algebraic properties of sequences
    5. bounded and monotone sequences converge
    6. Intermediate Value Theorem
    7. Extreme Value Theorem (just the f is bounded part)
    8. continuity implies uniform continuity on $[a, b]$
    9. Mean Value Theorem (the full details which involves three results)
    10. the monotonicity theorem for functions
    11. differentiability implies continuity
    12. the derivative rules (product rule, quotient rule, and chain rule)
    13. Riemann integrable implies bounded
    14. continuous implies Riemann integrable
    15. monotone implies Riemann integrable
    16. Fundamental Theorem of Calculus
    17. Mean Value Theorem for integrals
    18. Divergence Test
    19. Comparison Test
    20. any linearity proof
3. The final exam will consist of some subset of item 2 along with some new problems that involve the ideas we have considered and the problem solving skills you have acquired.
4. The next page provides some exercises you might want to consider after preparing for the exam. More likely than not, it is impossible for you to do all of these between now and the exam. Read through the exercises and start with those that seem to be at your level of understanding. The parenthetical label is intended to give you a rough idea about the level of difficulty of the problem.

1. (easy) Let r be a rational number. Prove that there exists a strictly decreasing sequence of irrational numbers that converges to r.
2. (easy) Exercise 2.1.17.
3. (medium) Exercise 2.2.39c
4. (medium) Exercise 2.4.2.
5. (medium) Exercise 2.4.23.
6. (easy) Exercise 3.1.4c.
7. (medium) Exercise 3.2.4.
8. (medium) Exercise 3.4.8. (Give a direct proof; do not use Theorem 3.30.)
9. (hard) Exercise 3.5.32.
10. (medium) Let $f\colon [0,4] \to \mathbb{R}$ be continuous on $[0,4]$ and suppose that $f(0) = f(4)$. Prove that there exists a point $c \in [0,1]$ such that $f(3c+1) = f(c)$.
11. (medium) Exercise 3.6.31.
12. (easy) Exercise 4.2.18.
13. (medium) Exercise 4.4.44.
14. (medium) Exercise 5.1.10.
15. (easy) Exercise 5.3.19. (Be certain to do this one.)
16. (hard) Exercise 5.6.5. The following remnant of scratchwork may be helpful.

$$M - 2\epsilon < (M-\epsilon)\sqrt[n]{d-c} \le v_n \le M\sqrt[n]{b-a} < M + \epsilon \quad \text{for } n \ge N$$

17. (medium) Exercise 5.6.18.
18. (medium) Evaluate $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \Big(\ln\Big(1 + \dfrac{i}{n}\Big) \cdot \dfrac{1}{3n-2} \Big)$.
19. (hard) Exercise 6.5.6.
20. (easy) Exercise 6.5.9.
21. (medium) Exercise 6.5.13.
22. (hard) Exercise 6.5.34. (Read the heading for this set of exercises and assume the results prior to this exercise.)