Assignments for Math 126, Fall 2014 (due on given date)

(9/2) 1. Read the syllabus (available online; I will not be passing out printed versions) carefully.

2. Spend some time reviewing Calculus I, perhaps beginning with the summary of differential calculus posted on the class website.

3. Look over some of the practice problems for differential calculus posted on the class website. I will be giving you a printed version of this assignment on the first day of class.

(9/4) 1. Reread the syllabus.

2. Do the practice problems for differential calculus, reviewing as necessary.

3. Turn in solutions for problems 1, 9, and 12. You should model your solutions based on the sample homework solutions posted on the class website.

4. Browse Section 2.1 of the textbook.

(9/5) 1. Read Section 2.1 carefully.

2. Do the problems in Section 2.1.

3. Turn in solutions for problems 5c, 6, and 7. Remember to write your solutions clearly.

4. Read the prelude to Chapter 3 carefully.

(9/9) 1. Read Section 3.1 carefully.

2. Do problems 1, 2, 3, and 5 in Section 3.1.

3. Read the first two pages of the Model Induction Proofs (see the Prelude to Chapter 3 for the appropriate link). You should be able to find two errors in each of the incorrect proofs.

4. Turn in a carefully written solution for problem 3.

(9/11) 1. Reread Section 3.1 if necessary. Spend some time thinking about Fibonacci numbers.

2. Do problems 4, 6, and 7 in Section 3.1. After working problem 4, read the third page of the Model Induction Proofs. You also should read the extra notes for Section 3.1, thinking carefully about each sentence of the examples given there.

3. Turn in a carefully written solution for problem 7d. A comment about this problem was discussed in class; be certain you understand what equation you are proving.

4. Browse Section 3.2 of the textbook.

(9/12) 1. Read Section 3.2 carefully.

2. Do the problems in Section 3.2.

3. Turn in solutions for problems 4e, 4g, 5c, and 6.

4. Browse Section 3.3 of the textbook.

(9/16)
1. Read Section 3.3 carefully.
2. Do the problems in Section 3.3.
3. Turn in solutions for problems 1b (there should be four steps), 2b, and 4.
4. Browse Section 3.4 of the textbook.

(9/18)
1. Read Section 3.4 carefully.
2. Do the problems in Section 3.4.
3. Turn in solutions for problems 2, 5, and 7. For 7, we have already proved that $1 \leq a_n \leq 3$ (see problem 5 in Section 3.1) so you just need to use induction to prove the increasing part, then (after concluding that the sequence converges) find the limit.
4. Browse Section 3.5 of the textbook.

(9/19)
1. Read Section 3.5 carefully.
2. Do the problems in Section 3.5.
3. Turn in solutions for problems 2b, 3e, 3i, and 5.
4. Browse Section 3.7 of the textbook.

(9/23)
1. Read Section 3.7 carefully.
2. Do the problems in Section 3.7. Problem 4 requires you to make some careful estimates.
3. Turn in solutions for problems 1b, 2a, 3a, and 3d.
4. Browse Section 3.8 of the textbook.

(9/25)
1. Read Section 3.8 carefully.
2. Do the problems in Section 3.8. Problem 8 is important since you must first decide which of the convergence tests to use. Even if you do not carry out all of the details, think carefully about what the series does and which test to use to verify your conjecture. Problems 5 and 9 require more thinking than the other problems.
3. Turn in solutions for problems 1c, 3, and 6.
4. Browse Section 3.9 of the textbook.

(9/26)
1. Read Section 3.9 carefully.
2. Do the problems in Section 3.9. Problem 6 is important for what is to come.
3. Turn in solutions for problems 2f, 3f, and 6c.
4. Browse Section 3.10 of the textbook.

(9/30) 1. Read Section 3.10 carefully.

2. Do the problems in Section 3.10.
3. Turn in solutions for the following three problems (read the appropriate headings)

$$2\text{d. } \sum_{k=1}^{\infty} \frac{3}{k4^k}\,(x-1)^k \qquad\qquad 6\text{e. } (4,8] \qquad\qquad 7\text{d. } \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2^{k+1}}\,(x+1)^k$$

4. Since we have a exam coming up very soon, you should start reviewing the material in Chapter 3. Go over the sections we have covered and review the key concepts. Practice some of the problems we have been doing, that is, do some of the problems again without looking at your previous solutions. Look at the exam on this material from a previous year (found on the website) and be certain the problems make sense and that you can do them with the time frame of an exam. For additional problems, you can work on problems 1, 14, 5, 6, 11, and 13 (in that order) in Section 3.14. When you feel you are ready, take the diagnostic quiz on Chapter 3. This quiz can be found on the website, along with complete solutions. We will review this material in class on Tuesday.

(10/2) 1. We have an exam covering Section 2.1 and the first 10 sections of Chapter 3.

(10/3) 1. Browse Section 2.2 of the textbook.

2. We will return to Chapter 3 toward the end of the semester and cover Sections 3.11 and 3.12. These are actually the pinnacles of the material on sequences and series but you will have to anxiously await their study. One of the advantages of waiting until later is so that you get a chance to review the material on sequences and series before the final exam. In addition, the theory of integration is more important for the last part of Chapter 3 so we want to go over it first. For many of you, integration will be more familiar material. However, I must warn you that the first few sections of Chapter 2 present more theoretical aspects of integration than you may have seen before. These sections contain the definition of the integral, a discussion of the properties of the integral, and a proof of the Fundamental Theorem of Calculus. Please do not dismiss these sections; give them careful thought.

(10/7) 1. Read Section 2.2 carefully.

2. Do the problems in Section 2.2. Problem 2 does require a careful sketch and then some thought about how to proceed; it is a good example of the type of thinking that is needed to solve non-routine problems. Do give problem 7 some careful thought.
3. Turn in solutions for problems 3 (x^3 only), 5d, and 6, where 5d is $y = 6 - |2x-3|$ on $[0,4]$. For problem 6, note that problem 7 in Section 2.1 comes into play.
4. Browse Section 2.3 of the textbook.

(10/9) 1. Read Section 2.3 carefully.

2. Do the problems in Section 2.3, omitting 7 and 8 if you run out of time.
3. Turn in solutions for problems 3, 4, and 2d, where problem 2d is $\lim_{n\to\infty}\sum_{i=1}^{n}\left(5+\frac{4i}{n}\right)^2\frac{8}{n}$.
4. Browse Section 2.4 of the textbook.
5. I will be out of town from October 8 through October 15 for the memorial service for my father. There will be no class this day (Thursday, October 9) but there will be class on Friday, October 10 with Professor Balof answering questions and discussing some of the new material. You should turn in the above assignment (item (3)) to Professor Balof on Friday.

(10/10) 1. Read Section 2.4 carefully.

2. Do the problems in Section 2.4. You should find that these problems go more quickly than the problems in previous sections. Do be careful with your notation (remember dx) and be aware of the linearity properties of the integral that you are using.
3. Turn in solutions for problems 2e, 4d, and 5d, where problem 4d is $\int_{-3}^{3}\left(8\sqrt{9-x^2}+10x^4\right)dx$ and problem 5d uses the interval $1 \le t \le 6$.
4. Browse Section 2.5 of the textbook.
5. Since there was no lecture on the material in Section 2.4, this means that you must learn this material on your own. Since one of the goals of the course is to learn how to read technical material, this is an opportunity to hone that skill. You should read both Section 2.4 in the textbook and the extra notes for Section 2.4 carefully and thoughtfully. For the record, only a rough grasp of this section is needed to understand the upcoming lecture on Section 2.5. This means that you can use some of the time during the October break to master Section 2.4. To give you some extra time, the above problems (item (3)) are due under my office door by midnight Wednesday, October 15.

(10/14) 1. No class today due to the October break.

(10/16) 1. Read Section 2.5 carefully.

2. Do the problems in Section 2.5.
3. Turn in solutions for problems 3 (giving clear details), 4d, and 5 (using appropriate steps), where problem 4d is $\int_{1}^{2}\left(3f(x)-6x^2\right)dx$.
4. Browse Section 2.6 of the textbook.

(10/17) 1. Read Section 2.6 carefully.

2. Do the problems in Section 2.6.
3. Turn in solutions for problems 1j, 2, and 3, where 1j is $f(x)=\int_{0}^{3x}\ln(1+t^2)\,dt$.
4. Browse Section 2.7 of the textbook.

(10/21) 1. Read Section 2.7 carefully.

2. Do the problems in Section 2.7.

3. Turn in solutions for problems 1m, 1n, 1o, and 2, given

$$\text{1m. } \int_2^4 \frac{8}{x^3}\,dx \qquad \text{1n. } \int_0^1 \left(2x^2 + \sqrt[3]{x}\,\right) dx \qquad \text{1o. } \int_0^4 \frac{3}{x+4}\,dx$$

4. Browse Section 2.8 of the textbook.

(10/23) 1. Read Section 2.8 carefully.

2. Do the problems in Section 2.8.

3. Turn in solutions for problems 1p, 2g, and 3′, given

$$\text{1p. } \int \frac{24}{(3x+1)^3}\,dx \qquad \text{2g. } \int_0^1 \frac{8x+4}{x^2+x+1}\,dx \qquad 3'.\ \int_0^2 (2x-3)\sqrt{4-x^2}\,dx$$

4. Browse Section 2.9 of the textbook.

(10/24) 1. Read Section 2.9 carefully.

2. You should look over the integrals in problem 1 and think about how you would solve each of them. Can you just write down the answer? Can you use guess and check? Do you need to make a substitution and, if so, what would u be? After making this assessment, do a few of each type beginning with 1b, 1e, 1i, 1j, 1k, and 1l. Repeat this process for problem 2, then begin with 2a, 2b, and 2e. Remember to change the limits for the definite integrals. Problems 3 and 4 indicate that there is more than one way to find an antiderivative while problem 5 shows how to determine the formula for the area of an ellipse. You can omit problem 6.

3. Turn in solutions for problems 1m, 2d, and 2g, where 1m. $\displaystyle\int \frac{x+3}{\sqrt{x-4}}\,dx$ 2g. $\displaystyle\int_0^4 \frac{1}{1+\sqrt{x}}\,dx$

4. Browse Section 2.10 of the textbook. The lecture on this material will be given by Professor Cotts as I will once again be out of town; this time for the wedding (taking place in Nipomo, CA) of one of my sons.

(10/28) 1. Read Section 2.10 carefully.

2. Do the problems in Section 2.10 beginning with 1a, 1b, 1c, 1e, 1i, 1j, 2b, 2c, 2d, 2e, 3, and 4.

3. Turn in solutions for problems 1i, 2b, and 3 in Section 2.10.

4. Read Section 2.11 carefully and perhaps look at the extra notes for this section of Chapter 2. We will not be discussing this section in class so this is another opportunity for you to practice reading and understanding technical material.

5. Do problems 1a, 1d, 1e, 1f, 2a, 2b, and 7 in Section 2.11. I will not be collecting any of these problems but you will be responsible for knowing how to evaluate improper integrals on the upcoming exam. This exam, by the way, is scheduled for next Friday, October 31.

6. Browse Section 2.12 of the textbook.

(10/30) 1. Read Section 2.12 carefully.

2. Do the problems in Section 2.12. For some of these, sketch a careful graph and think before you set up the integrals.
3. Turn in solutions for problems 1c, 4, and 6. For problem 4, sketch a careful graph and decide which direction most easily describes the region in question.
4. We have our second exam on Friday, covering Sections 2.1 through 2.12. We will review for this exam in class on Thursday, October 30. The questions on the exam will be similar to the homework problems you have been doing the past few weeks. You can find exams from a previous semester on the class website. In addition to the problems in each section (for the record, doing those problems again without your notes can be helpful), you can try problems 1 through 19 in Section 2.24.
3. As with the exam from the previous semester posted on the website, no calculators will be allowed during the exam. You will have 55 minutes for the exam.

(10/31) 1. We have an exam on Sections 2.1 through 2.12.

(11/4) 1. Browse Section 2.13 of the textbook.

(11/6) 1. Read Section 2.13 carefully.

2. Do the problems in Section 2.13.
3. Turn in solutions for problems 1c, 2, and 9.
4. Browse Section 2.14 of the textbook.

(11/7) 1. Read Section 2.14 carefully.

2. Do the problems in Section 2.14.
3. Turn in solutions for problems 1a, 6ab, and 8.
4. Browse Section 2.15 of the textbook.

(11/11) 1. Read Section 2.15.

2. Do problems 1, 2, and 3 in Section 2.15.
3. Turn in solutions for problems 2c and 2d.
4. Browse Section 2.17 of the textbook.

(11/13) 1. Read Section 2.17.

2. Do problems 3, 4, 5, 6, 8, and 13 in Section 2.17.
3. Turn in solutions for problems 4 and 8.
4. Browse Section 2.18 of the textbook.

(11/14)

1. Read Section 2.18.
2. Do problems 1, 2, 3, 4, and 5 in Section 2.18.
3. Turn in solutions for problems 3b and 4.
4. Browse Section 2.19 of the textbook.

(11/18)

1. Read Section 2.19 carefully.
2. Do the problems in Section 2.19. Remember that these problems involve more algebra than calculus.
3. Turn in solutions for the three problems listed below.

$$\int \frac{2x-1}{x^2+6x+13}\,dx, \qquad \int \frac{4x-3}{\sqrt{33+8x-x^2}}\,dx, \qquad \int \frac{6x^3+14x+7}{x^2+9}\,dx.$$

4. Browse Section 2.20 of the textbook. It will be helpful (in fact, almost essential) to have access to Appendix B of the textbook (the table of integrals) for class on this day.

(11/20)

1. Read Section 2.20 carefully.
2. Do the problems in Section 2.20. You can omit 7 if you run out of time.
3. Turn in solutions for problems 1g, 5, and 6f.
4. Browse Section 2.21 of the textbook.

(11/21)

1. Read Section 2.21 carefully.
2. Do (or at least make the appropriate trig substitution and determine the new integral for) the problems in Section 2.21.
3. Turn in solutions for problems 1b, 2d, and 1g. $\displaystyle\int \frac{x^4}{\sqrt{4-x^2}}\,dx.$
4. Browse Section 2.22 of the textbook.

(12/2)

1. Read Section 2.22 carefully.
2. Do (or at least determine the partial fraction decomposition for) the problems in Section 2.22.
3. Turn in solutions for problems 1b and 1f.
4. Browse Section 2.23 of the textbook.

(12/4)

1. Read Section 2.23 carefully.
2. Do problems 1–5 in Section 2.23. If you are so inclined, you can use Maple to do some of these calculations or to play around with these ideas. The next page will give you the relevant details.
3. Turn in solutions for problems 3 and 5.
4. Review for the exam on Friday.

A Computer Assignment

Maple is a powerful computer algebra system that can deal with symbols as well as numbers. One of the goals of Math 235 (Calculus Laboratory) is to learn how to use this software. However, the sole purpose of this exercise is to show you several commands in Maple to help you approximate the value of an integral. Enter the computer lab in Olin 248 and sit down at one of the computers. Use **maple** as the login and **cauchy** as the password. After a brief pause, a column of icons should show up on the left side of the screen. Search down this column until you see a colorful 16 with a maple leaf then click on this icon. A Maple session window should then appear in a few seconds. If you have any problems at this point, most anyone in the lab can help you or come find me or another member of the math department.

In the Maple window you will see a prompt that begins with the symbol >. Try typing the following commands. It is imperative that the with(student) command appear first. As this is a computer system, you must type things exactly, including the colon or semicolon.

```
> with(student);
> evalf(trapezoid(3*x^5,x=0..2,20));
> evalf(middlesum(3*x^5,x=0..2,20));
> evalf(simpson(3*x^5,x=0..2,20));
> int(3*x^5,x=0..2);
> evalf(simpson(sqrt(1+x^3),x=0..2,6));
> evalf(simpson(sin(x^2),x=0..Pi,4));
```

You can use the mouse and the cursor to return to previous lines and make changes if there is an error or you want to change the numbers.

These commands should be clear from the discussion in class and **evalf( )** tells Maple to evaluate the previous quantity in decimal form (floating point). Feel free to play with variations on this theme. The first part is the function, the numbers on either side of the two periods are the left and right endpoints of the interval (that is, the lower and upper limits of integration), and the last number is the number of subintervals. Pi is π. Be certain to use $*$ to indicate multiplication.

When you are finished with Maple, hold down the **Alt** key and press the **F4** key. If you really are ready to exit Maple, click the left mouse button on **no** and the Maple window should disappear. Finally, logout of the computer using the button in the top right corner of the screen.

(12/5) 1. We have an exam on the material we have covered since the last exam.

2. The exam specifically covers Sections 2.13 through 2.23 with the exception of 2.16. However, you do need to know some of the material from previous sections. In addition, you should be able to state the definition of the derivative, the definition of the integral, and both versions of the FTC, using the correct words and symbols. The first step in your review should be to go over the sections we have covered to make sure you understand the main ideas, redoing the problems in each section if necessary. After doing so, you can look at the following items on the website: (a) the second exam from Spring 2011, (b) problems 21, 23, 26, 27, 28, 29, 32, 33, 34, 35, 36, 37, and 38 in Section 2.24, and (c) the problems at the link 'Review for exam on integration'. We will review for the exam in class on Thursday and perhaps have a review session some evening during the week.

(12/9) 1. Look over the material in Chapter 3 that we covered earlier this semester.

2. Since our comprehensive final exam is very early in finals week and we have not looked at sequences and series for quite a few weeks, item (1) is very important. Please give it the attention it deserves.

3. Browse Section 3.11 of the textbook. If this material is not familiar, then repeat item (1) again with a focus on power series.

(12/11) 1. Read Section 3.11 carefully.

2. Do the problems in Section 3.11; I will not be collecting any of these but please do them in preparation for the final exam.

3. Browse Section 3.12 of the textbook.

(12/12) 1. Read Section 3.12 carefully.

2. Do (or at least think about how to do) problems 1–5 in Section 3.12.

3. Do at least one part of each of the problems 21, 28, and 29 in Section 3.14.

(12/15) 1. Our final exam is scheduled for 9–12 this morning. Details can be found on the next few pages.

As you may recall from reading the syllabus, the goals for this course are

- to develop quantitative reasoning skills;
- to learn how to read technical material;
- to learn to write technical information correctly and clearly;
- to take pride in your work and to avoid errors;
- to learn how to solve non-routine problems;
- to appreciate/understand how mathematicians view mathematics;
- to comprehend some aspects of calculus.

It is with these goals in mind that the final exam will be written. The exam is comprehensive and covers all of the sections that we have discussed this semester, with a slight emphasis on the more recent material. The final exam will require the skills and concepts that you have been practicing and pondering this semester. It is your responsibility to go back over the sections and make certain that you know how to do the types of problems we have encountered. Some of the problems on the final exam will be more involved than the sorts of problems that have appeared on the other exams that we have had. This should not be that much of a surprise because most of the test questions have been easier and shorter than homework problems due to the time constraints of a 50 minute exam. It is now time for you to put all your knowledge together and show me what you have learned this semester.

Here is the (most likely) introduction to the final exam that you will be taking.

Math 126 **Final Exam** **Fall 2014**

Write neat, concise, and accurate solutions to each of the problems in the space provided—I will not give any credit for steps I cannot follow. Your solutions should be written in the style expected for collected homework problems. Pay particular attention to correct use of notation and use sentences when appropriate. Each of the ten problems is worth eight points. No electronic devices or calculators are allowed for this exam.

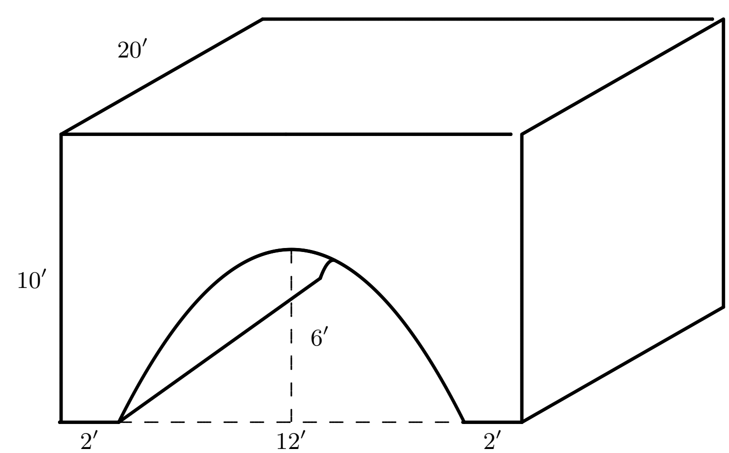
The problems will be graded as indicated in the heading so it is important that you work toward avoiding computational errors and that you pay attention to your writing and notation. The best advice is to review for the exam by looking over the sections in the text that we have covered, thinking carefully about the ideas we have discussed, and understanding the types of problems that have appeared on previous exams. You can redo problems from the sections in the textbook, you can look over your previous exams, and you can work on some of the review problems that appear on the next few pages. It is important that you arrive at the exam with a refreshed mind and body, and be prepared to stay positive and work hard for up to three hours. As just indicated, although the exam is written for a two hour period, you may have three hours for the exam. The exam time period is thus 9–12 on Monday morning in our usual classroom.

As should come as no surprise, it is expected that you can state the definition of the derivative, the definition of the integral, and both versions of the Fundamental Theorem of Calculus. Do not lose points by ignoring this fact. You should also be familiar with basic integration formulas and techniques of integration, be able to solve problems involving applications of the integral, understand the main ideas behind sequences, series, and power series, and know the Maclaurin series for e^x, $\sin x$, and $\cos x$. This is just a sampling of the things that you need to know for the exam; if you have been keeping up during the semester, it should not be too difficult to remember the common formulas and ideas that we have been using.

The following problems are not representative of the final exam. They are simply intended to give you some indication of the nature of a more difficult or novel problem that may appear on the final exam. Having said that, I do recommend that you give them some thought. However, keep in mind that most of the problems on the final exam will be (or at least should be) quite familiar to you. You can look at the final exam from Spring 2011 that is located on the course website but be aware that our final exam will not necessarily look like this.

1. Consider two different solids. The base of each solid is a triangle with vertices $(0, 0)$, $(2, 4)$, and $(6, 0)$. For solid A, each cross-section perpendicular to the y-axis is an equilateral triangle. For solid B, each cross-section perpendicular to the x-axis is a square. Find the ratio of the volume of solid A to the volume of solid B.

2. Find the number of cubic yards of concrete necessary to construct the culvert shown below. Assume that the arch of the culvert (which is empty space) has a parabolic shape.



3. Let $a_1 = 2$ and $a_{n+1} = 3 - (1/a_n)$ for each positive integer $n \geq 1$. Prove that $a_n = \dfrac{f_{2n+1}}{f_{2n-1}}$ for each positive integer n. Here f_n refers to the nth Fibonacci number.

4. For each positive integer n, let
$$y_n = \frac{1}{3n+2} + \frac{1}{3n+4} + \frac{1}{3n+6} + \cdots + \frac{1}{5n}.$$
Find the limit of the sequence $\{y_n\}$. (Try writing y_n in summation notation and think about integrals.)

5. Determine (using familiar calculus functions) the function represented by $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k(2k+1)}{4^k k!}\, x^{2k}$.

The problems that follow are more similar to the problems that you have been doing as homework. As you try these problems, put yourself in the mindset of an exam. That is, do not use your notes or look at the answer until you have finished the problem. Pay attention to problems you do not know how to start (these problems represent a lack of understanding) and problems you know how to start but get incorrect answers at the end (these problems indicate of lack of attention to detail).

Miscellaneous problems to try before the final exam

Since many of the answers are given right after the problem, you need to be careful to avoid using the answer as a hint for how to start the problem as this does not mimic a testing situation. Proceed without technology if at all possible.

1. Evaluate the limit $\lim\limits_{n\to\infty} \dfrac{n^4+n^2+1}{3^3+6^3+9^3+\cdots+(3n)^3}$. (4/27)

2. Use an integral to evaluate the limit $\lim\limits_{n\to\infty} \sum_{i=1}^{n} \left(5+\dfrac{3i}{n}\right)^2 \dfrac{3}{n}$. (129)

3. Evaluate each of the following integrals.

   a) $\displaystyle\int_{-1}^{2} (2x-3)(x-1)\,dx$ b) $\displaystyle\int_{0}^{3} (4x+2|x-1|)\,dx$ c) $\displaystyle\int_{0}^{1} \left(2t-3+2\sqrt{1-t^2}\right)dt$

   d) $\displaystyle\int_{1}^{8} \frac{x+2}{\sqrt[3]{x}}\,dx$ e) $\displaystyle\int_{1}^{4} \frac{1}{3x-2}\,dx$ f) $\displaystyle\int_{0}^{2} \frac{1}{4+x^2}\,dx$

   (The values are 15/2, 23, $\dfrac{\pi}{2}-2$, 138/5, $\frac{1}{3}\ln 10$, and $\pi/8$, respectively.)

4. Use a simpler function to approximate $\displaystyle\int_{1}^{2} \frac{1}{\sqrt{4x^6-1}}\,dx$. Is your estimate high or low? (3/16, low)

5. Find the derivative the function F defined by $F(x) = \displaystyle\int_{0}^{x^2} t\sqrt{t^3+4}\,dt$. $\left(F'(x) = 2x^3\sqrt{x^6+4}\right)$

6. Suppose that $v(t) = 3t - t^3$ gives the velocity in meters per second of a particle at time t seconds. Find the distance traveled by the particle for the time interval $0 \le t \le 4$. (44.5 meters)

7. Find the area of the region bounded by the curves $x^2y = 90$ and $40x + y = 130$. (40)

8. Find the area of the region bounded by the curves $y = 2\sqrt{x}$ and $y = x^3/16$. (20/3)

9. Find the volume of the solid that is generated when the region bounded by the curves $y = 4x$ and $y = x^2$ is revolved around (a) the x-axis (b) the y-axis. $\left(\left(\dfrac{2048}{15}\pi\right) \text{ and } \left(\dfrac{128}{3}\pi\right)\right)$

10. Suppose the base of a solid is the part of the parabola $y = 8 - 0.5x^2$ that lies above the x-axis and that each cross section perpendicular to the y-axis is a semicircle. Find the volume of this solid. (32π)

11. Find the volume of the solid that is generated when the region that lies below the curve $y = \ln x$ and above the x-axis on the interval $[1, e]$ is revolved around the y-axis. $\left(\pi(e^2+1)/2\right)$

12. Find the volume of the solid that is generated when the region that lies above the x-axis and below the curve $y = \sqrt{\dfrac{14(3-x)}{(x+1)(7-x)}}$ on the interval $[0, 3]$ is revolved around the x-axis. $\left(7\pi(\ln 16 - \ln 7)\right)$

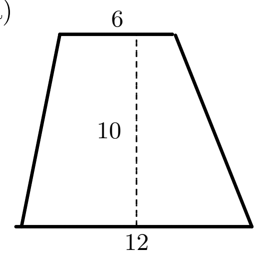
13. Find the center of mass of the region bounded by the curves $y = 2\sqrt{x}$ and $y = x^3/16$. $\left(\dfrac{48}{25}, \dfrac{12}{7}\right)$

14. Find the center of mass of the solid that is generated when the region below the curve $y = 4e^{-x/4}$ and above the x-axis on the interval $[0, \infty)$ is revolved around the x-axis. $(2, 0, 0)$

15. Find the length of the curve $y = 4x^{3/2}$ on the interval $[0, 10]$. (127)

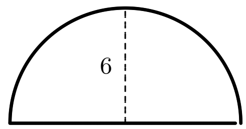
16. Find the force exerted by a liquid with weight density w on one side of each vertically submerged plate. The units on the figures are feet and the top of each plate is six feet beneath the surface of the liquid.

a)



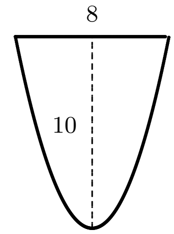
trapezoid

b)



semicircle

c)



parabola

(The forces are $1040w$, $(216\pi - 144)w$, and $1600w/3$ pounds, respectively.)

17. Evaluate each of the following definite integrals.

a) $\displaystyle\int_0^2 \frac{x}{4+x^2}\,dx$ b) $\displaystyle\int_0^2 \frac{x}{\sqrt{4+x^2}}\,dx$ c) $\displaystyle\int_0^2 \frac{x}{\sqrt{4+x}}\,dx$

d) $\displaystyle\int_1^\infty \frac{8}{(2x+5)^3}\,dx$ e) $\displaystyle\int_0^\infty \frac{6+e^{2x}}{e^{3x}}\,dx$ f) $\displaystyle\int_1^{\sqrt{2}} \frac{\sqrt{x^2-1}}{x^4}\,dx$

(The values are $\frac{1}{2}\ln 2$, $2\left(\sqrt{2}-1\right)$, $\dfrac{32}{3} - 4\sqrt{6}$, $2/49$, 3, and $\sqrt{2}/12$, respectively.)

18. Use the trapezoid rule and Simpson's rule with $n = 4$ to approximate $\displaystyle\int_0^1 e^{-x^2/2}\,dx$ to four decimal places. (The approximations are 0.8526 and 0.8557, respectively.)

19. Suppose that the following table represents the velocity of a particle moving in a straight line.

t	(sec)	0	1	2	3	4	5	6
v	(m/sec)	0	5	10	12	8	4	0

Use Simpson's rule to approximate the distance traveled by the particle. (40 meters)

20. Evaluate each of the following indefinite integrals.

a) $\displaystyle\int \left(2\sqrt{x}+1\right)^2 dx$ b) $\displaystyle\int \frac{3x}{(2x^2+5)^3}\,dx$

c) $\displaystyle\int \frac{12x}{(3x-1)^2}\,dx$ d) $\displaystyle\int \frac{3x+1}{\sqrt{12x-x^2}}\,dx$

e) $\displaystyle\int \frac{3x+8}{x^2+4x+6}\,dx$ f) $\displaystyle\int \frac{4x-7}{2x+1}\,dx$

g) $\displaystyle\int \arctan x\,dx$ h) $\displaystyle\int \frac{\sqrt{x^2+4}}{x^4}\,dx$

i) $\displaystyle\int \frac{3x+1}{\sqrt{13-12x-x^2}}\,dx$ j) $\displaystyle\int \frac{4x-1}{x^2+2x-15}\,dx$

k) $\displaystyle\int \frac{x^2+2x+4}{x^3+x^2+x+1}\,dx$ ℓ) $\displaystyle\int x\,e^{-x/2}\,dx$

m) $\displaystyle\int \frac{3x-7}{2x^2+7x-9}\,dx$ n) $\displaystyle\int \frac{2x^2+7x-9}{3x-7}\,dx$

The answers for these integrals are given below.

a) $2x^2 + \dfrac{8}{3}x^{3/2} + x + C$

b) $\dfrac{-3}{8(2x^2+5)^2} + C$

c) $\dfrac{4}{3}\Big(\ln|3x-1| - \dfrac{1}{3x-1}\Big) + C$

d) $-3\sqrt{12x - x^2} + 19\arcsin\Big(\dfrac{x-6}{6}\Big) + C$

e) $\dfrac{3}{2}\ln\big(x^2+4x+6\big) + \sqrt{2}\arctan\Big(\dfrac{x+2}{\sqrt{2}}\Big) + C$

f) $2x - \dfrac{9}{2}\ln|2x+1| + C$

g) $x\arctan x - \dfrac{1}{2}\ln(1+x^2) + C$

h) $\dfrac{-(x^2+4)^{3/2}}{12x^3} + C$

i) $-3\sqrt{13-12x-x^2} - 17\arcsin\Big(\dfrac{x+6}{7}\Big) + C$

j) $\dfrac{11}{8}\ln|x-3| + \dfrac{21}{8}\ln|x+5| + C$

k) $\dfrac{3}{2}\ln|x+1| - \dfrac{1}{4}\ln(x^2+1) + \dfrac{5}{2}\arctan x + C$

ℓ) $-2(x+2)e^{-x/2} + C$

m) $\dfrac{41}{22}\ln|2x+9| - \dfrac{4}{11}\ln|x-1| + C$

n) $\dfrac{1}{3}x^2 + \dfrac{35}{9}x + \dfrac{164}{27}\ln|3x-7| + C$

21. Prove the following statement: for each positive integer n, the integer $2^{5n-4} + 5^{2n-1}$ is divisible by 7.

22. Find the limit of the given sequence.

a) $\left\{\dfrac{k}{\sqrt{3k^2+4k+1}}\right\}$ b) $\left\{\sqrt{k^2-7k+15} - k\right\}$ c) $\left\{k\big(\sqrt[k]{10}-1\big)\right\}$

d) $\left\{\Big(1-\dfrac{2}{3n}\Big)^n\right\}$ e) $\left\{\dfrac{4^n+n^2}{2^{2n-3}+n^7}\right\}$ f) $\left\{\sqrt[n]{4n^2+n+3}\right\}$

(The limits are $1/\sqrt{3}$, $-7/2$, $\ln 10$, $e^{-2/3}$, 8, and 1.)

23. Define a sequence $\{x_n\}$ by $x_1 = 5$ and $x_{n+1} = 4 - (1/x_n)$ for $n \geq 1$. Prove that $1 \leq x_n \leq 5$ for all n, then prove that $\{x_n\}$ is a decreasing sequence. Conclude that $\{x_n\}$ converges and find its limit. $\big(2+\sqrt{3}\,\big)$

24. Find the sum of the given series

a) $\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}3^k}{4^{k-1}}$ b) $\displaystyle\sum_{k=1}^{\infty}\frac{3^k+5^k}{7^k}$ c) $\displaystyle\sum_{k=1}^{\infty}\frac{(-4)^k}{(2k+1)!}$

(The sums of the series are 12/7, 13/4, and $\frac{1}{2}\sin 2 - 1$, respectively.)

25. Let $\displaystyle\sum_{k=1}^{\infty} a_k$ be an infinite series and suppose that its sequence $\{s_n\}$ of partial sums is given by $s_n = \dfrac{n+1}{1-3n}$ for all $n \geq 1$. Find a_1, a_2, a_{10}, and the sum of the series.

(The values are -1, $2/5$, $2/377$, and $-1/3$, respectively.)

26. Determine whether or not the given series converges.

a) $\displaystyle\sum_{k=1}^{\infty}\frac{12}{3k+2}$ b) $\displaystyle\sum_{k=1}^{\infty}\frac{4k-1}{k^2+5k+2}$ c) $\displaystyle\sum_{k=1}^{\infty}\frac{2k^2+3}{k^4+7k-1}$

d) $\displaystyle\sum_{k=1}^{\infty}\frac{5^k}{2^k+6^k}$ e) $\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{\sqrt[k]{5}}$ f) $\displaystyle\sum_{k=1}^{\infty}\Big(\frac{k}{3k+1}\Big)^k$

(The series are D, D, C, C, D, and C, respectively.)

27. Classify the series $\sum_{k=1}^{\infty} \frac{(-3)^k k!}{3 \cdot 7 \cdot 11 \cdot \cdots \cdot (4k-1)}$ as AC, CC, or D. (It is AC.)

28. Show that each of the series $\sum_{k=1}^{\infty} \frac{3^k \sin k}{4^k}$ and $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k^2}{k^4 + 3k^2 + 10}$ is absolutely convergent.

29. Carefully prove that each of the series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{5k+4}$ and $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k}{k^2+1}$ is conditionally convergent.

30. Find the radius of convergence for the power series $\sum_{k=1}^{\infty} \frac{1}{k^2 3^k} (x-4)^k$. (3)

31. Find the interval of convergence for the power series $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)2^k} (x-1)^k$. $\big((-1, 3]\big)$

32. Give an example of a power series with $[4, 10)$ as its interval of convergence.

    One example is $\sum_{k=1}^{\infty} \frac{1}{k 3^k} (x-7)^k$.

33. Find (in more familiar terms) the function represented by the power series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2^k} (x+1)^k$.

    This is the power series for the function $f(x) = \frac{x+1}{x+3}$, valid on the interval $(-3, 1)$.

34. Find the Maclaurin series for the function $f(x) = \frac{1}{5-2x}$ and determine its interval of convergence.

    The Maclaurin series is $\sum_{k=0}^{\infty} \frac{2^k}{5^{k+1}} x^k$, with interval of convergence $(-2.5, 2.5)$.

35. By differentiating an appropriate power series (see problem 3.11.2), find the sum of the series $\sum_{k=1}^{\infty} k^3 x^k$.

    $$\sum_{k=1}^{\infty} k^3 x^k = \frac{x + 4x + x^3}{(1-x)^4}.$$

36. Use known series to find the Maclaurin series for the given function.

    a) $f(x) = e^{-x/3}$ **b)** $g(x) = \sin(x^2)$ **c)** $h(x) = \frac{1 - \cos x}{x}$

    $$f(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{3^k k!} x^k, \qquad g(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{4k+2}, \qquad h(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(2k)!} x^{2k-1}$$

37. Use known Maclaurin series to determine in more familiar terms the given function.

    a) $\sum_{k=0}^{\infty} \frac{1}{2^k k!} x^k$ **b)** $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k}$ **c)** $\sum_{k=0}^{\infty} \frac{(-9)^k}{(2k)!} x^{2k+1}$

    The functions are $e^{x/2}$, $\sin x / x$, and $x \cos(3x)$, respectively.

38. Find the Taylor series for the function $f(x) = 1/(2x-1)$ centered at $a = 6$.

    $$\frac{1}{2x-1} = \sum_{k=0}^{\infty} \frac{(-2)^k}{11^{k+1}} (x-6)^k \text{ with } \rho = 5.5.$$