Assignments for Math 126, Fall 2018

(due on the given date)

(8/29) 1. Read the syllabus (available on the website) very carefully; this may take 20 minutes or so.

2. Spend some time reviewing Calculus I, perhaps beginning with the summary of differential calculus posted on the class website.

3. Take the introductory quiz (available on the website) to check on your computational skills.

(8/31) 1. Reread the syllabus if necessary.

2. Do the practice problems for differential calculus, reviewing as necessary.

3. Turn in solutions for homework assignment 1. You should model your solutions based on the sample homework solutions posted on the class website.

4. Browse Section 2.1 of the textbook.

(9/3) 1. Read Section 2.1 carefully.

2. Do problems 1, 2, 3a–e, 4a–b, 5, 6, and 7 in Section 2.1.

3. Turn in solutions for homework assignment 2. Remember to write your solutions clearly.

4. Carefully read the prelude to Chapter 3 (available on the website).

(9/5) 1. Read Section 3.1 carefully.

2. Do problems 1, 2, 3, 4, and 5 in Section 3.1.

3. Read the first two pages of the Model Induction Proofs (see the Prelude to Chapter 3 for the appropriate link). You should be able to find two errors in each of the incorrect proofs. You will also find a solution to problem 4 there, but you should try the problem on your own first.

4. Turn in a carefully written solution for problem 3. You may use either the set S format or the informal style.

(9/7) 1. Reread Section 3.1 if necessary. Spend some time thinking about Fibonacci numbers.

2. Do problems 6 and 7 in Section 3.1. If you have not already done so, you should read the extra notes for Section 3.1, thinking carefully about each sentence of the examples given there.

3. Turn in carefully written solutions for problems 7b and 7d. A comment about problem 7d was discussed in class; be certain you understand what equation you are proving.

4. Browse Section 3.2 of the textbook.

(9/10) 1. Read Section 3.2 carefully.

2. Do the problems in Section 3.2. Some of these will go quickly but others will require some careful thought. Remember that there is a very strong emphasis on thinking as you study this material.

3. Turn in solutions for problems 4e, 4g, 5c, and 6. For 4e, there is no need to switch to x since algebra should be sufficient, whereas for 4g you do need to switch to x since you will most likely be using L'Hôpital's Rule. Be careful with the algebra for 5c; you might want to write out the first three terms of the sequence. (If you do not remember what $n!$ (n factorial) means, see Appendix A.) For 6, ask yourself how many terms are being added to get the general term of the sequence and then consider which of the added terms is the smallest; Example 3 in the extra notes may help to illustrate this idea.

4. Browse Section 3.3 of the textbook.

(9/12) 1. Read Section 3.3 carefully.

2. Do the problems in Section 3.3. For problem 1, you need to write out careful steps as in one of the examples in the section. For the other problems, you will want to do some algebra and/or use the results in Theorem 3.8 and/or use the Squeeze Theorem. The extra notes may provide some helpful examples.

3. Turn in solutions for problems 1b (there should be four steps), 2b, 2h, and 4. You should consider using the Squeeze Theorem for the last two of these problems.

4. Browse Section 3.4 of the textbook.

(9/14) 1. Read Section 3.4 carefully.

2. Do the problems in Section 3.4.

3. Turn in solutions for problems 2, 5, and 7. To prove that the sequence in problem 2 is bounded, consider using some over and under estimates for the terms in the sum. Note that there is no need to prove that the sequence in problem 5 converges since you are told to assume that it converges and all you have to do is find the limit. For problem 7, we have already proved that $1 \leq a_n \leq 3$ (see problem 5 in Section 3.1; just provide a reference in your solution as opposed to solving the problem again) so you just need to use induction to prove the increasing part, then (after concluding that the sequence converges) find the limit.

4. Browse Section 3.5 of the textbook.

(9/17) 1. Read Section 3.5 carefully.

2. Do the problems in Section 3.5. Think carefully about each problem type.
3. Turn in solutions for problems 2b, 3e, 3i, and 5. Note that some useful information for problem 3i is given in the heading for the exercise 3 problems.

(9/19) 1. Read Section 3.7 carefully. (We are omitting Section 3.6.)

2. Do the problems in Section 3.7. Problem 4 requires you to make some estimates about the sizes of the numbers (as we have done with other sums); as a start, determine how many two digit integers do not contain a 0 and how many three digit integers do not contain a 0.
3. Turn in solutions for problems 1b, 2a, 3a, and 3d. When using the Comparison Test, be very careful with the inequalities that you use to be sure they are correct.
4. Browse Section 3.8 of the textbook.

(9/21) 1. Read Section 3.8 carefully.

2. Do (or at least read them carefully) the problems in Section 3.8. Problem 8 is important since you must first decide which of the convergence tests to use. Even if you do not carry out all of the details, think carefully about what the series does and which test to use to verify your conjecture. Be aware that problems 5 and 9 require more thinking than the other problems.
3. Turn in solutions for problems 1b, 2b, 3, and 6 (problem 5 should give you a hint about what type of series to look for).
4. Browse Section 3.9 of the textbook.

(9/24) 1. Read Section 3.9 carefully.

2. Do problems 1, 2, 3, 5, and 6 in Section 3.9. For problem 3, think carefully about the last paragraph in the section as you decide which test to use to check for convergence.
3. Turn in solutions for problems 2f, 3f, and 6c.
4. Browse Section 3.10 of the textbook.
5. Since we have a test coming up very soon (September 28), you should start reviewing the material in Chapter 3.

(9/26) 1. Read Section 3.10 carefully.

2. Do the problems in Section 3.10. For problem 2, be certain to check the endpoints carefully. For problem 6, refer thoughtfully to the prototypes in problem 5.
3. Turn in solutions for the following three problems (read the appropriate headings)

   2d. $\displaystyle\sum_{k=1}^{\infty} \frac{3}{k2^k}\,(x+1)^k$ 6e. $[4, 10)$ 7d. $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{3^k}\,(x-1)^k$

4. See the next page for details concerning the exam on Friday.

(9/28)

1. We have an exam covering the first 10 sections of Chapter 3. See the next item for details.
2. Go over the sections we have covered and review the key concepts. You need to pay careful attention to the definitions and theorems, making certain that you understand them and remember them. Practice some of the problems we have been doing, that is, do some of the problems again without looking at your previous solutions. Look at the exam on this material from a previous year (found on the website; refer to the third exam) and be certain that the problems make sense and that you can do them within a time frame for a 50 minute exam. For additional problems, you can work on problems 1, 14, 5, 6, 11, and 13 (in that order) in Section 3.14. When you feel you are ready, take the diagnostic quiz on Chapter 3. This quiz can be found on the website, along with complete solutions.

(10/1)

1. Review Section 2.1 and browse Section 2.2 of the textbook.

(10/3)

1. Read Section 2.2 carefully.
2. Do problems 1–7 in Section 2.2. Problem 2 requires a careful sketch and then some thought about how to proceed; it is a good example of the type of thinking that is needed to solve nonroutine problems.
3. Turn in solutions for problems 3 (the $y = x^3$ case only), 5d, and 6. Problem 5d is $y = 6 - |2x - 3|$ on $[0, 4]$; apply the directions to problem 5 to this new function and interval. Problem 6 requires some comfort with abstract notation and the use of L'Hôpital's Rule.
4. Browse Section 2.3 of the textbook.

(10/5)

1. No class today due to the October Break.

(10/8)

1. Read Section 2.3 carefully.
2. Do problems 1–6 in Section 2.3.
3. Turn in solutions for problems 2d, 3, and 4, where 2d is $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \left(3 + \frac{4i}{n}\right)^3 \frac{4}{n}$.
4. Browse Section 2.4 of the textbook.

(10/10)

1. Read Section 2.4 carefully.
2. Do problems 2abce, 3, 4, 5, and 6 in Section 2.4. You should find that these problems go more quickly than the problems in previous sections. Do be careful with your notation (remember dx) and be aware of the linearity properties of the integral that you are using.
3. Turn in solutions for problems 2c, 4d, and 5d, where 4d is $\displaystyle\int_{-3}^{3} \left(8\sqrt{9 - x^2} + 10x^4\right) dx$ and 5d uses the interval $1 \le t \le 6$.
4. Browse Section 2.5 of the textbook.

(10/12) 1. Read Section 2.5 carefully.

2. Do problems 1ab, 2, 3, 4, 5, and 7 in Section 2.5.

3. Turn in solutions for problems 3 (giving clear details), 4d, and 5 (using appropriate steps), where problem 4d is $\int_1^2 \big(3f(x) - 6x^2\big)\,dx$.

4. Browse Section 2.6 of the textbook.

(10/15) 1. Read Section 2.6 carefully.

2. Do problems 1acdefh, 2, 3, 4, 5, and 6 in Section 2.6. You will find that some of these problems force you to think outside the box; do not give up on them too quickly.

3. Turn in solutions for problems 1j, 2, 3, and 5′, where the function for 1j is $f(x) = \int_0^{3x^2} \ln(1+t^2)\,dt$ and 5′ requests a function f so that $f(7) = 0$, $f'(x) = e^{-2x^2}$.

4. Browse Section 2.7 of the textbook.

(10/17) 1. Read Section 2.7 carefully.

2. Do problems 1, 2, and 3 in Section 2.7. Remember that we are not using any techniques of integration; we are simply thinking about differentiation in reverse. Since calculators will not be allowed on the exam, you should do all of these problems without the aid of an electronic device.

3. Turn in solutions for problems 1m, 1n, 1o, and 2, given

$$\text{1m. } \int_2^4 \frac{8}{x^3}\,dx \qquad \text{1n. } \int_0^1 \big(2x^2 + \sqrt[3]{x}\,\big)\,dx \qquad \text{1o. } \int_0^4 \frac{3}{x+4}\,dx$$

4. Browse Section 2.8 of the textbook.

(10/19) 1. Read Section 2.8 carefully.

2. Do the problems in Section 2.8; there are many integrals here but doing them is good practice. It is especially important that you know the general form for what the antiderivative should look like and that you check your answers to brush up on your differentiation skills.

3. Turn in solutions for problems 1p, 2g, and 3′, given

$$\text{1p. } \int \frac{24}{(3x+1)^3}\,dx \qquad \text{2g. } \int_0^1 \frac{8x+4}{x^2+x+1}\,dx \qquad 3'.\ \int_0^2 (2x-3)\sqrt{4-x^2}\,dx$$

Be careful with the distributive property as you do 3′.

(10/22) 1. Read Section 2.9 carefully.

2. You should look over the integrals in problem 1 and think about how you would solve each of them. Can you just write down the answer? Can you use guess and check? Do you need to make a substitution and, if so, what would u be? After making this assessment, do a few of each type beginning with 1b, 1e, 1i, 1j, 1k, and 1l. Repeat this process for problem 2, then begin with 2a, 2b, and 2e. Remember to change the limits for the definite integrals when using u-substitution. Problems 3 and 4 indicate that there is more than one way to find an antiderivative while problem 5 shows how to determine the formula for the area of an ellipse. You can omit problem 6.

3. Turn in solutions for problems 1h, 1m, 2d, and 2g, where the extra problems are

$$\text{1m.} \int \frac{x+3}{\sqrt{x-4}}\,dx \qquad\qquad \text{2g.} \int_0^4 \frac{1}{1+\sqrt{x}}\,dx$$

4. Browse Section 2.10 of the textbook.

(10/24) 1. Read Section 2.10 carefully.

2. Do problems 1a, 1b, 1c, 1e, 1i, 1j, 2b, 2c, 2d, 2e, 3, and 4 in Section 2.10.

3. Look over Section 2.11 as well as the extra notes for this section of Chapter 2.

4. Do problems 1a, 1d, 1e, 1f and 2a in Section 2.11.

5. Turn in solutions for problems 1i, 2b, and 3 in Section 2.10 and problem 1f in Section 2.11.

6. We have a test on Friday, October 26. We will review for the exam during Wednesday's class.

(10/26) 1. We have our second exam this day, covering Sections 2.2 through 2.11.

2. The questions on the test will involve ideas similar to the homework problems you have been doing the past few weeks. You can find exams (and solutions) from a previous semester on the class website. However, it is important to remember that this is NOT a practice exam; our exam may look very different than this one. In addition to the problems in Sections 2.2 to 2.11 (for the record, doing those problems again without consulting your notes can be helpful), you can try problems 4 through 12 in Section 2.24. For further practice, you can look over the integrals at the link 'Basic integration problems' on the website.

3. You need to be able to state the definition of the derivative, the definition of the integral, and both parts of the Fundamental Theorem of Calculus. All of these statements involve knowing all of the words, not just a few symbols.

4. No calculators or electronic devices will be allowed during the exam. You should plan your morning so that there is no need to leave the classroom during the exam. However, if it is necessary to do so, I ask that you leave your phone on your desk as you go. You will have 55 minutes for the exam. It is a good idea to show up a few minutes early if possible so that you are completely ready to begin the exam at the top of the hour. Remember to leave an empty seat between you and other students.

(10/29) 1. Browse Section 2.12 of the textbook.

(10/31) 1. Read Section 2.12 carefully.

2. Do the problems in Section 2.12. For some of these, sketch a careful graph and think about the problem before you set up the integrals.
3. Turn in solutions for problems 1e, 4, and 5b. For problem 4, sketch a careful graph and decide which direction most easily describes the region in question.
4. Browse Section 2.13 of the textbook.

(11/2) 1. Read Section 2.13 carefully.

2. Do the problems in Section 2.13.
3. Turn in solutions for problems 2, 7, and 9.
4. Browse Section 2.14 of the textbook.

(11/5) 1. Read Section 2.14 carefully.

2. Do the problems in Section 2.14.
3. Turn in solutions for problems 1b, 6a, and 8.
4. Browse Section 2.15 of the textbook.

(11/7) 1. Read Section 2.15.

2. Do problems 1, 2, and 3 in Section 2.15.
3. Turn in solutions for problems 2c and 2d.
4. Browse Section 2.17 of the textbook.

(11/9) 1. Read Section 2.17.

2. Do problems 3, 4, 5, 6, 8, and 13 in Section 2.17.
3. Turn in solutions for problems 4 and 8.
4. Browse Section 2.18 of the textbook.

(11/12) 1. Read Section 2.18.

2. Do problems 1, 2, 3, 4, and 5 in Section 2.18.
3. Turn in solutions for problems 2, 3b, and 4.
4. Browse Section 2.19 of the textbook.

(11/14) 1. Read Section 2.19 carefully.

2. Do the problems in Section 2.19. Remember that these problems involve more algebra than calculus.
3. Turn in solutions for the three problems listed below.

$$\int \frac{2x-1}{x^2+6x+13}\,dx, \qquad \int \frac{4x-3}{\sqrt{33+8x-x^2}}\,dx, \qquad \int \frac{6x^3+14x+7}{x^2+9}\,dx.$$

4. Browse Section 2.20 of the textbook. It will be helpful (in fact, almost essential) to have access to Appendix B of the textbook (the table of integrals) for class on this day.

(11/16)

1. Read Section 2.20 carefully.
2. Do problems 1, 2, 4, 5, and 6abef in Section 2.20.
3. Turn in solutions for problems 1g, 5, and 6f.
4. Browse Section 2.21 of the textbook.

(11/26)

1. Read Section 2.21 carefully.
2. Do (or at least make the appropriate trig substitution and determine the new integral for) the problems in Section 2.21.
3. Turn in solutions for problems 1b, 2d, and 1g. $\int \frac{x^4}{\sqrt{4-x^2}}\,dx$ (use a reduction formula after making the trig substitution).
4. After working on problem 1g, go to the website for Wolfram Alpha and try typing in this integration problem. How does the answer given there compare with your answer? Then replace the minus sign in the square root with a plus sign and ask Wolfram Alpha to find the new integral. Do you understand the answer in its given form?
5. Browse Section 2.22 of the textbook.

(11/28)

1. Read Section 2.22 carefully.
2. Do problems 1a,b,c,d,g,k in Section 2.22.
3. Turn in solutions for problems 1b and 1g.
4. We will review for the exam during this class period.

(11/30)

1. We have an exam on integration and the integral applications we have discussed.
2. For the exam, you need to be able to state the definition of the derivative, the definition of the integral, and both parts of the Fundamental Theorem of Calculus. All of these statements involve knowing all of the words, not just a few symbols.
3. The first step in your review should be to go over the new sections we have covered to make sure you understand the main ideas. You can then redo the problems in each section and you can work on problems 15, 17, 18, 27, 34, 35, and 36 in Section 2.24. You can also look at the second exam from Spring 2011 as well as the problems at the link 'Review for exam on integration.' For the latter, you can skip problem 30. There are lots and lots of problems here so you need to use discretion as you decide how many of these to try. You should consider spending more time on making certain that you know how to start the problem and/or set up the integral rather than getting lost in the simple but tedious details of calculations.

(12/3)

1. Review the first ten sections of Chapter 3.
2. Do take item 1 seriously. We have not looked at this material for a while and it will be included on the final exam. Flip through the sections to remind yourself of the topics, think about the various tests for the convergence of a series, and be certain that you are familiar with power series.
3. Browse Section 3.11 of the textbook.

(12/5) 1. Read Section 3.11 carefully.

2. Do problems 1, 2, 3, and 4 in Section 3.11.

3. I will not be collecting any of these problems. However, if you want some feedback, I am happy to look over your work on this material. If you are so inclined, turn in solutions for the following three problems (read the appropriate problem number headings). For problem $2'$, start with the formula for the series in problem 2 (found in the solutions). For problem 4d, use results in this section along with ideas we have already learned.

$$1g.\ f(x) = \frac{x^2}{4x+7} \qquad 2'.\ \sum_{k=1}^{\infty} k^3 x^k \qquad 4d.\ \sum_{k=1}^{\infty} \frac{2k^2+5}{3^k}$$

4. Browse Section 3.12.

(12/7) 1. Read Section 3.12 carefully.

2. Do problems 1, 2, 3, 4, and 5 in Section 3.12. Take your time on these problems and make sure that you understand what is going on.

3. We will discuss the final exam during class on this day, but we will most likely be spending the majority of our time on Taylor and Maclaurin series. Specific details concerning the final exam are provided in the following pages. Recall that the final exam schedule for all classes can be found on the Registrar's page.

As you may recall from reading the syllabus, the goals for this course are

- to develop quantitative reasoning skills;
- to learn how to read technical material;
- to learn to write technical information correctly and clearly;
- to take pride in your work and to avoid errors;
- to learn how to solve non-routine problems;
- to appreciate/understand how mathematicians view mathematics;
- to comprehend some aspects of calculus.

It is with these goals in mind that the final exam will be written. The exam is comprehensive and covers all of the sections that we have discussed this semester, with a slight emphasis on the more recent material. The final exam will require the skills and concepts that you have been practicing and pondering this semester. It is your responsibility to go back over the sections and make certain that you know how to do the types of problems we have encountered. A few of the problems on the final exam will be more involved than the sorts of problems that have appeared on the other exams that we have had. This should not be that much of a surprise because most of the test questions have been easier and shorter than homework problems due to the time constraints of a 50 minute exam. It is now time for you to put all your knowledge together and show me what you have learned this semester.

Here is the (most likely) introduction to the final exam that you will be taking.

Write neat, concise, and accurate solutions to each of the problems in the space provided—I will not give any credit for steps I cannot follow. Your solutions should be written in the style expected for collected homework problems. Pay particular attention to correct use of notation and use complete sentences when appropriate. Each of the * problems is worth * points (making a total of 80). No electronic devices or calculators are allowed for this exam.

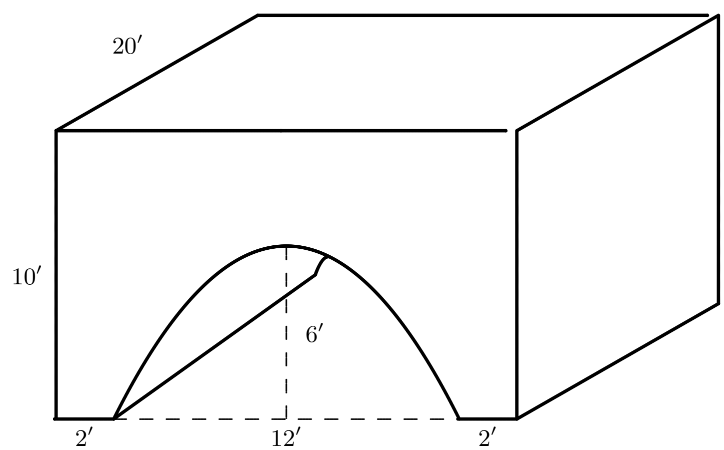
The problems will be graded as indicated in the heading so it is important that you work toward avoiding computational errors and that you pay attention to your writing and notation. The best advice is to review for the exam by looking over the sections in the text that we have covered, thinking carefully about the ideas we have discussed, and understanding the types of problems that have appeared on previous exams. You can redo problems from the sections in the textbook, you can look over your previous exams, and you can work on some of the review problems that appear on the next few pages. It is important that you arrive at the exam with a refreshed mind and body, and be prepared to stay positive and work hard for up to three hours. As just indicated, although the exam is written for a two hour period, you may have three hours for the exam. The exam time period is thus 9:00–12:00 or 2:00–5:00 on the appropriate day of our final exam.

As should come as no surprise, it is expected that you can state (as well as understand) the definition of the derivative, the definition of the integral, and both versions of the Fundamental Theorem of Calculus. Do not lose points by ignoring this fact. You should also be familiar with basic integration formulas and techniques of integration, be able to solve problems involving applications of the integral, understand the main ideas behind sequences, series, and power series, and know the Maclaurin series for e^x, $\sin x$, and $\cos x$. This is just a sampling of the things that you need to know for the exam; if you have been keeping up during the semester, it should not be too difficult to remember the common formulas and ideas that we have been using.

The following problems are not representative of the final exam. They are simply intended to give you some indication of the nature of a more difficult or novel problem that may appear on the final exam. Having said that, I do recommend that you give them some thought. However, keep in mind that most of the problems on the final exam will be (or at least should be) quite familiar to you. You can look at the final exam from Spring 2011 that is located on the course website but be aware that our final exam will not necessarily look like this.

1. Consider two different solids. The base of each solid is a triangle with vertices $(0, 0)$, $(2, 4)$, and $(6, 0)$. For solid A, each cross-section perpendicular to the y-axis is an equilateral triangle. For solid B, each cross-section perpendicular to the x-axis is a square. Find the ratio of the volume of solid A to the volume of solid B.

2. Find the number of cubic yards of concrete necessary to construct the culvert shown below. Assume that the arch of the culvert (which is empty space) has a parabolic shape.



3. Let $a_1 = 2$ and $a_{n+1} = 3 - (1/a_n)$ for each positive integer $n \geq 1$. Use mathematical induction to prove that $a_n = \dfrac{f_{2n+1}}{f_{2n-1}}$ for each positive integer n. Here f_n refers to the nth Fibonacci number.

4. For each positive integer n, let

$$y_n = \frac{1}{3n+2} + \frac{1}{3n+4} + \frac{1}{3n+6} + \cdots + \frac{1}{5n}.$$

   Find the limit of the sequence $\{y_n\}$. (Try writing y_n in summation notation and think about integrals.)

5. Determine (using familiar calculus functions) the function represented by $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k(2k+1)}{4^k k!}\, x^{2k}$.

The problems that follow are more similar to the problems that you have been doing as homework. As you try these problems, put yourself in the mindset of an exam. That is, do not use your notes or look at the answer until you have finished the problem. Pay attention to problems you do not know how to start (these problems represent a lack of understanding) and problems you know how to start but get incorrect answers at the end (these problems indicate of lack of attention to detail).

Miscellaneous problems to try before the final exam

Since many of the answers are given right after the problem, you need to be careful to avoid using the answer as a hint for how to start the problem as this does not mimic a testing situation. Proceed without technology if at all possible. Omit problems marked with an asterisk.

1. * Evaluate the limit $\lim_{n\to\infty} \dfrac{n^4+n^2+1}{3^3+6^3+9^3+\cdots+(3n)^3}$. (4/27)

2. Use an integral to evaluate the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \left(5+\dfrac{3i}{n}\right)^2 \dfrac{3}{n}$. (129)

3. Evaluate each of the following integrals.

   a) $\int_{-1}^{2} (2x-3)(x-1)\,dx$ b) $\int_{0}^{3} (4x+2|x-1|)\,dx$ c) $\int_{0}^{1} \left(2t-3+2\sqrt{1-t^2}\right) dt$

   d) $\int_{1}^{8} \dfrac{x+2}{\sqrt[3]{x}}\,dx$ e) $\int_{1}^{4} \dfrac{1}{3x-2}\,dx$ f) $\int_{0}^{2} \dfrac{1}{4+x^2}\,dx$

   (The values are 15/2, 23, $\dfrac{\pi}{2}-2$, 138/5, $\frac{1}{3}\ln 10$, and $\pi/8$, respectively.)

4. * Use a simpler function to approximate $\int_{1}^{2} \dfrac{1}{\sqrt{4x^6-1}}\,dx$. Is your estimate high or low? (3/16, low)

5. Find the derivative of the function F defined by $F(x) = \int_{0}^{x^2} t\sqrt{t^3+4}\,dt$. $\left(F'(x) = 2x^3\sqrt{x^6+4}\right)$

6. Suppose that $v(t) = 3t - t^3$ gives the velocity in meters per second of a particle at time t seconds. Find the distance traveled by the particle for the time interval $0 \le t \le 4$. (44.5 meters)

7. Find the area of the region bounded by the curves $x^2y = 90$ and $40x + y = 130$. (40)

8. Find the area of the region bounded by the curves $y = 2\sqrt{x}$ and $y = x^3/16$. (20/3)

9. Find the volume of the solid that is generated when the region bounded by the curves $y = 4x$ and $y = x^2$ is revolved around (a) the x-axis (b) the y-axis. $\left(\left(\dfrac{2048}{15}\pi\right) \text{ and } \left(\dfrac{128}{3}\pi\right)\right)$

10. Suppose the base of a solid is the part of the parabola $y = 8 - 0.5x^2$ that lies above the x-axis and that each cross section perpendicular to the y-axis is a semicircle. Find the volume of this solid. (32π)

11. Find the volume of the solid that is generated when the region that lies below the curve $y = \ln x$ and above the x-axis on the interval $[1, e]$ is revolved around the y-axis. $\left(\pi(e^2+1)/2\right)$

12. Find the volume of the solid that is generated when the region that lies above the x-axis and below the curve $y = \sqrt{\dfrac{14(3-x)}{(x+1)(7-x)}}$ on the interval $[0, 3]$ is revolved around the x-axis. $\left(7\pi(\ln 16 - \ln 7)\right)$

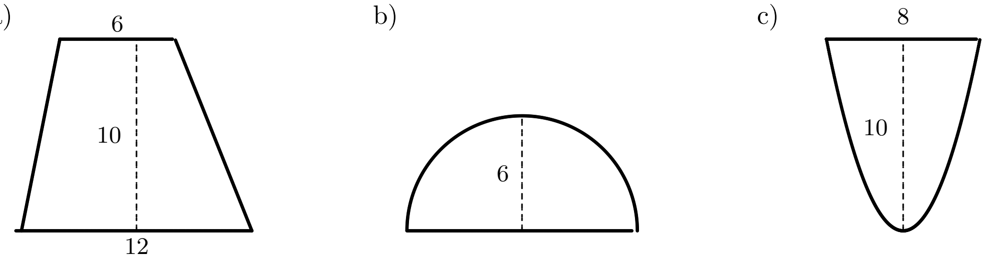
13. Find the center of mass of the region bounded by the curves $y = 2\sqrt{x}$ and $y = x^3/16$. $\left(\dfrac{48}{25}, \dfrac{12}{7}\right)$

14. Find the center of mass of the solid that is generated when the region below the curve $y = 4e^{-x/4}$ and above the x-axis on the interval $[0, \infty)$ is revolved around the x-axis. $(2, 0, 0)$

15. Find the length of the curve $y = 4x^{3/2}$ on the interval $[0, 10]$. (127)

16. Find the force exerted by a liquid with weight density w on one side of each vertically submerged plate. The units on the figures are feet and the top of each plate is six feet beneath the surface of the liquid.

a) b) c)



trapezoid semicircle parabola

(The forces are $1040w$, $(216\pi - 144)w$, and $1600w/3$ pounds, respectively.)

17. Evaluate each of the following definite integrals.

a) $\displaystyle\int_0^2 \frac{x}{4+x^2}\,dx$ b) $\displaystyle\int_0^2 \frac{x}{\sqrt{4+x^2}}\,dx$ c) $\displaystyle\int_0^2 \frac{x}{\sqrt{4+x}}\,dx$

d) $\displaystyle\int_1^\infty \frac{8}{(2x+5)^3}\,dx$ e) $\displaystyle\int_0^\infty \frac{6+e^{2x}}{e^{3x}}\,dx$ f) $\displaystyle\int_1^{\sqrt{2}} \frac{\sqrt{x^2-1}}{x^4}\,dx$

(The values are $\frac{1}{2}\ln 2$, $2\left(\sqrt{2}-1\right)$, $\dfrac{32}{3} - 4\sqrt{6}$, $2/49$, 3, and $\sqrt{2}/12$, respectively.)

18. * Use the trapezoid rule and Simpson's rule with $n = 4$ to approximate $\displaystyle\int_0^1 e^{-x^2/2}\,dx$ to four decimal places. (The approximations are 0.8526 and 0.8557, respectively.)

19. * Suppose that the following table represents the velocity of a particle moving in a straight line.

t	(sec)	0	1	2	3	4	5	6
v	(m/sec)	0	5	10	12	8	4	0

Use Simpson's rule to approximate the distance traveled by the particle. (40 meters)

20. Evaluate each of the following indefinite integrals.

a) $\displaystyle\int \left(2\sqrt{x}+1\right)^2 dx$ b) $\displaystyle\int \frac{3x}{(2x^2+5)^3}\,dx$

c) $\displaystyle\int \frac{12x}{(3x-1)^2}\,dx$ d) $\displaystyle\int \frac{3x+1}{\sqrt{12x-x^2}}\,dx$

e) $\displaystyle\int \frac{3x+8}{x^2+4x+6}\,dx$ f) $\displaystyle\int \frac{4x-7}{2x+1}\,dx$

g) $\displaystyle\int \arctan x\,dx$ h) $\displaystyle\int \frac{\sqrt{x^2+4}}{x^4}\,dx$

i) $\displaystyle\int \frac{3x+1}{\sqrt{13-12x-x^2}}\,dx$ j) $\displaystyle\int \frac{4x-1}{x^2+2x-15}\,dx$

k) $\displaystyle\int \frac{x^2+2x+4}{x^3+x^2+x+1}\,dx$ ℓ) $\displaystyle\int x\,e^{-x/2}\,dx$

m) $\displaystyle\int \frac{3x-7}{2x^2+7x-9}\,dx$ n) $\displaystyle\int \frac{2x^2+7x-9}{3x-7}\,dx$

The answers for these integrals are given below.

a) $2x^2 + \frac{8}{3}x^{3/2} + x + C$

b) $\frac{-3}{8(2x^2+5)^2} + C$

c) $\frac{4}{3}\Big(\ln|3x-1| - \frac{1}{3x-1}\Big) + C$

d) $-3\sqrt{12x - x^2} + 19\arcsin\Big(\frac{x-6}{6}\Big) + C$

e) $\frac{3}{2}\ln\big(x^2+4x+6\big) + \sqrt{2}\arctan\Big(\frac{x+2}{\sqrt{2}}\Big) + C$

f) $2x - \frac{9}{2}\ln|2x+1| + C$

g) $x\arctan x - \frac{1}{2}\ln(1+x^2) + C$

h) $\frac{-(x^2+4)^{3/2}}{12x^3} + C$

i) $-3\sqrt{13-12x-x^2} - 17\arcsin\Big(\frac{x+6}{7}\Big) + C$

j) $\frac{11}{8}\ln|x-3| + \frac{21}{8}\ln|x+5| + C$

k) $\frac{3}{2}\ln|x+1| - \frac{1}{4}\ln(x^2+1) + \frac{5}{2}\arctan x + C$

ℓ) $-2(x+2)e^{-x/2} + C$

m) $\frac{41}{22}\ln|2x+9| - \frac{4}{11}\ln|x-1| + C$

n) $\frac{1}{3}x^2 + \frac{35}{9}x + \frac{164}{27}\ln|3x-7| + C$

21. Prove the following statement: for each positive integer n, the integer $2^{5n-4} + 5^{2n-1}$ is divisible by 7.

22. Find the limit of the given sequence.

a) $\Big\{\frac{k}{\sqrt{3k^2+4k+1}}\Big\}$

b) $\big\{\sqrt{k^2-7k+15} - k\big\}$

c) $\big\{k\big(\sqrt[k]{10}-1\big)\big\}$

d) $\Big\{\Big(1-\frac{2}{3n}\Big)^n\Big\}$

e) $\Big\{\frac{4^n+n^2}{2^{2n-3}+n^7}\Big\}$

f) $\big\{\sqrt[n]{4n^2+n+3}\big\}$

(The limits are $1/\sqrt{3}$, $-7/2$, $\ln 10$, $e^{-2/3}$, 8, and 1.)

23. Define a sequence $\{x_n\}$ by $x_1 = 5$ and $x_{n+1} = 4 - (1/x_n)$ for $n \geq 1$. Prove that $1 \leq x_n \leq 5$ for all n, then prove that $\{x_n\}$ is a decreasing sequence. Conclude that $\{x_n\}$ converges and find its limit. $\big(2+\sqrt{3}\big)$

24. Find the sum of the given series.

a) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}3^k}{4^{k-1}}$

b) $\sum_{k=1}^{\infty} \frac{3^k+5^k}{7^k}$

c) $\sum_{k=1}^{\infty} \frac{(-4)^k}{(2k+1)!}$

(The sums of the series are 12/7, 13/4, and $\frac{1}{2}\sin 2 - 1$, respectively.)

25. Let $\sum_{k=1}^{\infty} a_k$ be an infinite series and suppose that its sequence $\{s_n\}$ of partial sums is given by $s_n = \frac{n+1}{1-3n}$ for all $n \geq 1$. Find a_1, a_2, a_{10}, and the sum of the series.

(The values are -1, 2/5, 2/377, and $-1/3$, respectively.)

26. Determine whether or not the given series converges.

a) $\sum_{k=1}^{\infty} \frac{12}{3k+2}$

b) $\sum_{k=1}^{\infty} \frac{4k-1}{k^2+5k+2}$

c) $\sum_{k=1}^{\infty} \frac{2k^2+3}{k^4+7k-1}$

d) $\sum_{k=1}^{\infty} \frac{5^k}{2^k+6^k}$

e) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt[k]{5}}$

f) $\sum_{k=1}^{\infty} \Big(\frac{k}{3k+1}\Big)^k$

(The series are D, D, C, C, D, and C, respectively.)

27. Classify the series $\sum_{k=1}^{\infty} \frac{(-3)^k k!}{3 \cdot 7 \cdot 11 \cdot \cdots \cdot (4k-1)}$ as AC, CC, or D. (It is AC.)

28. Show that each of the series $\sum_{k=1}^{\infty} \frac{3^k \sin k}{4^k}$ and $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k^2}{k^4 + 3k^2 + 10}$ is absolutely convergent.

29. Carefully prove that each of the series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{5k+4}$ and $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k}{k^2+1}$ is conditionally convergent.

30. Find the radius of convergence for the power series $\sum_{k=1}^{\infty} \frac{1}{k^2 3^k} (x-4)^k$. (3)

31. Find the interval of convergence for the power series $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)2^k} (x-1)^k$. $\big((-1, 3]\big)$

32. Give an example of a power series with $[4, 10)$ as its interval of convergence.

    One example is $\sum_{k=0}^{\infty} \frac{1}{(2k+1)3^k} (x-7)^k$.

33. Find (in more familiar terms) the function represented by the power series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2^k} (x+1)^k$.

    This is the power series for the function $f(x) = \frac{x+1}{x+3}$, valid on the interval $(-3, 1)$.

34. Find the Maclaurin series for the function $f(x) = \frac{1}{5-2x}$ and determine its interval of convergence.

    The Maclaurin series is $\sum_{k=0}^{\infty} \frac{2^k}{5^{k+1}} x^k$, with interval of convergence $(-2.5, 2.5)$.

35. By differentiating an appropriate power series (see problem 3.11.2), find the sum of the series $\sum_{k=1}^{\infty} k^3 x^k$.

    $$\sum_{k=1}^{\infty} k^3 x^k = \frac{x + 4x^2 + x^3}{(1-x)^4}.$$

36. Use known series to find the Maclaurin series for the given function.

    a) $f(x) = e^{-x/3}$ **b)** $g(x) = \sin(x^2)$ **c)** $h(x) = \frac{1 - \cos x}{x}$

    $f(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{3^k k!} x^k$, $g(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{4k+2}$, $h(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(2k)!} x^{2k-1}$

37. Use known Maclaurin series to determine in more familiar terms the given function. (See exercises 28 and 29 in Section 3.14 if you want more practice for these types of problems.)

    a) $\sum_{k=0}^{\infty} \frac{1}{2^k k!} x^k$ **b)** $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k}$ **c)** $\sum_{k=0}^{\infty} \frac{(-9)^k}{(2k)!} x^{2k+1}$

    The functions are $e^{x/2}$, $\sin x/x$, and $x \cos(3x)$, respectively.

38. Find the Taylor series for the function $f(x) = 1/(2x-1)$ centered at $a = 6$.

    $$\frac{1}{2x-1} = \sum_{k=0}^{\infty} \frac{(-2)^k}{11^{k+1}} (x-6)^k \text{ with } \rho = 5.5.$$