Assignments for Math 126, Fall 2020

(due on the given date)

Zoom office hour link (3–4 pm): `https://whitman.zoom.us/j/98199577545`

(8/17)

1. Read the introduction to the class (available on the website) and respond to the email request there.
2. Turn in solutions for homework assignment 0 found at the link 'Assignments To Turn In for Fall 2020'. This is a sample assignment so that we can check how homework submission and homework grading is going to work and identify any problems before classes actually begin. It should not take too long and I am hoping you can get it done before you get busy with orientation materials and the like. Here is one option for how to handle these assignments. (1) Print a copy of the assignment. (2) Write your solutions on the copy. (3) Convert the completed assignment into a pdf file, either by the phone app (see the Introduction) or by some other method. (4) Send the pdf file to me via email (gordon@whitman.edu) as an attachment. If you cannot print the assignment, you can simply copy the problems onto your own paper and write your solutions, then do items (3) and (4) above. You may have other means to turn your completed work into a pdf file and this may work better than the phone app.
3. This assignment is not intended to freak you out or send you into worry mode about the course. In fact, I will not be recording any grades for this assignment. The problems are essentially applications of the quadratic formula, with a partial check on your recall of logarithms. A calculator can easily give you the answers so you can use that as a check on your work. However, the intention is to obtain exact answers ($\sqrt{2}$ is an exact answer while 1.414 is an approximate answer). I also want you to start practicing writing your answers more carefully. I DO NOT want some scribbled numbers/equations and a circled answer. Model your solutions on the sample solution given as problem 0 on the assignment.
4. The purpose of the assignment is threefold: (1) a check on how grading will work for the semester; (2) a chance for you to get your brain into math mode again; (3) a brief review of some ideas from precalculus. Let me know if you have questions.

(8/24)

1. Read the syllabus (available on the website) very carefully; this may take 20 minutes or so.
2. Spend some time reviewing Calculus I, perhaps beginning with the summary of differential calculus posted on the class website.
3. Watch the 20 minute youtube video `https://youtu.be/IljF9qGEUXo` on the history of calculus. [minor correction: the Pythagorean Theorem is Proposition 47 of Book I (not 46)]

(8/26)

1. Reread the syllabus to pick up any tidbits you missed the first time.
2. Watch the 30 minute youtube video `https://youtu.be/2pLi43efXAM`. [As is evident, I need more practice with the tablet.]
3. Do the practice problems for differential calculus (refer to the website), reviewing as necessary.
4. Turn in solutions for homework assignment 1 (available on the website), submitting your work as a pdf file to gordon@whitman.edu. You should model your solutions based on the sample homework solutions posted on the class website. As mentioned in the syllabus, it is recommended that you sketch a solution on a separate page before actually writing your final version for submission.
5. As mentioned in an email, make sure that you attach your pdf file directly to your email to me. It is quite helpful if the subject line of your email is (in this case) HW assg 1 (or something similar) and the name of the attached file has the following exact form `email_hw_01.pdf`, where email means the part of your whitman email address prior to the @ symbol. For example, I would name my homework file `gordon_hw_01.pdf`.
6. Carefully read the brief introduction to Chapter 2.

(8/28)

1. Look over Sections 2.1, 2.2, and 2.3, but do not get lost in the details.
2. Watch the 37 minute youtube video `https://youtu.be/6Muu_e_h1X0`.
3. Do exercises 1, 3ad, and 4ab in Section 2.1, exercise 5 in Section 2.2 (you need to use some geometry for part (c)), and exercises 1, 2, and 6 in Section 2.3.
4. Turn in solutions for homework assignment 2 (available on the website). See the previous assignment for how to submit your assignment. As mentioned there, the subject line should be something like HW assg 2 and the attached file should have the form `email_hw_02.pdf`, where email means the part of your whitman email address prior to the @ symbol.

(8/31)

1. Look over Sections 2.4 and 2.5, but do not get lost in the details.
2. Watch the 33 minute youtube video `https://youtu.be/zkffC0pIxSU`.
3. Do exercises 3, 4b, and 5 in Section 2.4. You can use the properties of the integral and area to solve these problems. Also, do exercises 1b, 2, 3, 4, and 7 in Section 2.5. These problems should not take too long so do not spend too much time on any one problem.
4. Turn in solutions for homework assignment 3 (available on the website). See the previous assignment for how to submit your assignment. As mentioned there, the subject line should be something like HW assg 3 and the attached file should have the form `email_hw_03.pdf`, where email means the part of your whitman email address prior to the @ symbol.

(9/2)

1. Read Section 2.6 carefully.
2. Watch the 30 minute youtube video `https://youtu.be/R5HryLeOHVU`.
3. Do problems 1acdefh, 2, 3, 4, 5, and 6 in Section 2.6. You will find that some of these problems force you to think outside the box; do not give up on them too quickly.
4. Turn in solutions for homework assignment 4 (available on the website), remembering to follow the guidelines for the subject line and the name of the pdf file.

(9/4)

1. Read Section 2.7 carefully.
2. Watch the 28 minute youtube video `https://youtu.be/yz-ahbr0cLM`.
3. Do problems 1, 2, and 3 in Section 2.7. Remember that we are not using any techniques of integration; we are simply thinking about differentiation in reverse. Since calculators may not be allowed on the exam, you should do all of these problems without the aid of an electronic device.
4. Turn in solutions for homework assignment 5 (available on the website), remembering to follow the guidelines for the subject line and the name of the pdf file.

(9/7)

1. Read Section 2.8 carefully.
2. Watch the 46 minute (I guess I had a lot to say!) youtube video `https://youtu.be/ZsyMkbIo23Y`.
3. Do the problems in Section 2.8; there are many integrals here but doing them is good practice. You should focus on learning to quickly recognize the general form of the antiderivative and thus how to get started on the problem.
4. Turn in solutions for homework assignment 6 (available on the website), remembering to follow the guidelines for the subject line and the name of the pdf file.

(9/9)

1. Read Section 2.9 carefully.
2. Watch the 41 minute youtube video `https://youtu.be/pstbYEd6-oo`.
3. You should look over the integrals in problem 1 and think about how you would solve each of them. Can you just write down the answer? Can you use guess and check? Do you need to make a substitution and, if so, what would u be? After making this assessment, do a few of each type beginning with 1b, 1e, 1i, 1j, 1k, and 1l. Repeat this process for problem 2, then begin with 2a, 2b, and 2e. Remember to change the limits for the definite integrals when using u-substitution. Problems 3 and 4 indicate that there is more than one way to find an antiderivative while problem 5 shows how to determine the formula for the area of an ellipse. You can omit problem 6.
4. Turn in solutions for homework assignment 7.
5. We will have a class meeting during our assigned class time on this day; a Zoom link will be sent to you on Monday or Tuesday.

(9/11)

1. We have our first exam this day, covering Sections 2.1 through 2.9.
2. The questions on the exam will be similar to the homework problems you have been doing the past three weeks and/or involve the ideas that have been presented thus far. You can find exams (and solutions) from a previous semester on the class website. However, it is important to remember that this is NOT a practice exam; our exam may look rather different than this one. For the Spring 2011 First Exam only problems 1, 2, 3, 5, 6, 7 (the antiderivative only), and 9, along with the extra problems 1 to 4 are relevant. In addition to the problems that were assigned in Sections 2.1 to 2.9 (for the record, doing those problems again without consulting your notes can be helpful), you can try problems 5, 6, 9, 10, 11, and 12 in Section 2.24. For further practice evaluating integrals using ideas that we have discussed thus far, you can look over the integrals at the link 'Basic integration problems' on the website, omitting problems 16, 23, 26, 33, and 36. Looking at these problems and deciding the best approach for solving each integral (even if you do not carry out the details) is a valuable skill to acquire.
3. The exam will be posted at 7:30 am (Pacific time) Friday and it is due by 5:00 pm (Pacific time) Friday. You are on the honor system to spend less than 90 minutes on the exam and to work independently on the problems. To be specific concerning what it means to work alone, you may not seek help from other students, you may not seek help from any other person, you may not seek help from any textbooks or notes, you may not seek help from the Internet, and you may not seek help from Maple, Wolfram Alpha, or any electronic devices (with the obvious exceptions for creating and sending files). Even talking/texting briefly to someone about the problems or sharing notes is a violation of the policy. Any violation will result in a case of academic dishonesty (refer to the academic honesty form that you signed when you first arrived at Whitman). Except for the printing, scanning, and submitting aspects, taking the exam would be as if you had been in a classroom on campus. By signing your name on the exam, you agree to these guidelines.
4. You need to be able to state the definition of the derivative (Definition 1.7 in Section 1.7), the definition of the integral (Definition 2.1 in Section 2.3), and both parts of the Fundamental Theorem of Calculus (Theorem 2.4 in Section 2.6 and Theorem 2.5 in Section 2.7). All of these statements involve knowing all of the words (as in ALL of the words), not just a few symbols. For the FTC, you should be aware of the focus of each part of the theorem and which mathematician is more closely linked to each version. You also need to know and be able to use the basic antiderivative formulas and the techniques of integration we have been practicing the last few sections.

(9/14)

1. There is no specific assignment for this day.
2. Hopefully, you will have received your graded exams so you can go over these. Also, remember to start work on the Wednesday assignment.

(9/16)
1. Read Section 2.10 carefully.
2. Watch the 47 minute youtube video `https://youtu.be/qR_KW6gTkLg`.
3. Do problems 1b, 1c, 1e, 1i, 1j, 2b, 2c, 2d, 3, and 4 in Section 2.10.
4. Look over Section 2.11 as well as the extra notes for this section of Chapter 2.
5. Do problems 1a, 1d, 1e, and 1f in Section 2.11.
6. Turn in solutions for homework assignment 8.

(9/18)
1. Read Section 2.12 carefully.
2. Watch the 42 minute youtube video `https://youtu.be/JOpImF9rQO4`.
3. Do the problems in Section 2.12. For some of these, sketch a careful graph and think about the problem before you set up the integrals.
4. Turn in solutions for homework assignment 9. Note that these problems are from the textbook so you can check your answers if so desired.

(9/21)
1. Read Section 2.13 carefully.
2. Watch the 50 minute youtube video `https://youtu.be/tR6stjokA4Y`. This is one lecture where not being in person really creates difficulties. It also illustrates how we all can make mistakes by getting distracted; the goal is to decrease their number and recognize them when they occur.
3. Do the problems in Section 2.13.
4. Turn in solutions for homework assignment 10.

(9/23)
1. Read Section 2.14 carefully.
2. Watch the 41 minute youtube video `https://youtu.be/F8TmMMJMHJU`.
3. Do the problems in Section 2.14.
4. Turn in solutions for homework assignment 11.

(9/25)
1. Read Section 2.15 carefully.
2. Watch the 38 minute youtube video `https://youtu.be/mPm6dpgxnlY`.
3. Do the problems 2 and 3 in Section 2.15.
4. Turn in solutions for homework assignment 12.

(9/28)
1. Read Section 2.17 carefully.
2. Watch the 41 minute youtube video `https://youtu.be/L-r5_9zLeho`.
3. Do problems 3, 4, 5, 6, 8, and 13 in Section 2.17. Problem 13 is a good example of a multiple step problem.
4. Turn in solutions for homework assignment 13.

(9/30)
1. Read Section 2.18 carefully.
2. Watch the 27 minute youtube video `https://youtu.be/XjE40II1ciQ`.
3. Do problems 2 and 3bc in Section 2.18.
4. Turn in solutions for homework assignment 14.

(10/2)
1. No class this day due to the October break.

(10/5)
1. Read Section 2.19 carefully.
2. Watch the 51 minute youtube video `https://youtu.be/Ii-i1Og8xwg`. Sorry for the length; you may be able to skim parts of it. Also, see if you can catch the error in one of the antiderivatives.
3. Do the problems in Section 2.19. Remember that these problems involve more algebra than calculus.
4. Turn in solutions for homework assignment 15.

(10/7)
1. Read Section 2.22 carefully.
2. Watch the 44 minute youtube video `https://youtu.be/9N03TPtm8L8`. Feel free to skim parts of this; you won't hurt my feelings.
3. Do problems 1a,c,d,g,h,k in Section 2.22.
4. Turn in solutions for homework assignment 16.

(10/9)
1. We have an exam covering the material since the first exam. See the next item for details.
2. You need to know how to set up integrals related to the various applications that we have considered and you need to be able to evaluate integrals using any of the techniques of integration that have been considered this semester; this includes knowing the basic antiderivative formulas. In addition, you need to know all of the things mentioned in item 4 of the 9/11 assignment. READ THAT LAST SENTENCE AGAIN! You should also read item 3 of the 9/11 assignment since the test taking guidelines will be the same.
3. The first step in your review should be to go over the new sections we have covered to make sure you understand the main ideas. You can then redo the problems in each section and you can work on problems 24, 34, 35abcijlpst, 36, and 38 in Section 2.24. For 36, you should set up the integral that represents the quantity and make sure that you know what steps to take to perform the integration, but it is probably best not to carry out the details. Use an electronic device to check that you have the correct integral. At some point, you should practice evaluating a few integrals but do not let this take away from the more abstract skill of setting up the integrals. You can also look at the second exam from Spring 2011 (omit question 12) as well as the problems at the link 'Review for exam on integration' (especially 1–12). For the latter, you can skip problem 19–22, 25–29, and 31. There are lots and lots of problems here (way more than you have time to do but you should at least ponder how you would get started on them even if you do not carry out the details) so you need to use discretion as you decide how many of these to try. You should consider spending more time on making certain that you know how to start the problem and/or set up the integral rather than getting lost in the simple but tedious details of calculations.

(10/12)

1. Carefully read the prelude to Chapter 3 (available on the website).
2. It is very important that you read the prelude slowly and thoughtfully, perhaps spending 20 or more minutes thinking about the material. It might also be a good idea to spend 10–15 minutes looking over Section 3.1 of the textbook.

(10/14)

1. Read Section 3.1 carefully.
2. Watch the 41 minute youtube video `https://youtu.be/f0vFj2JB1E0`.
3. Do problems 1, 2, 3, 4, 5, 7b, and 7d in Section 3.1.
4. Read the first two pages of the Model Induction Proofs (see the Prelude to Chapter 3 for the appropriate link). You should be able to find two errors in each of the incorrect proofs. You will also find a solution to problem 4 there, but you should try the problem on your own first.
5. Turn in solutions for homework assignment 17.
5. You can also watch the 38 minute youtube video `https://youtu.be/VagOTqxsRuA` for some insight into the Fibonacci numbers.

(10/16)

1. Read Section 3.2 carefully.
2. Watch the 48 minute youtube video `https://youtu.be/SE3wNzTWCwU`.
3. Do the problems in Section 3.2. Some of these problems will go quickly, but other problems may require some careful thought. Remember that there is a very strong emphasis on thinking about concepts as you study this material. You may find the extra notes for the sections in Chapter 3 enlightening.
4. Turn in solutions for homework assignment 18.

(10/19)

1. Read Section 3.3 carefully; work mindfully and deliberately on your technical reading skills.
2. Watch the 50 minute youtube video `https://youtu.be/FwAsReAuxt0`. (I need to work on being more animated on these. In class, there is time to pause for questions from students, do more examples on confusing topics and fewer examples on simple topics, and go off on (nonmathematical) tangents. In striving for clarity in the videos, I err on the side of going slow and methodical, which could translate into boring.)
3. Do the problems in Section 3.3. For problem 1, you need to write out careful steps as in one of the examples presented in the reading for this section. For the other problems, you will want to do some algebra and/or use the results in Theorem 3.8 and/or use the Squeeze Theorem. The extra notes may provide some helpful examples.
4. Turn in solutions for homework assignment 19.

(10/21)

1. Read Section 3.4 carefully.
2. Watch the 44 minute youtube video `https://youtu.be/W6udAU-1zXc` in which I essentially read through the section with you.
3. Do the problems in Section 3.4.
4. Turn in solutions for homework assignment 20.

(10/23)

1. Read Section 3.5 carefully.
2. Watch the 53 minute youtube video `https://youtu.be/m9eiNyxbmsU`. You may need to take a break or two during this rather lengthy video. The first part does mention some previous homework problems on sequences.
3. Do the problems in Section 3.5. Think carefully about each problem type.
4. Turn in solutions for homework assignment 21.

(10/26)

1. Read Section 3.6 carefully, making certain you understand the inequalities next to the graph.
2. Watch the 52 minute youtube video `https://youtu.be/VsbLCbqhiD8`.
3. Do the problems in Section 3.6; some of these appear on homework assignment 22. Make certain that you use correct notation for improper integrals and use these problems as an opportunity to review some integration.
4. Turn in solutions for homework assignment 22.

(10/28)

1. Read Section 3.7 carefully.
2. Watch the 48 minute youtube video `https://youtu.be/I3b8bx7fKtM`.
3. Do the problems in Section 3.7. When using the Comparison Test, it is important that you be very careful with the inequalities that you use; check them a second time! Problem 4 requires you to make some estimates about the sizes of the numbers (as we have done with other sums); as a start, determine how many two digit integers do not contain a 0 and how many three digit integers do not contain a 0. This is a good example of a nonroutine problem involving ideas that you have learned recently.
4. Turn in solutions for homework assignment 23.

(10/30)

1. Read Section 3.8 carefully.
2. Watch the 53 minute youtube video `https://youtu.be/2loFV8UT_lk`.
3. Do the problems in Section 3.8. For problems 1 and 2, it might be best to look at all of the series first, deciding what steps you would need to take to solve the problem. You can then fill in a few details to make certain you know how to write your solutions. Problems 4 through 7 help you think about the concepts; take them seriously. Problem 8 is important since you must first decide which of the convergence tests to use. Even if you do not carry out all of the details, think carefully about what the series does and which test to use to verify your conjecture. For problem 9, you may find it helpful to write out the first eight terms of each series.
4. Turn in solutions for homework assignment 24.

(11/2)

1. Read Section 3.9 carefully, paying particular attention to the last paragraph.
2. Watch the 42 minute youtube video `https://youtu.be/Vzva0rPVL88`.
3. Do problems 2, 3, 5, and 6 in Section 3.9. For problem 3, think carefully about the last paragraph in the section as you decide which test to use to check for convergence. Problem 6 is important for what is to come.
4. Turn in solutions for homework assignment 25.

(11/4)

1. Read Section 3.10 carefully.
2. Watch the 48 minute youtube video `https://youtu.be/kPbJNAqWIks`.
3. Do the problems in Section 3.10. For problem 2, be certain to check the endpoints carefully. For problem 6, refer thoughtfully to the prototypes in problem 5.
4. Turn in solutions for homework assignment 26.
5. We will meet during our scheduled class time to review for the exam. I will send a Zoom link for this meeting on Monday or Tuesday.

(11/6)

1. We have an exam covering the first 10 sections of Chapter 3. See the next item for details.
2. Go over the sections we have covered and review the key concepts. You need to pay careful attention to the definitions and theorems, making certain that you understand them, remember them, and know how to use them. Practice some of the problems we have been doing, that is, do some of the problems again without looking at your previous solutions. Look at the exam on this material from a previous year (found on the website) and be certain that the problems make sense. Note that some of the questions on the exam ask for examples of sequences or series with specific properties. As we have seen, using correct notation is extremely important for sequences and series so go over your turned-in homework solutions to identify any such errors you may have made. For additional problems, you can work on problems 1, 14, 5, 6, and 11 (in that order) in Section 3.14. When you feel you are ready, take the diagnostic quiz on Chapter 3. This quiz can be found on the website, along with complete solutions.
3. Things to know for the exam include how to do a proof by induction, adjectives for sequences, limits of certain forms of sequences, the Completeness Axiom, the two sequences associated with a series, geometric series (and their sums) and p-series, all seven tests for convergence, the distinction between absolute convergence and conditional convergence, and how to work with power series.
4. Please reread item 3 of the 9/11 assignment to remind yourself of the test taking guidelines that must be followed for this exam.

(11/9)

1. There is no specific assignment for this day.
2. Hopefully, you will have received your graded exams so you can go over these and learn from any mistakes that you made. You might also spend a few minutes reviewing power series since we will continue our study of them this week.

(11/11)

1. Read Section 3.11 carefully, pondering this new way of representing functions.
2. Watch the 46 minute youtube video `https://youtu.be/yWa3CQvv-to`.
3. Do problems 1, 2, 3, and 4 in Section 3.11.
4. Turn in solutions for homework assignment 27.

(11/13)

1. Read Section 3.12 carefully.
2. Watch the 54 minute youtube video `https://youtu.be/VghaDbwU5vo`. There is a typo in the last example in this lecture. The problem as stated should have $(2k)!$ for the denominator rather than the expression $(2k!)$ as written. Note that $2k! = 2 \cdot k!$, which is very different that $(2k)!$. Sorry for any confusion this may have caused.
3. Do problems 1, 2, and 5 in Section 3.12.
4. Turn in solutions for homework assignment 28.

(11/16)

1. Reread Section 3.12 if necessary.
2. Watch the 34 minute youtube video `https://youtu.be/uRQrEN4jgwc`.
3. Do problems 3 and 4 in Section 3.12 and problems (or at least one part of each problem) 21, 24, 28, and 29 in Section 3.14. Be patient as you do these problems to avoid computational errors.
4. Turn in solutions for homework assignment 29.

(11/18)

1. Read Section 1.21 carefully.
2. Watch the 35 minute youtube video `https://youtu.be/lhVTU9pKuvU`.
3. Do problems 1, 2, and 3 in Section 1.21.
4. Turn in solutions for homework assignment 30.

(11/20)

1. Read Section 1.22 carefully.
2. Watch the 23 minute youtube video `https://youtu.be/9V9AMQzyXjk`.
3. Do problems 2, 4, 6, 8, and 9 in Section 1.22.
4. Turn in solutions for homework assignment 31.

(11/23)

1. Look over the sections of the textbook that we have covered this semester.
2. Watch the 56 minute youtube video `https://youtu.be/XcPnkSs0kgE`. This video considers, and outlines solutions for, some nonroutine problems related to the material from this semester.
3. Watch the 47 minute youtube video `https://youtu.be/zBy2eVUaVh0` which presents an overview of the material that we have covered this semester.
4. Final exams are December 1 to December 5: see the Registrar's page.
5. See the next pages for information concerning the final exam; some of this information will need to be updated, but it gives you a sense for the exam.

As you may recall from reading the syllabus, the goals for this course are

- to develop quantitative reasoning skills;
- to learn how to read technical material; [reading the textbook and notes on your own]
- to learn to write technical information correctly and clearly; [via feedback on collected HW problems]
- to take pride in your work and to avoid errors; [see item (4) on the syllabus]
- to learn how to solve non-routine problems; [see the second paragraph of the syllabus]
- to appreciate/understand how mathematicians view mathematics;
- to comprehend some aspects of calculus.

It is with these goals in mind that the final exam will be written. The exam is comprehensive and covers all of the sections that we have discussed this semester, with a slight emphasis on the more recent material. The final exam will require the skills and concepts that you have been practicing and pondering this semester. It is your responsibility to go back over the sections and make certain that you know how to do the types of problems we have encountered. One or two of the problems on the final exam will be more involved than the sorts of problems that have appeared on the other exams that we have had. This should not be that much of a surprise because most of the test questions have been somewhat easier and shorter than homework problems due to the time constraints of a 50 minute exam. It is now time for you to put all your knowledge together and show me what you have learned this semester.

Here is the (most likely) introduction to the final exam that you will be taking.

Write neat, concise, and accurate solutions to each of the problems in the space provided—I will not give any credit for steps I cannot follow. Your solutions should be written in the style expected for collected homework problems. Pay particular attention to correct use of notation and use complete sentences when appropriate. Each of the * problems is worth * points (making a total of 80). No electronic devices or calculators are allowed for this exam.

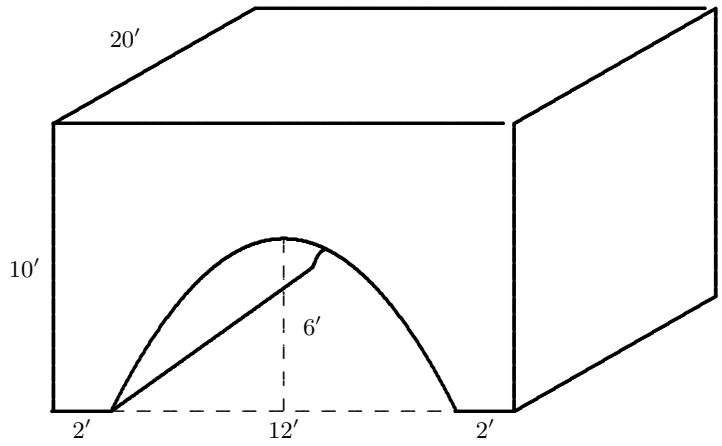
The problems will be graded as indicated in the heading so it is important that you work toward avoiding computational errors and that you pay attention to your writing and notation. The best advice is to review for the exam by looking over the sections in the text that we have covered, thinking carefully about the ideas we have discussed, and understanding the types of problems that have appeared on previous exams. You can redo the assignments problems (there were 31 of these), redo (or do?!) the assigned problems from the sections in the textbook, look over your previous exams, and work on some of the review problems that appear on the next few pages. It is important that you arrive at the exam with a refreshed mind and body, and be prepared to stay positive and work hard for up to three hours. As just indicated, although the exam is written for a two hour period, you may have three hours for the exam. The exam time period is thus either 9:00–12:00 or 2:00–5:00 on the appropriate day of the final exam.

As should come as no surprise, it is expected that you can state (as well as understand) the definition of the derivative, the definition of the integral, and both versions of the Fundamental Theorem of Calculus. Do not lose points by ignoring this fact. You should also be familiar with basic integration formulas and techniques of integration, be able to solve problems involving applications of the integral, understand the main ideas behind sequences, series, and power series, and know the Maclaurin series for e^x, $\sin x$, and $\cos x$. This is just a sampling of the things that you need to know for the exam; if you have been keeping up during the semester, it should not be too difficult to remember the common formulas and ideas that we have been using somewhat regularly.

The following problems are not representative of the final exam. They are simply intended to give you some indication of the nature of a more difficult or novel problem that may appear on the final exam. Having said that, I do recommend that you give them some thought. However, keep in mind that most of the problems on the final exam will be (or at least should be) quite familiar to you. You can look at the final exam from Spring 2011 that is located on the course website but be aware that our final exam will not necessarily look like this.

1. Consider two different solids. The base of each solid is a triangle with vertices $(0,0)$, $(2,4)$, and $(6,0)$. For solid A, each cross-section perpendicular to the y-axis is an equilateral triangle. For solid B, each cross-section perpendicular to the x-axis is a square. Find the ratio of the volume of solid A to the volume of solid B.

2. Find the number of cubic yards of concrete necessary to construct the culvert shown below. Assume that the arch of the culvert (which is empty space) has a parabolic shape.



3. Let $a_1 = 2$ and $a_{n+1} = 3 - (1/a_n)$ for each positive integer $n \geq 1$. Use mathematical induction to prove that $a_n = \dfrac{f_{2n+1}}{f_{2n-1}}$ for each positive integer n. Here f_n refers to the nth Fibonacci number.

4. For each positive integer n, let

$$y_n = \frac{1}{3n+2} + \frac{1}{3n+4} + \frac{1}{3n+6} + \cdots + \frac{1}{5n}.$$

Find the limit of the sequence $\{y_n\}$. (Try writing y_n in summation notation and think about integrals.)

5. Determine (using familiar calculus functions) the function represented by $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k(2k+1)}{4^k k!}\, x^{2k}$.

The problems that follow are more similar to the problems that you have been doing as homework. As you try these problems, put yourself in the mindset of an exam. That is, do not use your notes or look at the answer until you have finished the problem. Pay attention to problems you do not know how to start (these problems represent a lack of understanding) and problems you know how to start but get incorrect answers at the end (these problems indicate of lack of attention to detail).

Miscellaneous problems to try before the final exam

Since many of the answers are given right after the problem, you need to be careful to avoid using the answer as a hint for how to start the problem as this does not mimic a testing situation. Proceed without technology if at all possible. Omit problems marked with an asterisk.

1. * Evaluate the limit $\lim\limits_{n\to\infty} \dfrac{n^4+n^2+1}{3^3+6^3+9^3+\cdots+(3n)^3}$. (4/27)

2. Use an integral to evaluate the limit $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \Big(5+\dfrac{3i}{n}\Big)^2 \dfrac{3}{n}$. (129)

3. Evaluate each of the following integrals.

   a) $\displaystyle\int_{-1}^{2} (2x-3)(x-1)\,dx$ b) $\displaystyle\int_{0}^{3} (4x+2|x-1|)\,dx$ c) $\displaystyle\int_{0}^{1} \big(2t-3+2\sqrt{1-t^2}\big)\,dt$

   d) $\displaystyle\int_{1}^{8} \frac{x+2}{\sqrt[3]{x}}\,dx$ e) $\displaystyle\int_{1}^{4} \frac{1}{3x-2}\,dx$ f) $\displaystyle\int_{0}^{2} \frac{1}{4+x^2}\,dx$

   (The values are $15/2$, 23, $\dfrac{\pi}{2}-2$, $138/5$, $\frac{1}{3}\ln 10$, and $\pi/8$, respectively.)

4. * Use a simpler function to approximate $\displaystyle\int_{1}^{2} \frac{1}{\sqrt{4x^6-1}}\,dx$. Is your estimate high or low? (3/16, low)

5. Find the derivative of the function F defined by $F(x)=\displaystyle\int_{0}^{x^2} t\sqrt{t^3+4}\,dt$. $\big(F'(x)=2x^3\sqrt{x^6+4}\big)$

6. Suppose that $v(t)=3t-t^3$ gives the velocity in meters per second of a particle at time t seconds. Find the distance traveled by the particle for the time interval $0\le t\le 4$. (44.5 meters)

7. Find the area of the region bounded by the curves $x^2y=90$ and $40x+y=130$. (40)

8. Find the area of the region bounded by the curves $y=2\sqrt{x}$ and $y=x^3/16$. (20/3)

9. Find the volume of the solid that is generated when the region bounded by the curves $y=4x$ and $y=x^2$ is revolved around (a) the x-axis (b) the y-axis. $\Big(\Big(\dfrac{2048}{15}\,\pi\Big)$ and $\Big(\dfrac{128}{3}\,\pi\Big)\Big)$

10. Suppose the base of a solid is the part of the parabola $y=8-0.5x^2$ that lies above the x-axis and that each cross section perpendicular to the y-axis is a semicircle. Find the volume of this solid. (32π)

11. Find the volume of the solid that is generated when the region that lies below the curve $y=\ln x$ and above the x-axis on the interval $[1,e]$ is revolved around the y-axis. $\big(\pi(e^2+1)/2\big)$

12. Find the volume of the solid that is generated when the region that lies above the x-axis and below the curve $y=\sqrt{\dfrac{14(3-x)}{(x+1)(7-x)}}$ on the interval $[0,3]$ is revolved around the x-axis. $\big(7\pi(\ln 16-\ln 7)\big)$

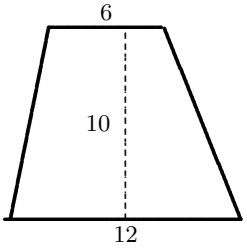
13. Find the center of mass of the region bounded by the curves $y=2\sqrt{x}$ and $y=x^3/16$. $\Big(\dfrac{48}{25},\dfrac{12}{7}\Big)$

14. Find the center of mass of the solid that is generated when the region below the curve $y=4e^{-x/4}$ and above the x-axis on the interval $[0,\infty)$ is revolved around the x-axis. $(2,0,0)$

15. Find the length of the curve $y=4x^{3/2}$ on the interval $[0,10]$. (127)

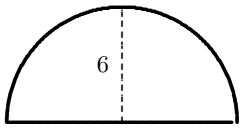
16. Find the force exerted by a liquid with weight density w on one side of each vertically submerged plate. The units on the figures are feet and the top of each plate is six feet beneath the surface of the liquid.

a)



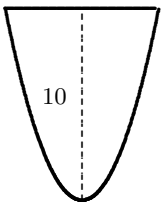
trapezoid

b)



semicircle

c)



parabola

(The forces are $1040w$, $(216\pi - 144)w$, and $1600w/3$ pounds, respectively.)

17. Evaluate each of the following definite integrals.

a) $\displaystyle\int_0^2 \frac{x}{4+x^2}\,dx$ b) $\displaystyle\int_0^2 \frac{x}{\sqrt{4+x^2}}\,dx$ c) $\displaystyle\int_0^2 \frac{x}{\sqrt{4+x}}\,dx$

d) $\displaystyle\int_1^\infty \frac{8}{(2x+5)^3}\,dx$ e) $\displaystyle\int_0^\infty \frac{6+e^{2x}}{e^{3x}}\,dx$ * f) $\displaystyle\int_1^{\sqrt{2}} \frac{\sqrt{x^2-1}}{x^4}\,dx$

(The values are $\frac{1}{2}\ln 2$, $2\big(\sqrt{2}-1\big)$, $\dfrac{32}{3}-4\sqrt{6}$, $2/49$, 3, and $\sqrt{2}/12$, respectively.)

18. * Use the trapezoid rule and Simpson's rule with $n = 4$ to approximate $\displaystyle\int_0^1 e^{-x^2/2}\,dx$ to four decimal places. (The approximations are 0.8526 and 0.8557, respectively.)

19. * Suppose that the following table represents the velocity of a particle moving in a straight line.

t	(sec)	0	1	2	3	4	5	6
v	(m/sec)	0	5	10	12	8	4	0

Use Simpson's rule to approximate the distance traveled by the particle. (40 meters)

20. Evaluate each of the following indefinite integrals.

a) $\displaystyle\int \big(2\sqrt{x}+1\big)^2\,dx$ b) $\displaystyle\int \frac{3x}{(2x^2+5)^3}\,dx$

c) $\displaystyle\int \frac{12x}{(3x-1)^2}\,dx$ d) $\displaystyle\int \frac{3x+1}{\sqrt{12x-x^2}}\,dx$

e) $\displaystyle\int \frac{3x+8}{x^2+4x+6}\,dx$ f) $\displaystyle\int \frac{4x-7}{2x+1}\,dx$

g) $\displaystyle\int \arctan x\,dx$ * h) $\displaystyle\int \frac{\sqrt{x^2+4}}{x^4}\,dx$

i) $\displaystyle\int \frac{3x+1}{\sqrt{13-12x-x^2}}\,dx$ j) $\displaystyle\int \frac{4x-1}{x^2+2x-15}\,dx$

k) $\displaystyle\int \frac{x^2+2x+4}{x^3+x^2+x+1}\,dx$ ℓ) $\displaystyle\int x\,e^{-x/2}\,dx$

m) $\displaystyle\int \frac{3x-7}{2x^2+7x-9}\,dx$ n) $\displaystyle\int \frac{2x^2+7x-9}{3x-7}\,dx$

The answers for these integrals are given below.

a) $2x^2 + \frac{8}{3}\,x^{3/2} + x + C$

b) $\dfrac{-3}{8(2x^2+5)^2} + C$

c) $\frac{4}{3}\Big(\ln|3x-1| - \dfrac{1}{3x-1}\Big) + C$

d) $-3\sqrt{12x-x^2} + 19\arcsin\Big(\dfrac{x-6}{6}\Big) + C$

e) $\frac{3}{2}\ln\big(x^2+4x+6\big) + \sqrt{2}\arctan\Big(\dfrac{x+2}{\sqrt{2}}\Big) + C$

f) $2x - \frac{9}{2}\ln|2x+1| + C$

g) $x\arctan x - \frac{1}{2}\ln(1+x^2) + C$

h) $\dfrac{-(x^2+4)^{3/2}}{12x^3} + C$

i) $-3\sqrt{13-12x-x^2} - 17\arcsin\Big(\dfrac{x+6}{7}\Big) + C$

j) $\frac{11}{8}\ln|x-3| + \frac{21}{8}\ln|x+5| + C$

k) $\frac{3}{2}\ln|x+1| - \frac{1}{4}\ln(x^2+1) + \frac{5}{2}\arctan x + C$

ℓ) $-2(x+2)e^{-x/2} + C$

m) $\frac{41}{22}\ln|2x+9| - \frac{4}{11}\ln|x-1| + C$

n) $\frac{1}{3}\,x^2 + \frac{35}{9}\,x + \frac{164}{27}\ln|3x-7| + C$

21. Prove the following statement: for each positive integer n, the integer $2^{5n-4} + 5^{2n-1}$ is divisible by 7.

22. Find the limit of the given sequence.

a) $\Big\{\dfrac{k}{\sqrt{3k^2+4k+1}}\Big\}$

b) $\big\{\sqrt{k^2-7k+15} - k\big\}$

c) $\big\{k\big(\sqrt[k]{10} - 1\big)\big\}$

d) $\Big\{\Big(1 - \dfrac{2}{3n}\Big)^n\Big\}$

e) $\Big\{\dfrac{4^n + n^2}{2^{2n-3} + n^7}\Big\}$

f) $\big\{\sqrt[n]{4n^2+n+3}\big\}$

(The limits are $1/\sqrt{3}$, $-7/2$, $\ln 10$, $e^{-2/3}$, 8, and 1.)

23. Define a sequence $\{x_n\}$ by $x_1 = 5$ and $x_{n+1} = 4 - (1/x_n)$ for $n \geq 1$. Prove that $1 \leq x_n \leq 5$ for all n, then prove that $\{x_n\}$ is a decreasing sequence. Conclude that $\{x_n\}$ converges and find its limit. $\big(2+\sqrt{3}\,\big)$

24. Find the sum of the given series.

a) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}3^k}{4^{k-1}}$

b) $\displaystyle\sum_{k=1}^{\infty} \frac{3^k + 5^k}{7^k}$

c) $\displaystyle\sum_{k=1}^{\infty} \frac{(-4)^k}{(2k+1)!}$

(The sums of the series are 12/7, 13/4, and $\frac{1}{2}\sin 2 - 1$, respectively.)

25. Let $\displaystyle\sum_{k=1}^{\infty} a_k$ be an infinite series and suppose that its sequence $\{s_n\}$ of partial sums is given by $s_n = \dfrac{n+1}{1-3n}$ for all $n \geq 1$. Find a_1, a_2, a_{10}, and the sum of the series.

(The values are -1, 2/5, 2/377, and $-1/3$, respectively.)

26. Determine whether or not the given series converges.

a) $\displaystyle\sum_{k=1}^{\infty} \frac{12}{3k+2}$

b) $\displaystyle\sum_{k=1}^{\infty} \frac{4k-1}{k^2+5k+2}$

c) $\displaystyle\sum_{k=1}^{\infty} \frac{2k^2+3}{k^4+7k-1}$

d) $\displaystyle\sum_{k=1}^{\infty} \frac{5^k}{2^k+6^k}$

e) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt[k]{5}}$

f) $\displaystyle\sum_{k=1}^{\infty} \Big(\frac{k}{3k+1}\Big)^k$

(The series are D, D, C, C, D, and C, respectively.)

27. Classify the series $\displaystyle\sum_{k=1}^{\infty} \frac{(-3)^k k!}{3 \cdot 7 \cdot 11 \cdot \cdots \cdot (4k-1)}$ as AC, CC, or D. (It is AC.)

28. Show that each of the series $\displaystyle\sum_{k=1}^{\infty} \frac{3^k \sin k}{4^k}$ and $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k^2}{k^4 + 3k^2 + 10}$ is absolutely convergent.

29. Carefully prove that each of the series $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{5k+4}$ and $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k}{k^2+1}$ is conditionally convergent.

30. Find the radius of convergence for the power series $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2 3^k}(x-4)^k$. (3)

31. Find the interval of convergence for the power series $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)2^k}(x-1)^k$. $\big((-1,3]\big)$

32. Give an example of a power series with $[4, 10)$ as its interval of convergence.

    One example is $\displaystyle\sum_{k=0}^{\infty} \frac{1}{(2k+1)3^k}(x-7)^k$.

33. Find (in more familiar terms) the function represented by the power series $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2^k}(x+1)^k$.

    This is the power series for the function $f(x) = \dfrac{x+1}{x+3}$, valid on the interval $(-3, 1)$.

34. Find the Maclaurin series for the function $f(x) = \dfrac{1}{5-2x}$ and determine its interval of convergence.

    The Maclaurin series is $\displaystyle\sum_{k=0}^{\infty} \frac{2^k}{5^{k+1}} x^k$, with interval of convergence $(-2.5, 2.5)$.

35. By differentiating an appropriate power series (see problem 3.11.2), find the sum of the series $\displaystyle\sum_{k=1}^{\infty} k^3 x^k$.

    $\displaystyle\sum_{k=1}^{\infty} k^3 x^k = \frac{x + 4x^2 + x^3}{(1-x)^4}$.

36. Use known series to find the Maclaurin series for the given function.

    a) $f(x) = e^{-x/3}$ **b)** $g(x) = \sin(x^2)$ **c)** $h(x) = \dfrac{1 - \cos x}{x}$

    $f(x) = \displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{3^k k!} x^k$, $g(x) = \displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{4k+2}$, $h(x) = \displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(2k)!} x^{2k-1}$

37. Use known Maclaurin series to determine in more familiar terms the given function. (See exercises 28 and 29 in Section 3.14 if you want more practice for these types of problems.)

    a) $\displaystyle\sum_{k=0}^{\infty} \frac{1}{2^k k!} x^k$ **b)** $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k}$ **c)** $\displaystyle\sum_{k=0}^{\infty} \frac{(-9)^k}{(2k)!} x^{2k+1}$

    The functions are $e^{x/2}$, $\sin x / x$, and $x \cos(3x)$, respectively.

38. Find the Taylor series for the function $f(x) = 1/(2x-1)$ centered at $a = 6$.

    $\dfrac{1}{2x-1} = \displaystyle\sum_{k=0}^{\infty} \frac{(-2)^k}{11^{k+1}} (x-6)^k$ with $\rho = 5.5$.