Math 240 Assignments for Fall 2024 (due on given date)

for Wednesday, September 4

1. Read the syllabus carefully.
2. Read 'A Note to Students' that appears in the introductory pages of the textbook and spend a few minutes looking over the first few pages of Section 1.1.
3. Turn in a carefully written solution for the following problem:

   Let ℓ_1 be the line $x + y + 61 = 0$ and let ℓ_2 be the line tangent to the curve $y = x^2$ when $x = 4$. Find the point of intersection of the lines ℓ_1 and ℓ_2.

for Friday, September 6

1. Carefully read Section 1.1; this textbook is written in a style that is intended for students. It is absolutely imperative that you learn how to read this textbook; I will not be reading it to you or just repeating it in a lecture. Reading a textbook is an acquired skill so do not become discouraged if it is difficult and frustrating for a while. You must read each word and phrase carefully, you need to be certain to understand every calculation, you may need to pause to think or write out some details, and you may need to read some parts of a section several times. If there is something that you do not understand, make a note in the margin. You may find an answer later or upon a second reading; if not, ask a question during a class meeting, in office hours, or via email.
2. Pay attention to all of the new terms that are defined in this section and be absolutely certain that you understand how to perform row operations on a matrix.
3. Do all four of the practice problems that appear at the end of the section. After (and only after) doing so, read the solutions that appear at the very end of the section (after the exercises).
4. Do Exercises 1, 5, 9, 11, 13, 19, 23, 24, 25, and 31 in Section 1.1. Most of these should go rather quickly. Note that answers to most of the odd numbered problems can be found in an appendix to the textbook. Do the true–false questions in Exercises 23 and 24 very carefully.
5. Turn in carefully written solutions for Exercises 2, 14, 16, and 22; these are due at the beginning of class. For Exercises 2, 14, and 16, I request that you carefully row reduce the augmented matrix for each problem all the way to the form where you can immediately read off the solutions; see the final version of the matrix in Example 1. Granted, the problems can be solved without doing so, but it is important that you practice this technique. In addition, find the full solution (that is, completely solve the system) for Exercise 16. Remember to do the steps slowly AND to check your final answer.

for Monday, September 9

1. Carefully read Section 1.2.
2. Do the two practice problems that appear at the end of the section. After (and only after) doing so, read the solutions that appear at the very end of the section (after the exercises).
3. Do Exercises 1, 3, 9, 13, 16, 21–27, 29, and 31 in Section 1.2. Do the true–false questions in Exercises 21 and 22 very carefully.
4. Turn in carefully written solutions for Exercises 14, 20, and 33; these are due at the beginning of class.

for Wednesday, September 11

1. Carefully read Section 1.3. Vectors are an abstract concept; the geometry in $\mathbb{R}^2$ and $\mathbb{R}^3$ can be helpful, but you need to become comfortable thinking about vectors in general terms. In particular, pay attention to the algebraic properties of vectors in $\mathbb{R}^n$. The concept of span is very important as is the connection between linear combinations of vectors and linear systems of equations.
2. Do the two practice problems that appear at the end of the section. After (and only after) doing so, read the solutions that appear at the very end of the section (after the exercises).
3. Do Exercises 1, 5, 9, 11, 13, 15, 16, 22, 23, and 24 in Section 1.3. Do the true–false questions in Exercises 23 and 24 very carefully.
4. Turn in carefully written solutions for Exercises 12, 14, and 26. For Exercise 14, write $\mathbf{b}$ as an explicit linear combination of the columns of A. For Exercise 26, carefully explain your answers to parts (a) and (b). You should row reduce a matrix for part (a) and write an explicit linear combination for part (b).

for Friday, September 13

1. Carefully read Section 1.4. Note that we now have three different ways to express the same problem.
2. Do the two practice problems that appear at the end of the section.
3. Do Exercises 1, 3, 5, 7, 9, 23, 24, 31, 34, and 35 in Section 1.4.
4. Turn in carefully written solutions for Exercises 12, 14, and 26.

for Monday, September 16

1. Carefully read Section 1.5.
2. Do the two practice problems that appear at the end of the section.
3. Do Exercises 1, 2, 5, 7, 11, 13, 17, 23, 24, 27, 29, 31, 35, and 40 in Section 1.5.
4. Turn in carefully written solutions for Exercises 8, 18, and 36. Be sure to write the parametric form of solutions carefully. For Exercise 36, briefly describe your thought process that led to your solution.

for Wednesday, September 18

1. Carefully read Section 1.7. The concept of linear independence is extremely important.
2. Do the four practice problems that appear at the end of the section.
3. Do Exercises 1, 3, 5, 7, 9, 15, 17, 19, 21, 22, and 33–38 in Section 1.7. Most of these problems should go very quickly; remember to focus on the ideas.
4. Turn in carefully written solutions for Exercises 14, 28, 32, and 34. Explain why the answer to 34 is false and then show how to carefully tweak the wording so that the statement is true.

for Friday, September 20

1. Carefully read Section 1.8.
2. Do Exercises 1, 3, 5, 7, 17, 20, 21, 22, 30, 36, 38, 39, and 40 in Section 1.8. Note that you can do Exercises 38 and 40 at the same time. As a first step, multiply the second row by 3, then start row reducing. We will not be doing many highly computational problems like this, but it is good practice to do a few and focus on working error-free. The answers for these two problems come out rather simply and note that you can always check your answers. For Exercise 39, see if you can spot a solution by inspection by looking at just the first two columns of the matrix.
3. Turn in carefully written solutions for Exercises 8, 10, 12, and 31. For Exercise 8, be certain to explain your answer. You can do Exercises 10 and 12 concurrently in order to save computational work. Your solution for Exercise 31 should be a carefully written proof.
4. The first of our special assignments (mentioned in the syllabus) is due next Wednesday. Section 1.6 discusses three different applications of linear systems of equations. Look over the section and choose one of the applications that strikes your interest: if you select economics, then turn in a solution for Exercise 4; if you select chemistry, then turn in a solution for Exercise 8; if you select network flow, then turn in a solution for Exercise 12. You are to work alone on these problems with no help from other students or the Internet. In addition, I will only answer questions concerning clarity; the idea is for you to work on these problems independently. Please follow these guidelines to avoid cases of academic dishonesty. After choosing an application and reading the relevant material in Section 1.6, write a very careful and clear solution to your chosen problem, using the ideas and notation that we have developed thus far in the course.

for Monday, September 23

1. Carefully read Section 1.9. Unless you are interested, you can ignore all of the graphs is Tables 1–4. The concepts of onto and one-to-one are important.
2. Do Exercises 1, 15, 19, 21, 23, 24, and 29 in Section 1.9.
3. Turn in carefully written solutions for Exercises 2, 16, 26, and 34. You can combine Exercises 2 and 26 since they involve the same transformation. Once again, a careful proof is needed for Exercise 34.

for Wednesday, September 25

1. Carefully read Section 2.1.
2. Do Exercises 1, 3, 5, 7, 9, 10, 15, 16, 21, 26, 27, 32, and 40 in Section 2.1. There are not a lot of calculations here; the key is to understand the concept of matrix multiplication. Note that the hint for Exercise 32 is relevant for Exercise 40. Also, for Exercise 40, you might find it easier to represent the matrices using column notation and the vectors $\mathbf{e}_i$. As usual, make sure that you do all of these exercises, recording any questions that you have.
3. Turn in carefully written solutions for Exercises 12, 17, and 22. For Exercise 17, consider the process needed to find B. For Exercise 22, think about previous theorems that are relevant.
4. The first special assignment is also due this day; see the September 20 assignment for details.
5. The second special assignment (due 9/30) will be handed out on this day.

for Friday, September 27

1. Carefully read Section 2.2.
2. Do Exercises 1, 3, 5, 9, 10, 18, 19, 29, 30, 31, 32, 35, and 38 in Section 2.2. As usual, make sure that you do all of these exercises, recording any questions that you have.
3. Turn in carefully written solutions for Exercises 18, 30, 32, and 38.

for Monday, September 30

1. Read Section 2.3; you should pay more attention to the results and concepts rather than get bogged down in the proofs.
2. Do the three practice problems that appear at the end of the section.
3. Do Exercises 3, 5, 11, 12, 17, 19, 26, and 27 in Section 2.3; I will not be collecting any of these problems. However, you should make note of any questions you have concerning the material in the first three sections of Chapter 2; we will next be moving on to Chapter 4.
4. The second special assignment is due this day.
5. We will review for the exam during class.

for Wednesday, October 2

1. We have an exam on all of the material that has been covered thus far, namely Sections 1–5 and 7–9 in Chapter 1 and Sections 1–3 in Chapter 2. Reviewing these sections and looking over the assigned homework problems is the best place to start. You need to know the terminology for the various concepts we have discussed (this is where the true/false questions are helpful) and you need to be able to perform the computations efficiently and accurately. Note that for many of the computational problems, it is possible to check your answers so partial credit will be minimal for such problems.
2. If you want further problems to solve prior to the exam, you can work on problems 1, 6, 7, 13, 18, and 22 in the Chapter 1 supplementary exercises, and problems 1abenop, 2, 8, and 9 in the Chapter 2 supplementary exercises.
3. Here is the full solution to a problem we did in class on Monday.
   Problem: Let A and B be $n \times n$ matrices for which AB is invertible. Prove that A is invertible.
   Solution: Suppose that $B\mathbf{x} = \mathbf{0}$. It follows that $AB\mathbf{x} = A\mathbf{0} = \mathbf{0}$. Since AB is invertible, we know that $\mathbf{x}$ must be $\mathbf{0}$. Since the equation $B\mathbf{x} = \mathbf{0}$ only has the trivial solution, the matrix B is invertible. Noting that $A = (AB)B^{-1}$ is the product of two invertible matrices, we conclude that A is invertible.

for Friday, October 4

1. No assignment is due this day; we will begin Chapter 4.

for Monday, October 7

1. Carefully read Section 4.1, pondering some of the more abstract vector spaces.
2. Do Exercises 1, 5, 6, 7, 8, 9, 12, 13, 14, 15, 18, 21, 22, 23, 24, and 33 in Section 4.1.
3. Turn in carefully written solutions for Exercises 12, 18, 22, and 33.

for Wednesday, October 9

1. Carefully read Section 4.2.
2. Do Exercises 2, 3, 7, 9, 15, 17, 21, 23, 25, 26, 32, and 33 in Section 4.2.
3. Turn in carefully written solutions for Exercise 32 and the following exercise:

   Consider the matrix $A = \begin{bmatrix} 1 & 12 & 3 & 1 \\ 2 & 9 & 1 & 7 \\ 4 & 27 & 5 & 11 \end{bmatrix}$.

   a. Find vectors $\mathbf{u}$ and $\mathbf{v}$ so that $\operatorname{Nul} A = \operatorname{span}\{\mathbf{u}, \mathbf{v}\}$. Choose vectors with integer entries.

   b. Show that both of the vectors $\begin{bmatrix} 11 \\ 2 \\ 16 \end{bmatrix}$ and $\begin{bmatrix} 17 \\ 19 \\ 47 \end{bmatrix}$ belong to $\operatorname{Col} A$.

for Friday, October 11

1. No class today due to the October Break.

for Monday, October 14

1. Carefully read Section 4.3.
2. Do Exercises 3, 5, 7, 11, 13, 19, 21, 22, 26, 32, 34, and 37 in Section 4.3. For Exercise 37, try the t values 0, π, $\pi/2$, and $\pi/4$, in that order.
3. Turn in carefully written solutions for Exercises 10, 26, 32, and 34.

for Wednesday, October 16

1. Carefully read Section 4.4.
2. Do Exercises 1, 4, 5, 8, 11, 13, 14, 15, 16, 20, 21, 27, 30, 31, and 32 in Section 4.4.
3. Turn in carefully written solutions for Exercises 14, 20, 30, and 32.

for Friday, October 18

1. Carefully read Section 4.5.
2. Do Exercises 1, 5, 8, 9, 11, 13, 14, 19, 20, 22, 24, 25, 29, and 30 in Section 4.5.
3. Turn in carefully written solutions for Exercises 8 and 24.
4. The third special assignment is due at the beginning of class this day.

for Monday, October 21

1. Carefully read Section 4.6.
2. Do Exercises 1, 3, 5, 7, 15, 17, 18, 20, 21, 24, 26, 29, 30, 32, and 33 in Section 4.6.
3. Turn in carefully written solutions for Exercises 20, 24, 26, and 32.

for Wednesday, October 23

1. Carefully read Section 3.1 and Section 3.2 through Example 3.
2. Do Exercises 3, 8, 9, 10, 11, 19, 20, 22, 25, 27, 29, 38, 39, and 40 in Section 3.1 and Exercises 1, 2, 5, 9, 12, 15, 17, 19, 21, and 24 in Section 3.2. Most of these exercises should take very little time or writing.
3. Turn in carefully written solutions for Exercises 8, 10, and 22 in Section 3.1, and Exercises 12 and 24 in Section 3.2.

for Friday, October 25

1. Carefully read the rest of Section 3.2 and Section 3.3 through page 179; we'll skip the rest of 3.3.
2. Do Exercises 29, 31, 33, 36, and 40 in Section 3.2 and do Exercises 1, 4, 7, 11, and 12 in Section 3.3.
3. Turn in carefully written solutions for Exercises 36 and 40 in Section 3.2 and Exercises 4 and 12 in Section 3.3.

for Monday, October 28

1. Carefully read Section 5.1.
2. Do Exercises 1, 5, 7, 11, 13, 16, 17, 19, 21, 22, 25, 26, and 29 in Section 5.1.
3. Turn in carefully written solutions for Exercises 16, 19 (as modified, see below), and 26 in Section 5.1. The modification for Exercise 19 is the following: Find a basis for $\mathbb{R}^3$ made up of eigenvectors of the matrix A; you should be able to do this with barely any computations.

for Wednesday, October 30

1. Carefully read Section 5.2.
2. Do Exercises 1, 4, 9, 11, 15, 18, 21, 22, 23, and 24 in Section 5.2.
3. Turn in carefully written solutions for Exercises 4, 11 (as modified, see below), 23, and 24 in Section 5.2. The modification for Exercise 11 is the following: Find a basis for $\mathbb{R}^3$ made up of eigenvectors of the matrix. (Record your answer in your notes to use for the Friday assignment.)

for Friday, November 1

1. Carefully read Section 5.3.
2. Do Exercises 1, 5, 12, 13, 16, 21, 22, 26, 27, 28, 31, and 32 in Section 5.3.
3. Turn in carefully written solutions for Exercises 12, 28, and 32 in Section 5.3 plus the following two exercises: diagonalize the matrix in Exercise 5.1.19 and also the matrix in Exercise 5.2.11. Note that you have essentially solved these two problems in previous homework assignments.
4. Since we have a test coming up, you should look at the November 6 assignment for some information about the test. In addition, a handout with some review problems will be distributed in class.

for Monday, November 4

1. In addition to reading pages 84 and 85 of Section 1.10 and Example 1 in Section 4.9, carefully read Section 5.6 through the paragraph following Example 1.
2. Do Exercises 3, 4, 5, and 15 in Section 5.6.
3. Turn in a carefully written solution for Exercise 4 in Section 5.6.
4. We will review for the Wednesday exam during our class time.

for Wednesday, November 6

1. We have an exam on all of the material that has been covered since the first exam. However, note that many of the ideas and techniques from the first exam are still important to know. Rereading the sections and making sure that you know how to do all of the types of assigned problems is the best place to start.
2. If you want further study problems, you can work on problems 1 (skip 1m) and 13 (just use cofactor expansion) in the Chapter 3 supplementary exercises, problems 1a–s, 2, 5, and 15 in the Chapter 4 supplementary exercises, and problems 1abcde and 18 in the Chapter 5 supplementary exercises.
3. **IMPORTANT:** Our fourth special assignment (due 11/13) will be handed out this day.

for Friday, November 8

1. No assignment is due this day.

for Monday, November 11

1. Carefully read Section 6.1 and do the practice problems at the end of the section.
2. Do Exercises 1, 5, 7, 11, 14, 15, 16, 19, 20, 24, 29, and 30 in Section 6.1.
3. Turn in carefully written solutions for Exercises 14, 16, 29, and 30 in Section 6.1. For Exercise 30, it is not necessary to follow the steps given in the problem unless you find them helpful.

for Wednesday, November 13

1. Carefully read Section 6.2 and do the practice problems at the end of the section.
2. Do Exercises 1, 5, 7, 10, 13, 14, 15, 21, 23, 24, 29, and 34 in Section 6.2.
3. Turn in carefully written solutions for Exercises 10 and 14 in Section 6.2.
4. Special assignment 4 is also due this day.

for Friday, November 15

1. Carefully read Section 6.3 and do the practice problem at the end of the section.
2. Do Exercises 1, 7, 9, 11, 12, 15, 16, 21, 22, and 23 in Section 6.3. For Exercise 23, you will find it helpful to consider Theorem 3 on page 335.
3. Turn in carefully written solutions for Exercises 12 and 16 in Section 6.3, along with the extra problem: Given $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 7 \end{bmatrix}$, let $H = \operatorname{span}\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$. Find $\operatorname{proj}_H \mathbf{x}$ and the distance from $\mathbf{x}$ to H.

for Monday, November 18

1. Read Section 6.4.
2. Do Exercises 3, 5, 9, 13, 17, 18, 19, and 20 in Section 6.4.
3. Turn in carefully written solutions for Exercise 20 in Section 6.4, along with the extra problem:

   Find an orthogonal basis for $\quad \text{span}\left\{\begin{bmatrix}1\\0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\2\\2\\0\end{bmatrix}, \begin{bmatrix}4\\3\\2\\1\end{bmatrix}\right\}$.