#### for Wednesday, August 29

1. Read the syllabus for the course (available on the website), then read the following parts of the textbook: the preface, Appendix A.1, Appendix A.7, and the short introduction to Chapter 1.

# for Friday, August 31

- 1. Read Section 1.1. Except for the last part of this section, the material should read rather quickly. (If you have not yet taken abstract algebra, then you may skim the discussion of fields.) Start now to really learn what it means to read and understand a mathematics textbook as you will be doing quite a bit of this over the next four months.
- 2. Do exercises 4b, 5, 9, 10, 12, and 15 in Section 1.1. Many of the solutions to these exercises involve proof by contradiction.
- 3. Turn in carefully written solutions for exercises 10 and 15. Be certain to include a (perhaps abbreviated) statement of the exercise and to use words and complete sentences in your proof. I ask that you work alone on these two exercises (getting no help from any source other than the textbook) so that I can get a sense of your level of preparation for solving problems and writing proofs.

#### for Monday, September 3

- 1. Read Section 1.2 through the discussion of geometric sums.
- 2. Do exercises 2, 7, 9, 12, and 18ad in Section 1.2. For exercise 7, the solution should basically consist of one multi-step equation and just a few words. Recall that we used the result of exercise 12 in our solution for exercise 17. The fact that previous results can often be used to simplify the proofs of later results is an important observation; keep this in mind as you do later assignments.
- 3. Turn in solutions for exercises 7 and 18d.

# for Wednesday, September 5

- 1. Read Section 1.3 through the paragraph following the proof of Theorem 1.18. The Completeness Axiom and the Archimedean Property are extremely important so be certain to ask questions if you find anything confusing about them.
- 2. Do exercises 1, 2, 12, 16, 17, 19, and 20 in Section 1.3.
- 3. Turn in solutions for exercises 17 and 19.

# for Friday, September 7

- 1. Read Section 1.4. Hopefully, much of this material is familiar to you from Math 260 or previous math courses. Study carefully the proof that the set of real numbers is uncountable and note the use of the Completeness Axiom since this approach is probably different than the one you have seen.
- 2. Do exercises 1fg, 2, 9, 20, and 23abc in Section 1.4.
- 3. Turn in solutions for exercises 9, 20, and 23c.

## for Monday, September 10

- 1. Read Section 1.5. Much of this material should be familiar to you but be certain that you have the vocabulary down; this includes being able to state the definitions if requested or needed.
- 2. Do exercises 11, 23, 24, 25, 26, 39, 41, 42, 43, and 45 in Section 1.5. Some of these exercises are relatively easy but a few involve a little more thought. The somewhat unusual functions that appear in these problems are important to learn. Be certain you can visualize the graphs of the functions that appear in exercise 11 as they will be the basis for important examples later in the semester. The functions in exercises 42 and 45 may seem rather bizarre, but it is important to realize that functions can behave in strange ways so definitions of properties (such as continuity) must be stated very carefully.
- 3. Turn in solutions for exercises 23 (assume that the two functions are defined on the interval [a, b]), 43, and 45e (use the result of exercise 1.4.6).

### for Wednesday, September 12

- 1. Read Section 2.1 through the proof of Theorem 2.4. Become very familiar with the adjectives for sequences, being able to state their definitions and giving examples of sequences with or without a given property. This will most likely be your first introduction to proofs that involve "Let  $\epsilon > 0$  be given" so study the example carefully.
- 2. Do exercises 4, 5, 7, 9, 10, 11, and 15 in Section 2.1. For exercises that ask for examples, try to find several different sequences with each property.
- 3. Turn in solutions for exercises 5, 9b, and 10.

# for Friday, September 14

- 1. Finish reading Section 2.1. Be certain to make note of any questions that you have.
- 2. Do exercises 14, 18, 19, 21, 24, 26, and 42 in Section 2.1.
- 3. Turn in solutions for exercises 18 and 24.

#### for Monday, September 17

- 1. Read Section 2.2 through the proof of Theorem 2.13. The two main results presented here are very important so read this material carefully and note any questions you have.
- 2. Do exercises 2, 5, 8, 12, 14, 15, 16, and 20 in Section 2.2.
- 3. Turn in solutions for exercises 2 and 15 in Section 2.2.

#### for Wednesday, September 19

- 1. Finish reading Section 2.2.
- 2. Do exercises 21, 25, 26, 31, 32, 36, and 37 in Section 2.2. For the second part of exercise 25, you might want to consider the special case  $a_0 = 0$  and  $a_1 = 1$  first.
- 3. Turn in solutions for exercises 25 and 37 in Section 2.2.
- 4. Carl will present a solution to Exercise 2.1.26 and Henrique will present a solution to Exercise 2.4.12a.

# for Friday, September 21

- 1. Read Section 2.3 through the statement of the Bolzano-Weierstrass Theorem; we will not consider any of the rest of this section at this time.
- 2. Do exercises 2, 3, 10, and 14 in Section 2.3.
- 3. Turn in solutions for exercises 2.4.17 and 2.4.25. For exercise 17, you need to provide a proof of your answer to the question. With some thought, you should find that exercise 25 has a fairly simple solution. This is the first of our no help assignments; see the syllabus for the guidelines on these types of assignments.
- 4. Sam will present a solution to Exercise 2.3.1 and Chaoyi will present a solution to Exercise 2.4.6.

#### for Monday, September 24

- 1. Review Chapters 1 and 2, looking over the main results that we have discussed and the exercises that have been assigned. See the Wednesday assignment for some comments concerning the exam that will be given on that day.
- 2. Do exercises 5, 7, 9, and 16 in Section 2.4.
- 3. Turn in a solution for exercise 2.4.2. This is the second of our no help assignments; see the syllabus for the guidelines on these types of assignments. You may find it helpful to first consider the case in which x = 0, then show how the general case reduces to this special case.
- 4. Sarah will present a solution to Exercise 2.4.11ac.

#### for Wednesday, September 26

- 1. We have a test on Chapters 1 and 2. You need to be familiar with the concepts and theorems that are included in these chapters. You should be able to state definitions of commonly used terms (such as supremum and Cauchy sequence) and provide examples to illustrate these terms. By flipping through the pages of these two chapters, you should be able to identify the main results we have considered. You should be able to prove the Archimedean property of the real numbers, the linearity properties of sequences, that convergent sequences are bounded, and that bounded monotone sequences converge.
- 2. On the test, you will be asked to state some definitions or results, give some examples, solve some familiar problems, and solve one or two problems that you have not seen. The best preparation for the test is to have been keeping up with the material during these first few weeks of the semester.

#### for Friday, September 28

- 1. Read the introduction to Chapter 3 and Section 3.1 through the proof of Theorem 3.2.
- 2. Do exercises 1, 2, and 4ab in Section 3.1.
- 3. Turn in a solution for exercise 4a.

# for Monday, October 1

- 1. Finish reading Section 3.1.
- 2. Do exercises 7, 10, 13, 16, 34, and 35bc in Section 3.1.
- 3. Turn in solutions for exercises 7, 34c, and 35b.
- 4. Sam will present a solution to exercise 10, Chaoyi will present a solution to exercise 13, and Sarah will present a solution to exercise 16.

#### for Wednesday, October 3

- 1. Read Section 3.2.
- 2. Do exercises 1c, 2, 4, 7, 8, 10, and 15 in Section 3.2.
- 3. Turn in solutions for exercises 4 and 8.
- 4. Carl will present a solution to exercise 25 and Henrique will present a solution to exercise 30.
- 5. The third no-help assignment is due next Monday, but you might want to start working on it sooner rather than later. Do exercises 3.1.29 and 3.2.31. For the first of these exercises, proceed by contradiction and generate an unbounded sequence. Noting that this sequence has a monotone subsequence should lead to a contradiction. For the second, start by showing that the function has a limit of 0 at every point. You may use the following fact. If S is a finite set and  $a \notin S$ , then the number  $d = \min\{|x - a| : x \in S\}$  is positive and  $(a - d, a + d) \cap S = \emptyset$ .

### for Friday, October 5

1. No class today due to the October Break.

#### for Monday, October 8

- 1. Read Section 3.3 through the proof of Theorem 3.20.
- 2. Do exercises 1, 5, 6, 7, and 9 in Section 3.3.
- 3. Students will present a solution for the problem listed by their name: Sarah (1), Sam (5), Chaoyi (6), Henrique (7), and Carl (9).
- 4. Special assignment 3 is due at the beginning of class. This assignment is listed on the 10/3 date.

# for Wednesday, October 10

- 1. Finish reading Section 3.3.
- 2. Do exercises 22, 27, 30, 36, 37, 39, and 43 in Section 3.3.
- Turn in solutions for exercises 22 (you only need to do the supremum part) and 43. For the other exercises, I will request volunteers to discuss their ideas for solutions.

#### for Friday, October 12

- 1. Read Section 3.4; pay careful attention to the proofs that appear here.
- 2. Do exercises 1, 2, 3, 4, and 5 in Section 3.4. Be prepared to present solutions for the functions in Exercise 2.
- 3. Turn in solutions for exercises 4 and 5. For Exercise 5, do not use the fact that Cauchy sequences converge.
- 4. The fourth no-help assignment is due Monday. Do exercises 3.4.7 and 3.5.30.

#### for Monday, October 15

- 1. Read Section 3.5 through page 117; we will not be discussing any more of this section. However, if you are curious, then you should read the rest of the section to get an introduction to the concept of bounded variation. This idea, along with that of absolute continuity (see the last set of exercises in Section 3.6) are extremely important in higher level analysis courses.
- 2. Do exercises 7, 8, and 9 in Section 3.5. The reading and exercises are not intended to take too much time.
- 3. Special assignment 4 is due at the beginning of class. This assignment is listed on the 10/12 date.

# for Wednesday, October 17

- 1. Review Chapter 3.
- 2. Do exercises 1, 4, 8, 12, 18, and 34–38 in Section 3.6.
- 3. Turn in a solution for exercise 3.6.8.
- 4. The fifth no-help assignment is due Monday. Do exercises 3.6.21, 3.6.29, and 3.6.31.

#### for Friday, October 19

- 1. Read the introduction to Chapter 4 and Section 4.1.
- 2. Do exercises 1, 2, 4, 5, 6, 7, and 9 in Section 4.1. Note that many of these exercises are Calculus I problems so you should be able to do them easily at this point in your mathematical career.
- 3. Turn in solutions for exercises 8 and 13 in Section 4.1.

### for Monday, October 22

- 1. Do exercises 12, 18, 19, 21, 22, and 31 in Section 4.1. Be prepared to offer solutions to these problems at the board.
- 2. Special assignment 5 is due at the beginning of class. This assignment is listed on the 10/17 date.

### for Wednesday, October 24

- 1. We have a test over the material we have covered since the first exam but be aware that you do need to know much of the content from earlier chapters. If you have been keeping up (doing the reading, working on the problems, thinking about the concepts), you should be in good shape for the exam.
- 2. You should be able to state basic definitions (various limit forms, continuity, uniform continuity, derivative, and the like) and the three "value" theorems (IVT, EVT, and even the MVT which is in Section 4.2). You should be able to prove the IVT, the EVT, the uniform continuity theorem, and the fact that differentiability implies continuity. As usual, you should be prepared to give examples and solve problems related to the material that we have discussed.

### for Friday, October 26

- 1. Read Section 4.2 up to the paragraph prior to Theorem 4.13.
- 2. Do exercises 14 and 15 in Section 4.2.

#### for Monday, October 29

- 1. Finish reading Section 4.2.
- 2. Do exercises 18, 20, 24, 29, and 30 in Section 4.2.
- 3. Students will present a solution for the problem listed by their name: Carl (21), Sarah (27), Henrique (28), Sam (37), and Chaoyi (39).
- 4. Turn in solutions for exercises 18 and 20 in Section 4.2.

#### for Wednesday, October 31

- 1. Read Section 4.3 through the proof of Theorem 4.24.
- 2. Do exercises 4, 5, 6, 7, 11, and 13 in Section 4.3.
- 3. Turn in solutions for exercises 8 and 12 in Section 4.3.
- 4. The sixth no-help assignment is due Monday. Do exercises 4.1.19, 4.2.26, and the following: if the function  $f: \mathbb{R} \to \mathbb{R}$  is twice differentiable on  $\mathbb{R}$  and has at least five distinct real roots, then the function g defined by g(x) = f(x) + 2f'(x) + f''(x) has at least three distinct real roots. You may use the usual properties of  $e^x$  for this last problem (which gives you quite a hint for the solution).

#### for Friday, November 2

- 1. Spend some time reviewing the portions of Chapter 4 that we have discussed.
- 2. Do exercises 12, 13, and 15 in Section 4.4.
- 3. Turn in solutions for exercises 12 and 13 in Section 4.4.
- Students will present a solution for the Section 4.4 problem listed by their name: Carl (4), Sam (5), Sarah (9), Chaoyi (11), and Henrique (20).

#### for Monday, November 5

- 1. Do exercises 31, 34, and 42 in Section 4.4.
- 2. Special assignment 6 is due at the beginning of class. This assignment is listed on the 10/31 date.

## for Wednesday, November 7

- 1. Read the introduction to Chapter 5 and Section 5.1.
- 2. Do exercises 2, 4, 8, 9, 10, 11, and 12 in Section 5.1. Be prepared to discuss these problems in class.
- 3. Turn in solutions for exercises 8 and 12 in Section 5.1. Give two proofs for exercise 12; one directly from the definition and another using the results of Theorem 5.5.

### for Friday, November 9

- 1. Reread Section 5.1 to be certain that you understand the definitions and the notation.
- 2. Do exercises 16, 17, 18, 19, 20, and 21 in Section 5.1. Be prepared to discuss these problems in class.
- 3. Turn in solutions for exercises 15, 20, and 19b (using exercise 20 in the solution for exercise 19b) in Section 5.1. The first of these problems will be worth 10 points and the remaining two problems will be worth 5 points each so the total will be out of 20. This is the seventh of our no help assignments.

# for Monday, November 12

- 1. Read Section 5.2 through the proof of Theorem 5.10; this reading may take 90 minutes or so.
- 2. Turn in solutions for exercises 2 and 4 in Section 5.2.

# for Wednesday, November 14

- 1. Finish reading Section 5.2.
- 2. Do exercises 10, 11, 12, and 14 in Section 5.2.
- 3. Turn in solutions for exercises 10 and 14a in Section 5.2.

### for Friday, November 16

- 1. Read Section 5.3 through the proof of the Fundamental Theorem of Calculus.
- 2. Do exercises 6, 8, 10, 11, 12, (you should NOT use the Fundamental Theorem of Calculus for any of these exercises) 15, and 16 in Section 5.3.
- 3. Turn in solutions for exercises 11 and 12 in Section 5.3.

#### for Monday, November 26

- 1. Finish reading Section 5.3.
- 2. Do exercises 13, 17, 18, 26, and 29 in Section 5.3.
- 3. Turn in solutions for exercises 18 and 20 in Section 5.2 (yes, the section is correct). You may use Theorem 5.15 for exercise 18. These problems will be worth 10 points each so the total will be out of 20. This is the eighth of our no help assignments; see the syllabus for the guidelines on these types of assignments.
- 4. Review the material that that we have discussed since the last exam and look over the exercises that have been assigned. Bring any questions you have to class so that we can discuss them.

### for Wednesday, November 28

1. We have a test focused on the material that we have covered thus far in Chapters 4 and 5 but, as usual, you do need to know some of the content from earlier chapters. If you have been keeping up (doing the reading, working on the problems, thinking about the concepts and notation), you should be in good shape for the exam. You should be able to state the definitions of the common terms we have been using and give examples of functions exhibiting various properties. As an example, if you are asked to state the definition of the Riemann integral, you may assume that tagged partitions and Riemann sums have already been defined. You should be prepared to prove the Mean Value Theorem, that derivatives have the IVP, the linearity properties of the integral, that Riemann integrable functions are bounded, that continuity implies Riemann integrability, that monotonicity implies Riemann integrability, and both parts of the Fundamental Theorem of Calculus.

#### for Friday, November 30

- 1. Read Section 5.4 through the proof of Theorem 5.22.
- 2. Do exercises 4, 6, and 7 in Section 5.4.

### for Monday, December 3

- 1. Continue reading Section 5.4 through the proof of Theorem 5.24.
- 2. Do exercises 9, 14, 16, and 17 in Section 5.4.
- 3. Turn in solutions for exercises 5.4.12 and 5.6.4, thus pretending that you are in a calculus class again.
- 4. This is the ninth (and final) of our no help assignments; see the syllabus for the guidelines on these types of assignments. It is due on Friday, December 7. Turn in solutions for exercises 3 and 18 in Section 5.6. For exercise 3, note that F' may not exist at the points where  $F'(x) \neq f(x)$ . Exercise 18 is not too difficult once you recognize what needs to be done. The key issue is that the points  $s_i$  and  $t_i$  (which technically also depend on n but the notation then becomes very awkward) are different points (tags) in the subinterval. These problems will be worth 10 points each so the total will be out of 20.

### for Wednesday, December 5

- 1. Read Sections 6.1 and 6.2. This material should look familiar from Calculus II but you should find that you have a much a deeper understanding of it now.
- 2. Be prepared to discuss (as in go to the board) problems 5, 12, 13, 15, and 20 in Section 6.1 and problems 1, 3, 8, 14, and 21 in Section 6.2.
- 3. Turn in solutions for problems 3 and 5cd in Section 6.5.

# for Friday, December 7

- 1. Read Section 6.3. You can ignore the lim inf and lim sup symbols for now and treat them as ordinary limits. Since the general ideas should be somewhat familiar to you from calculus, focus on the proofs and concepts rather than just the computational details.
- 2. Be prepared to discuss problems 3, 5, 6, and 12 in Section 6.3.
- 3. Special assignment 9 is due at the beginning of class. This assignment is listed on the 12/3 date.
- 4. Information regarding the final exam, which takes place in our usual classroom from 2:00 to 5:00 pm on Wednesday, December 12, is provided on the next page.

- 1. A good place to start is to spend an hour or two looking over the sections that we have covered and reviewing the problems that have been assigned.
- 2. You should be able to give the definition of any concept that we have used regularly during the semester, be able to state any major result we have considered, and be able to generate examples that illustrate these ideas and concepts. You should also be prepared (as in know the basic idea so that you can reconstruct a proof) to prove the following results.
  - 1. Archimedean property of the real numbers
  - 2. there exists a rational number in any interval
  - 3. convergent sequences are bounded
  - 4. algebraic properties of sequences
  - 5. bounded monotone sequences converge
  - 6. Intermediate Value Theorem
  - 7. Extreme Value Theorem (just the f is bounded part)
  - 8. continuity implies uniform continuity on [a, b]
  - 9. differentiability implies continuity
  - 10. Mean Value Theorem (the full details which involve three results)
  - 11. the monotonicity theorem for differentiable functions
  - 12. the product rule and quotient rule for derivative
  - 13. Riemann integrable implies bounded
  - 14. continuous implies Riemann integrable
  - 15. monotone implies Riemann integrable
  - 16. Fundamental Theorem of Calculus
  - 17. Mean Value Theorem for integrals
  - 18. Divergence Test
  - 19. Comparison Test
  - 20. any linearity proof
- 3. The final exam will consist of some subset of item 2 (statements, examples, and proofs) along with some new problems that involve the ideas we have considered and the problem solving skills you have acquired.
- 4. The next page provides some exercises you might want to consider after preparing for the exam. More likely than not, it is impossible for you to do all of these between now and the exam. Read through the exercises and start with those that seem to be at your level of understanding. The parenthetical label is intended to give you a rough idea about the level of difficulty of the problem.

- 1. (easy) Let r be a rational number. Prove that there exists a strictly decreasing sequence of irrational numbers that converges to r.
- 2. (easy) Exercise 2.1.17.
- 3. (medium) Exercise 2.2.39c
- 4. (medium) Exercise 2.4.2.
- 5. (medium) Exercise 2.4.23.
- 6. (easy) Exercise 3.1.4c.
- 7. (medium) Exercise 3.2.4.
- 8. (medium) Exercise 3.4.10. (Some care is required here.)
- 9. (hard) Exercise 3.5.32.
- 10. (medium) Let  $f: [0,4] \to \mathbb{R}$  be continuous on [0,4] and suppose that f(0) = f(4). Prove that there exists a point  $c \in [0,1]$  such that f(3c+1) = f(c).
- 11. (medium) Exercise 3.6.31.
- 12. (easy) Exercise 4.2.18.
- 13. (medium) Exercise 4.4.44.
- 14. (medium) Exercise 5.1.10. (Try  $h(x) = x^2$  for a bit more of a challenge.)
- 15. (easy) Exercise 5.3.19.
- 16. (medium) Exercise 5.6.18.
- 17. (medium) Evaluate  $\lim_{n \to \infty} \sum_{i=1}^{n} \left( \ln \left( 1 + \frac{i}{n} \right) \cdot \frac{1}{3n-2} \right).$
- 18. (hard) Exercise 6.5.6.
- 19. (easy) Exercise 6.5.9.
- 20. (medium) Exercise 6.5.13.