Math 455: Real Analysis **Fall 2020**

for Monday, August 24

1. Read the syllabus for the course (available on the website) and the preface to the textbook. It would also be a good idea to read through the three appendices of the textbook. In addition to giving you a sense for the style of the book, it provides a brief review of Math 260. (This is another source of preparation materials for those of you taking the senior written exam as math majors.) If time is limited, you should focus on sections A.1, A.5, and A.7. At some point soon, you should watch the 23 minute youtube video `https://youtu.be/iim3gDdD_5k` giving a brief introduction to the history of real analysis.

for Wednesday, August 26

1. Read the introduction to Chapter 1 and Section 1.1. Except for the last part of this section, the reading should go rather quickly. (If you have not yet taken abstract algebra, then you may skim the discussion of fields.) Start now to really learn what it means to read and understand a mathematics textbook as you will be doing quite a bit of this over the next four months.
2. Watch the 28 minute youtube video `https://youtu.be/Q-KHtYtrvM8`. [I still need some practice.]
3. Do exercises 6, 7, 9, 10, 12, and 15 in Section 1.1. No integers should appear in your solutions for exercises 6 and 7. You may use the result of Exercise 11 for Exercise 12. Many of the solutions to these exercises involve proof by contradiction.
4. Turn in carefully written solutions for exercises 10 and 15. Be certain to include a (perhaps abbreviated) statement of the exercise and to use words and complete sentences in your proof.

for Friday, August 28

1. Read Section 1.2 through the discussion of geometric sums, including the paragraph following the proof of Theorem 1.10. The triangle inequality plays an important role in real analysis.
2. Watch the 23 minute youtube video `https://youtu.be/SkmXYbEW5NE`. [This time I sound a little low energy.]
3. Do exercises 1, 2, 6, 7, 14, 15, 17, and 19 in Section 1.2. For Exercise 7, be certain to use the triangle inequality; there should be no use of double inequalities related to absolute values. Note carefully that Exercise 12 can be used to solve Exercise 17. The fact that previous results can often be used to simplify the proofs of later results is an important observation; keep this in mind as you do later assignments.
4. Turn in solutions (due by noon Pacific time) for exercises 7 and 17.

for Monday, August 31

1. Read Section 1.3 through the paragraph following the proof of Theorem 1.18. The Completeness Axiom and the Archimedean Property are extremely important so be certain to ask questions if you find anything confusing about these concepts or proofs involving them.
2. Watch the 34 minute youtube video `https://youtu.be/vMBIY_O8724`.
3. Do exercises 5, 6, 12, 15, 16, 17, 19, and 20 in Section 1.3.
4. Turn in solutions (due by noon Pacific time) for exercises 12 and 15. You may use Theorem 1.17 in your solution for Exercise 15.

for Wednesday, September 2

1. Read Section 1.4. Hopefully, much of this material is familiar to you from Math 260 or previous math courses. Study carefully the proof that the set of real numbers is uncountable, making note of the use of the Completeness Axiom, since this approach is probably different than the one you have seen.
2. Watch the 36 minute (so much for keeping it short) youtube video `https://youtu.be/oJ4ExKrmzCw`.
3. Do exercises 1adfg, 7, 8, 17, 20, and 23 in Section 1.4. For part (f), try to find a function that does not require transcendental functions; a hand-waving argument that it has the necessary properties is sufficient. Exercise 8 provides good practice with injective and surjective functions. Think carefully about options for Exercise 17 that use previous results. For Exercise 20, you may use the following fact: if a set A is in a one-to-one correspondence with a subset of a countable set, then A is a countable set. Finally, do not spend too much time on part (d) of Exercise 23 if you happen to get stuck.
4. Turn in solutions (due by noon Pacific time) for exercises 8 and 20.

for Friday, September 4

1. Read Section 1.5. Much of this material should be familiar to you but be certain that you have the vocabulary down; this includes being able to state the definitions if requested or needed. Think carefully about, and ponder the implications of, the rather bizarre functions we discussed in class. It is important to realize that functions can behave in strange and unexpected ways so definitions of properties (such as continuity) must be stated very carefully.
2. Watch the 27 minute youtube video `https://youtu.be/yXYGOK4pzdk`.
3. Do exercises 23, 24, 25, 27, 28, 34, 35, and 43 in Section 1.5.
4. Turn in solutions (due by noon Pacific time) for exercises 27 (you may assume that the two functions are both defined for all real numbers) and 43.

for Monday, September 7

1. Read Section 2.1 through the paragraph following the proof of Theorem 2.5. Become very familiar with the adjectives for sequences, being able to state their definitions and giving examples of sequences with or without a given property. This will most likely be your first introduction to proofs that involve "Let $\epsilon > 0$ be given" so study the examples in the textbook and your notes carefully.
2. Watch the 43 minute youtube video `https://youtu.be/RBheTDgxZP8`.
3. Do exercises 5, 7, 9, 11, 14, 15, and 16 in Section 2.1. For exercises that ask for examples, try to find several different sequences with each property.
4. Turn in solutions (due by noon Pacific time) for exercises 5, 9b, and 14.

for Wednesday, September 9

1. Finish reading Section 2.1. Be certain to make note of any questions that you have.
2. Watch the 38 minute youtube video `https://youtu.be/6Inix1hTUKk`.
3. Do exercises 17, 20, 21, 22, 24, 31, 33a, and 42 in Section 2.1.
4. Turn in solutions (due by noon Pacific time) for exercises 17, 20c, and 24.

for Friday, September 11

1. Read Section 2.2 through the proof of Theorem 2.13. The two main results presented here are very important so read this material carefully and note any questions you have.
2. Watch the 40 minute (slow paced) youtube video `https://youtu.be/DY5nz-r09vs`.
3. Do exercises 2, 8, 14, 15, 16, and 20 in Section 2.2.
4. Turn in solutions (due by noon Pacific time) for exercises 2 and 15 in Section 2.2.

for Monday, September 14

1. Finish reading Section 2.2.
2. Watch the 36 minute youtube video `https://youtu.be/5YcGZ-ccHf0`.
3. Do exercises 21, 25, 28, 32, 36, and 37 in Section 2.2. For the second part of exercise 25, you might want to consider the special case $a_0 = 0$ and $a_1 = 1$ first.

3. Turn in solutions (due by noon Pacific time) for exercises 28, 32 (only for the more general case), and 37 in Section 2.2.

for Wednesday, September 16

1. Read Section 2.3 through the statement of the Bolzano-Weierstrass Theorem; we will not consider any of the rest of this section at this time.
2. Do exercises 2, 3, and 14 in Section 2.3.

for Friday, September 18

1. We have an exam on Chapters 1 and 2. You need to be familiar with the concepts and theorems that are included in these chapters. You should be able to state definitions of commonly used terms (such as supremum and Cauchy sequence) and provide examples to illustrate these terms. By flipping through the pages of these two chapters, you should be able to identify the main results we have considered. You should be able to prove the Archimedean property of the real numbers, the linearity properties of sequences, that convergent sequences are bounded, and that bounded monotone sequences converge. The intention here is that you know the basic idea behind the proof and then are able to write out the details; this is not an exercise in rote memorization.
2. On the exam, you will be asked to state some definitions or results, give some examples, prove some of the results listed in item (1), solve some familiar problems, and solve one or two problems that you have not seen. The best preparation for the exam is to have been keeping up with the material during these first few weeks of the semester; pondering concepts, learning various ways to approach problems, and knowing why certain results are true.
3. The exam will be posted at 7:30 am (Pacific time) Friday and it is due by 5:00 pm (Pacific time) Friday. You are on the honor system to spend less than 90 minutes on the exam and to work independently on the problems. To be specific concerning what it means to work alone, you may not seek help from other students, you may not seek help from any other person, you may not seek help from any textbooks or notes, you may not seek help from the Internet, and you may not seek help from Maple, Wolfram Alpha, or any electronic devices (with the obvious exceptions for creating and sending files). Even talking/texting briefly to someone about the problems or sharing notes is a violation of the policy. Any violation will result in a case of academic dishonesty (refer to the academic honesty form that you signed when you first arrived at Whitman). Except for the printing, scanning, and submitting aspects, taking the exam would be as if you had been in a classroom on campus. By signing your name on the exam, you agree to these guidelines.

for Monday, September 21

1. There is no specific assignment for this day. However, you should watch the youtube video for Section 3.1 some time during this day. You might also want to read item (2) and take a look at those problems.
2. The first of our special assignments is due next Monday. Turn in solutions for exercises 2, 20, 21, and 25 from Section 2.4. You may use your notes, our class website, and the textbook but no other resources, as in none whatsoever. (Refer to the guidelines for the first exam.) In other words, you must work alone on these problems. Name your solutions file `email_sp_1.pdf`

for Wednesday, September 23

1. Read the introduction to Chapter 3 and Section 3.1 through the end of page 85.
2. Watch the 33 minute youtube video `https://youtu.be/dSBF8-hZaXw`.
3. Do exercises 1, 2, 4abdf, 5, and 7 in Section 3.1.
4. Turn in solutions (due by noon Pacific time) for exercises 4b and 7.

for Friday, September 25

1. Finish reading Section 3.1.
2. Watch the 35 minute youtube video `https://youtu.be/eud9EHxHmFQ`.
3. Do exercises 10, 13, 16, 22, 34, and 39 in Section 3.1.
4. Turn in solutions (due by noon Pacific time) for exercises 22, 34c, and 39b (part (ii) only).

for Monday, September 28

1. The first special assignment (named `email_sp_1.pdf`) is due on this day (see the 9/21 assignment).
2. Read Section 3.2. There will be no youtube video for this section.
3. Do exercises 2, 4, 7, 8, 10, 15, and 29 in Section 3.2.

for Wednesday, September 30

1. Read Section 3.3 through the paragraph following Theorem 3.18.
2. Watch the 54 minute (I had no idea I talked that long) youtube video `https://youtu.be/CHzzTq8gekw`. There are some Section 3.2 solutions at the end.
3. Do exercises 1 (add $S \neq \emptyset$), 5, 6, 7, 9, 12, and 13 in Section 3.3.
4. Turn in solutions (due by noon Pacific time) for exercises 7 and 9. For Exercise 7, you may use the results in Exercise 3.1.40 (without proving them), but you should then provide careful details for the solutions to Exercise 7.

for Friday, October 2

1. No class today due to the October Break.

for Monday, October 5

1. Finish reading Section 3.3.
2. Watch the 36 minute youtube video `https://youtu.be/K3vb0huQOG8`. I spend a little time at the beginning discussing the first special assignment. As I reviewed the video a bit, I noticed that I made several errors in the r, s specific example by reversing the roles of r and s and not thinking as I went. The important point here is to try some specific examples to get a sense of what is going on.
3. Do exercises 14, 16, 17, 36, 37, and 39 in Section 3.3.
4. Turn in solutions (due by noon Pacific time) for exercises 3.3.16 (note that it has two parts; also, give reference to the theorems or exercises that you need) and 3.6.23. (For the record, this is hw assignment 13.)
5. The second of our special assignments is due this coming Friday (10/9). Turn in solutions for exercises 3.1.29, 3.2.31, and 3.6.18. For Exercise 3.1.29, one option is to proceed by contradiction and consider using Theorem 2.18. Also, as mentioned at the end of Section 3.1, you may use analogous results that have been proven for two-sided limits for one-sided limits without additional proofs. Think carefully about the function defined in Exercise 3.2.31 as we will see it several more times this semester. For Exercise 3.6.18, write your solution as a student in real analysis as opposed to a calculus student. You may use your notes, our class website, and the textbook but no other resources, as in none whatsoever, including other students in the class. (Refer to the guidelines for the first exam. As a reminder, you may work with other students and/or ask me questions on all of the regular assignments.) In other words, you must work alone on these special assignment problems. Name your solutions file `email_sp_2.pdf`

for Wednesday, October 7

1. Read Section 3.4; pay very careful attention to the proofs of Theorems 3.28 and 3.29.
2. Watch the 32 minute youtube video `https://youtu.be/6wqli4XyCgs`.
3. Do exercises 1, 2, 3, 4, and 5 in Section 3.4.
4. Turn in solutions (due by noon Pacific time) for exercises 4 and 5. For Exercise 5, do not use the fact that Cauchy sequences converge.

for Friday, October 9

1. The second special assignment is due by noon Pacific time on this day (see the 10/5 assignment).
2. Read Section 3.5 through page 117; we will not be discussing any more of this section. However, if you are curious, then you should read the rest of the section to get an introduction to the concept of bounded variation. This idea, along with that of absolute continuity (see the last set of exercises in Section 3.6) are extremely important in higher level analysis courses.
3. Watch the 41 minute youtube video `https://youtu.be/i98UWgdIVns`.
4. Do exercises 6, 7, 8, and 9 in Section 3.5. The reading and exercises are not intended to take too much time.

for Monday, October 12

1. Review Chapter 3.
2. Do exercises 1, 4, 8, 31, and 34–38 in Section 3.6.
3. Turn in solutions (due by noon Pacific time) for exercises 1 and 31.
4. Spend some time reviewing for the exam on Friday.

for Wednesday, October 14

1. Read the introduction to Chapter 4 and Section 4.1.
2. There will not be a youtube video for this section.
3. Do exercises 1, 2, 4, 6, 9, 12, 18, and 19 in Section 4.1. Note that many of these exercises are Calculus I problems so you should be able to do them easily at this point in your mathematical career.
4. Turn in solutions (due by noon Pacific time) for exercises 6b and 19 in Section 4.1.

for Friday, October 16

1. Read Section 4.2 up to the paragraph prior to Theorem 4.13.
2. There may not be a youtube video for this material.
3. Do exercises 8, 11, 13, 14, 15, and 18 in Section 4.2.
4. Turn in solutions (due by noon Pacific time) for exercises 15 and 18 in Section 4.2. Your proof for Exercise 18 should be very short and involve reducing the problem to an exercise you have already solved.

for Monday, October 19

1. Finish reading Section 4.2.
2. Watch the 45 minute youtube video `https://youtu.be/iDyObOW_Nsg`.
3. Do exercises 21, 22, 24, 26, 27, 31, 37, and 39. Pay particular attention to the function in Exercise 26.
4. Turn in solutions (due by noon Pacific time) for exercises 20, 24, and 37 in Section 4.2.

for Wednesday, October 21

1. We have a test over the material we have covered since the first exam but be aware that you do need to know much of the content from earlier chapters. If you have been keeping up (doing the reading, working on the problems, thinking about the concepts), you should be in good shape for the exam.

2. You should be able to state basic definitions (various limit forms, continuity, uniform continuity, derivative, and the like) and the three "value" theorems (IVT, EVT, and MVT). You should be able to prove the IVT, the EVT, the uniform continuity theorem, and the fact that differentiability implies continuity. As usual, you should be prepared to give examples and solve problems related to the material that we have discussed.

3. The exam will be posted at 7:30 am (Pacific time) Wednesday and it is due by 5:00 pm (Pacific time) Wednesday. (Let me know if there are issues with this time frame.) You are on the honor system to spend less than 90 minutes on the exam and to work independently on the problems. To be specific concerning what it means to work alone, you may not seek help from other students, you may not seek help from any other person, you may not seek help from any textbooks or notes, you may not seek help from the Internet, and you may not seek help from Maple, Wolfram Alpha, or any electronic devices (with the obvious exceptions for creating and sending files). Even talking/texting briefly to someone about the problems or sharing notes is a violation of the policy. Any violation will result in a case of academic dishonesty (refer to the academic honesty form that you signed when you first arrived at Whitman). Except for the printing, scanning, and submitting aspects, taking the exam would be as if you had been in a classroom on campus. By signing your name on the exam, you agree to these guidelines.

for Friday, October 23

1. Do exercises 4, 5, 6, 8, 11, and 20 in Section 4.4. These problems should not take too long.

2. Turn in a solution (due by noon Pacific time) for exercise 20. For the example, we want f to be unbounded as $x \to \infty$.

3. The third of our special assignments is due this coming Wednesday (10/28). Turn in solutions for exercises 3.4.10, 4.4.31 (just the cylinder part), 4.4.44, and the problem stated below.

   Suppose that f is continuous on $[a, b]$ and differentiable on (a, b). Assume that the set $\{x \in (a, b) : f'(x) \leq 0\}$ is countably infinite. Prove that f is strictly increasing on $[a, b]$.

   For Exercise 3.4.10, start by choosing $\delta > 0$ for the interval $[0, 2p]$, then reducing all of the cases down to this interval using the periodic nature of the function. For Exercise 4.4.44, you need to read the previous problem for the definition of the strong derivative. For the extra problem, there are two steps: first prove that f is increasing on $[a, b]$, then prove that f is actually strictly increasing on $[a, b]$. You may use your notes, our class website, and the textbook but no other resources, as in none whatsoever, including other students in the class. (Refer to the guidelines for the first exam. As a reminder, you may work with other students and/or ask me questions on all of the regular assignments.) In other words, you must work alone on these special assignment problems. Name your solutions file `email_sp_3.pdf`

for Monday, October 26

1. Read Section 4.3 through the proof of Theorem 4.24.
2. Do exercises 2, 3, 4, 7, 11, 12, and 13 in Section 4.3.
3. Turn in solutions (due by noon Pacific time) for exercises 7 and 12.
4. The 18 minute youtube video `https://youtu.be/JGA-iatgYZ4` gives the solutions to several exercises in Chapter 4, including ideas for how to approach such problems.

for Wednesday, October 28

1. The third special assignment is due by noon Pacific time on this day (see the 10/23 assignment).
2. Spend some time reviewing the portions of Chapter 4 that we have discussed.
3. Do exercises 7 and 21 in Section 4.4.

for Friday, October 30

1. Read the introduction to Chapter 5 and Section 5.1.
2. Watch the 20 minute youtube video `https://youtu.be/bijylOrki_E`.
3. Do exercises 1, 2, 3, 4, 9, 10, and 11 in Section 5.1.
4. Turn in solutions (due by noon Pacific time) for exercises 8 and 12 in Section 5.1. Give two proofs for exercise 12; one directly from the definition and another using the results of Theorem 5.5.
5. The fourth of our special assignments is due this coming Wednesday (11/4). Turn in solutions for Exercises 3.4.10, 5.1.20, 5.1.19, and 5.2.7. Name your solutions file `email_sp_4.pdf`

   (3.4.10) Note that $f(x + np) = f(x)$ for each integer n. Explain why f is uniformly continuous on $[0, 2p]$. Let $\epsilon > 0$. Get an appropriate $\delta > 0$ and be sure that $\delta < p$. Suppose that x and $y > x$ are real numbers such that $|y - x| < \delta$. Explain why there is an integer n such that $n \leq x/p < n + 1$. Show that both $x - np$ and $y - np$ belong to the interval $[0, 2p]$. Now finish the proof.

   (5.1.20) Give a letter name to each of the three suprema and then carefully use the definition.

   (5.1.19) Use Exercise 5.1.20 for this problem as it makes things much simpler.

   (5.2.7) Use the Cauchy criterion for integrals.

for Monday, November 2

1. Reread Section 5.1 to be certain that you understand the definitions and the notation.
2. Do exercises 15, 16, 17, and 18 in Section 5.1.
3. Turn in a solution (due by noon Pacific time) for Exercise 15 (note that it has three parts). Think carefully in order to find a solution with very little need of tagged partitions by taking advantage of previous results.

for Wednesday, November 4

1. The fourth special assignment is due by noon Pacific time on this day (see the 10/30 assignment).
2. Read Section 5.2 through the proof of Theorem 5.10; this reading may take quite a while.
3. Do exercises 2 and 4 in Section 5.2.
4. The fifth of our special assignments is due next Monday (11/9). Turn in solutions for Exercises 5.2.18, 5.2.20, and 5.3.13. Name your solutions file `email_sp_5.pdf`

   (5.2.18) You may use Theorem 5.15 for this exercise and you definitely need to use part (c) of Theorem 5.5.

   (5.2.20) The beginning of the Section 5.3 video may be helpful here.

   (5.3.13) As indicated, you may use Exercise 5.3.12 (without providing a proof) in your solution to this exercise.

for Friday, November 6

1. Finish reading Section 5.2.
2. Do exercises 10, 11, 12, 13, and 15 in Section 5.2.
3. Turn in solutions (due by noon Pacific time) for exercises 10 and 11 in Section 5.2.

for Monday, November 9

1. The fifth special assignment is due by noon Pacific time on this day (see the 11/4 assignment).
2. Read Section 5.3.
3. Watch the 54 minute youtube video `https://youtu.be/bH99oxEUYyQ`.
4. Do exercises 10, 11, 12, (you should NOT use the Fundamental Theorem of Calculus for any of these exercises) and 15 in Section 5.3.

for Wednesday, November 11

1. Read Section 5.4 through the proof of Theorem 5.24.
2. Do exercises 6, 9, and 12 in Section 5.4.
3. Turn in a solution (due by noon Pacific time) for exercise 9 in Section 5.4.

for Friday, November 13

1. We have a test focused on the material that we have covered thus far in Chapters 4 and 5 but, as usual, you do need to know some of the content from earlier chapters. If you have been keeping up (doing the reading, working on the problems, thinking about the concepts and notation), you should have a good start for preparing for the exam.

2. You should be able to state the definitions of the common terms we have been using and give examples of functions exhibiting various properties. As an example, if you are asked to state the definition of the Riemann integral, you may assume that tagged partitions and Riemann sums have already been defined. You should also be able to state the main theorems that we have covered. You should be prepared to prove the Mean Value Theorem, that derivatives have the IVP, the linearity properties of the integral, that Riemann integrable functions are bounded, that continuity implies Riemann integrability, that monotonicity implies Riemann integrability, and both parts of the Fundamental Theorem of Calculus.

3. The exam will be posted at 7:30 am (Pacific time) Friday and it is due by 5:00 pm (Pacific time) Friday. (Let me know if there are issues with this time frame.) You are on the honor system to spend less than 100 minutes on the exam and to work independently on the problems. To be specific concerning what it means to work alone, you may not seek help from other students, you may not seek help from any other person, you may not seek help from any textbooks or notes, you may not seek help from the Internet, and you may not seek help from Maple, Wolfram Alpha, or any electronic devices (with the obvious exceptions for creating and sending files). Even talking/texting briefly to someone about the problems or sharing notes is a violation of the policy. Any violation will result in a case of academic dishonesty (refer to the academic honesty form that you signed when you first arrived at Whitman). Except for the printing, scanning, and submitting aspects, taking the exam would be as if you had been in a classroom on campus. By signing your name on the exam, you agree to these guidelines.

4. The sixth (and final) of our special assignments is due next Friday (11/20). Turn in solutions for Exercises 6.1.23, 6.2.15, 6.5.5, and 6.5.22. These problems only involve ideas from Sections 6.1 and 6.2. Name your solutions file `email_sp_6.pdf`

   (6.1.23) Just turn in proofs for parts (b) and (c), using the other parts for your own clarity about the problem.

   (6.2.15) Consider also the case in which $0 < a < 1$.

   (6.5.5) This exercise is not as scary as it may first appear. The sequence $\{t_n\}$ has some helpful properties and there is a simple connection between the numbers a_n and t_n. Make note of telescoping sums as you proceed through your scratchwork. You do not need to turn in a solution for part (e), but you might find it helpful for your understanding.

   (6.5.22) A theorem from Chapter 4 should be pleading with you to let him out.

for Monday, November 16

1. There is no specific assignment. You can perhaps look over your exam (assuming I was able to grade them in time) or start reading Chapter 6 material.

for Wednesday, November 18

1. Read Section 6.1. This material should look familiar from Calculus II, but you should find that you have a much a deeper understanding of it now.
2. Watch the 34 minute youtube video `https://youtu.be/MwZ4cVxHbTw`. You can also go to my Math 126 page and refer to the assignments page there. The (lengthy) videos listed for Sections 3.5 and 3.7 might be helpful.
3. Do exercises 1c, 2b, 4, 5, 6, 11, 12, 13, 16, and 18 in Section 6.1.
4. Turn in solutions (due by noon Pacific time) for exercises 4, 13, and 16 in Section 6.1.

for Friday, November 20

1. The sixth special assignment is due by noon Pacific time on this day (see the 11/13 assignment).
2. Read Section 6.2.
3. Watch the 57 minute youtube video `https://youtu.be/CuSA7c5AvtQ`. There is a rather terrible section in the 35 to 40 minute mark where I screw up a simple problem. (Clearly, I do not know how to use the pause key on these videos.) Other parts of the video may be too basic as well, but there are a few important details that you may find helpful. So, you will need to use some discretion, and probably some skimming, with this video.
4. Do exercises 3, 8, 14, 18, 20, and 21 in Section 6.2.

for Monday, November 23

1. Read Section 6.3.
2. Watch the 55 minute youtube video `https://youtu.be/3JXj6I0-Jzw`.
3. Do exercises 4, 6, 8, 9, 10, 12, 19, 21, and 24 in Section 6.3.
4. Turn in solutions (due by noon Pacific time) for exercises 8c and 14bf in Section 6.2 and exercises 4, 12f, and 21 in Section 6.3. These are essentially calculus problems so they should not take too much time.
5. Information regarding the final exam, which is scheduled for 2:00 to 5:00 pm on Tuesday, December 1, is provided on the next two pages. The plan would be the following: I send you the final exam as a pdf file attached to an email just before 2:00 pm and you would return it by 5:00 pm that same day.

Review for Final Exam

1. A good place to start is to spend an hour or two looking over the sections that we have covered and reviewing the exercises that have been assigned.

2. You should be able to give the definition of any concept that we have used regularly during the semester, be able to state any major result we have considered, and be able to generate examples that illustrate these ideas and concepts. You should also be prepared (as in know the basic idea so that you can reconstruct a proof) to prove the following results.

    1. Archimedean property of the real numbers
    2. there exists a rational number in any interval
    3. convergent sequences are bounded
    4. algebraic properties of sequences
    5. bounded monotone sequences converge
    6. Intermediate Value Theorem
    7. Extreme Value Theorem (just the f is bounded part)
    8. continuity implies uniform continuity on $[a, b]$
    9. differentiability implies continuity
    10. Mean Value Theorem (the full details which involve three results)
    11. the monotonicity theorem for differentiable functions
    12. the product rule and quotient rule for derivatives
    13. Riemann integrable implies bounded
    14. continuous implies Riemann integrable
    15. monotone implies Riemann integrable
    16. Fundamental Theorem of Calculus (both parts)
    17. Mean Value Theorem for integrals
    18. Divergence Test
    19. Comparison Test
    20. any linearity proof (sequences, series, limits, integrability)

3. The final exam will consist of some subset of item 2 (statements, examples, and proofs) along with some new problems that involve the ideas we have considered and the problem solving skills you have acquired.

4. The next page provides some exercises (some of which were assigned during the semester) you might want to consider after preparing for the exam. More likely than not, it is impossible for you to do all of these between now and the exam. Read through the exercises and start with those that seem to be at your level of understanding. The parenthetical label is intended to give you a rough idea about the level of difficulty of the problem, but this sort of label is highly subjective.

1. (easy) Let r be an irrational number. Prove that there exists a strictly decreasing sequence of rational numbers that converges to r.
2. (easy) Exercise 2.1.17 (without using the squeeze theorem).
3. (medium) Exercise 2.2.39c
4. (medium) Exercise 2.4.2.
5. (medium) Exercise 2.4.23.
6. (easy) Exercise 3.1.4c.
7. (medium) Exercise 3.2.4.
8. (medium) Exercise 3.4.10. (Some care is required here.)
9. (medium) Let $f\colon [0,4] \to \mathbb{R}$ be continuous on $[0,4]$ and suppose that $f(0) = f(4)$. Prove that there exists a point $c \in [0,1]$ such that $f(3c+1) = f(c)$.
10. (medium) Exercise 3.6.31.
11. (easy) Exercise 4.2.18.
12. (medium) Exercise 4.4.44.
13. (medium) Exercise 5.1.10. (Try $h(x) = x^2$ for a bit more of a challenge.)
14. (easy) Exercise 5.3.19.
15. (hard) Exercise 5.6.5. The following remnant of scratchwork may be helpful.

$$M - 2\epsilon < (M - \epsilon)\sqrt[n]{d-c} \leq v_n \leq M\sqrt[n]{b-a} < M + \epsilon \quad \text{for } n \geq N$$

16. (medium) Exercise 5.6.18.
17. (medium) Evaluate $\lim_{n\to\infty} \sum_{i=1}^{n} \Big(\ln\Big(1 + \frac{i}{n}\Big) \cdot \frac{1}{3n-2}\Big)$.
18. (hard) Exercise 6.5.6.
19. (easy) Exercise 6.5.9.
20. (medium) Exercise 6.5.13.