for Wednesday, August 30

1. Read the syllabus for the course (available on the website), then read the following parts of the textbook: Appendix A.1, the preface, and the short introduction to Chapter 1.
2. Turn in a carefully written solution to the following problem:

   Prove that $\sum_{i=1}^{n}(-1)^{i+1}i^2 = \frac{1}{2}(-1)^{n+1}n(n+1)$ for each positive integer n.

   You should use mathematical induction for your proof. I ask that you work alone on this problem (getting no help from any source other than the textbook or your Math 260 notes) so that I can get a sense of your level of preparation for solving problems and writing proofs.

for Friday, September 1

1. Read Section 1.1. Except for the last part of this section, the material should read rather quickly. (If you have not yet taken abstract algebra, then you may skim the discussion of fields.) Start now to really learn what it means to read and understand a mathematics textbook as you will be doing quite a bit of this over the next four months.
2. Do exercises 7, 10, 12 (use Exercise 11), 14, and 15 in Section 1.1. Many of the solutions to these exercises involve proof by contradiction.
3. Turn in a carefully written solution for exercise 15. Be certain to include a (perhaps abbreviated) statement of the exercise and to use words and complete sentences in your proof.

for Monday, September 4

1. Read Section 1.2 through the discussion of geometric sums.
2. Do exercises 6, 7, 9, 12, and 17 in Section 1.2. Note carefully that Exercise 12 can be used to help solve Exercise 17. The fact that previous results can often be used to simplify the proofs of later results is an important observation; keep this in mind as you do later assignments.
3. Turn in a solution for exercise 7; think carefully about how you approach this problem.

for Wednesday, September 6

1. Read Section 1.3 through the paragraph following the proof of Theorem 1.18. The Completeness Axiom and the Archimedean Property are extremely important so be certain to ask questions if you find anything confusing about them.
2. Do exercises 12, 15, 17, 19, and 20 in Section 1.3.
3. Turn in a solution for exercise 15; you may use Theorem 1.17 in your solution.

for Friday, September 8

1. Read Section 1.4, focusing primarily on the proof of Theorem 1.29 since it gives a different proof that the set of real numbers is uncountable than the one you have most likely seen. Note the use of the Completeness Axiom. Hopefully, much of the rest of the material in this section is familiar to you from Math 260 or previous math courses.
2. Do exercises 4, 9, and 17 in Section 1.4.
3. Skim Section 1.5. Much of this material should be familiar to you but be certain that you know the vocabulary. Think carefully about and ponder the implications of the rather bizarre functions we discussed in class. It is important to realize that functions can behave in strange and unexpected ways so definitions of properties (such as continuity) must be stated very carefully.
4. Do exercises 23, 24, 25, 27, 34, 35, and 43 in Section 1.5. Be prepared to discuss these exercises in class.

for Monday, September 11

1. Read Section 2.1 through the proof of Theorem 2.4. Become very familiar with the adjectives for sequences, being able to state their definitions and giving examples of sequences with or without a given property. This will most likely be your first introduction to proofs that involve "Let $\epsilon > 0$ be given" so study the examples in the textbook and your notes carefully.
2. Do exercises 5, 7, 9, 10, 11, and 15 in Section 2.1. For exercises that ask for examples, try to find several different sequences with each property. Be prepared to share your examples in class as well as outline your solutions to the exercises that require proofs.
3. Turn in a solution to the following problem: use the definition of convergence to prove that the sequence $\left\{\dfrac{3n+11}{4n+7}\right\}$ converges. Try to keep your computations as simple as possible.
4. Student presentations at the board for exercises in Section 2.1 are Yidan (5), Awa (15), Jonathan (7e–7g), Thomas (9c), Olivia (10), Emilia (7a–7d), and Eli (11).

for Wednesday, September 13

1. Finish reading Section 2.1. Be certain to make note of any questions that you have.
2. Do exercises 17, 19, 20, 21, 33, and 42 in Section 2.1.
3. Turn in solutions for exercises 17 (do not use the Squeeze Theorem) and 20c in Section 2.1.
4. Student presentations at the board for exercises in Section 2.1 are Yidan (31), Awa (22), Jonathan (24), Thomas (41), Olivia (16), Emilia (32), and Eli (43).

for Friday, September 15

1. Read Section 2.2 through the proof of Theorem 2.13. The two main results presented here are very important so read this material carefully and note any questions you have.
2. Do exercises 2, 8, 12, 14, 15, 16, and 20 in Section 2.2.
3. Turn in solutions for exercises 2 and 15 in Section 2.2.
4. Student presentations for exercises in Section 2.2 are Eli (8), Awa (14), Yidan (16), and Thomas (20).
5. **IMPORTANT:** Make note of the assignment for next Wednesday. You should begin thinking about these problems soon.

for Monday, September 18

1. Finish reading Section 2.2.
2. Do exercises 21, 31, 32, 36, and 37 in Section 2.2.
3. Read Section 2.3 through the statement of the Bolzano-Weierstrass Theorem; we will not consider any of the rest of this section at this time.
4. Do exercises 3, 5, and 6 in Section 2.3.
5. Turn in solutions for exercises 31b and 37 in Section 2.2.
6. Student presentations for exercises in Section 2.2 are Olivia (21), Jonathan (32), and Emilia (36).

for Wednesday, September 20

1. Review Chapter 2, looking over the main results that we have discussed and the exercises that have been assigned.
2. Do exercises 5, 7, 16, 17, and 19 in Section 2.4.
3. Turn in solutions for exercises 2, 20, and 25 in Section 2.4; for Exercise 25, you may find the result of Exercise 1.3.19 useful. This is the first of our no help assignments. **IMPORTANT:** Please refer to the syllabus for the guidelines on these types of assignments. For the record, following these guidelines falls under the umbrella of the academic dishonesty policy of Whitman College.

for Friday, September 22

1. Read the introduction to Chapter 3 and Section 3.1 through the discussion after the proof of Theorem 3.2.
2. Do exercises 1, 2, 4abcf, and 8 in Section 3.1.
3. Turn in a solution for exercise 4b.

for Monday, September 25

1. Finish reading Section 3.1.
2. Do exercises 10, 13, 16, 18, 22, and 34 in Section 3.1.
3. Turn in solutions for exercises 22 and 34c.

for Wednesday, September 27

1. Read Section 3.2.
2. Do exercises 1c, 2, 4, 7, 8, and 10 in Section 3.2.
3. Turn in solutions for exercises 4 and 10.

for Friday, September 29

1. We have a test on the material we have covered thus far. You need to be familiar with the concepts and theorems that are included in Sections 1.1 to 3.2. You should be able to state definitions of commonly used terms (such as supremum and Cauchy sequence) and provide examples to illustrate these terms. By flipping through the pages of these sections, you should be able to identify the main results we have considered. You should be able to prove the Archimedean property of the real numbers, the linearity properties of sequences and continuous functions, that convergent sequences are bounded, and that bounded monotone sequences converge.
2. On the test, you may be asked to state some definitions or results, give some examples, prove some of the results listed in item (1), solve some familiar problems, and solve one or two problems that you have not seen. The best preparation for the test is to have been keeping up with the material during these first few weeks of the semester; pondering concepts, learning various ways to approach problems, and knowing why certain results are true.

for Monday, October 2

1. There is no assignment due this day as you recover from the exam. However, it would be a good idea to review Sections 3.1 and 3.2.

for Wednesday, October 4

1. Read Section 3.3 through the paragraph following Theorem 3.18.
2. Do exercises 1 (add $S \neq \emptyset$), 5, 6, 7, 9, 12, 13, and 14 in Section 3.3.
3. Turn in solutions for exercises 6 and 9.
4. There is a special assignment due next Wednesday (10/11); see the assignment for that date.

for Friday, October 6

1. No class today due to the October Break.

for Monday, October 9

1. Finish reading Section 3.3.
2. Do exercises 20, 27, 36, 37, 39, and 44b in Section 3.3.
3. Turn in solutions for exercises 20 (do not use transcendental functions for your example) and the portion of 44b that is omitted below.

   Let $S = \{x \in [a, b] : f \text{ is bounded on } [a, x]\}$. Since f is locally bounded at a, there exist positive numbers M_a and $\delta_a < b - a$ such that $|f(x)| \leq M_a$ for all $x \in [a, a + \delta_a)$. It follows that $[a, a + \delta_a) \subseteq S$. Since the set S is nonempty and bounded above, the Completeness Axiom asserts that S has a supremum, call it β.

   Suppose that $\beta \notin S$. (get a contradiction)

   Now suppose that $\beta < b$. Then there exist positive numbers M_2 and δ_2 such that $|f(x)| \leq M_2$ for all $x \in [\beta, \beta + \delta_2]$. Since f is bounded on both of the intervals $[a, \beta]$ and $[\beta, \beta + \delta_2]$, we find that $\beta + \delta_2 \in S$, a contradiction to the fact that $\beta = \sup S$. It follows that $\beta = b$. We conclude that $b \in S$, that is, the function f is bounded on $[a, b]$.

for Wednesday, October 11

1. Read Section 3.4; pay careful attention to the proofs that appear here.
2. Do exercises 1, 2, 3, 4, and 5 in Section 3.4.
3. Turn in a solution for exercise 5; do not use the fact that Cauchy sequences converge.
4. Turn in solutions for exercises 3.2.31, 3.3.43, and 3.6.13. Exercise 3.6.13 is rather easy. If you prefer more of a challenge (or would like extra credit) turn in a solution for Exercise 3.6.21. It is preferred that you give a direct proof for this exercise rather than a proof by contradiction. This is the second of our no help assignments; see the syllabus for the guidelines on these types of assignments. For the record, following these guidelines falls under the umbrella of the academic dishonesty policy of Whitman College.

for Friday, October 13

1. Read Section 3.5 through page 117; we will not be discussing any more of this section. However, if you are curious, then you should read the rest of the section to get an introduction to the concept of bounded variation. This idea, along with that of absolute continuity (see the last set of exercises in Section 3.6) are extremely important in higher level analysis courses.
2. Do exercises 7, 8, and 9 in Section 3.5. Be prepared to present solutions to the class for the requested functions in Exercise 9.
3. Turn in a solution for Exercise 3.6.31.
4. **IMPORTANT:** Make note of the assignment for next Friday.

for Monday, October 16

1. Review Chapter 3, making sure you know the definitions of key concepts and the statements of important theorems, along with the basic ideas behind their proofs.
2. Do exercises 1, 4, 8, and 34–38 in Section 3.6. Be prepared to discuss these exercises in class.
3. Turn in a solution for exercise 22 in Section 3.6.

for Wednesday, October 18

1. Read the introduction to Chapter 4 and Section 4.1.
2. Do exercises 1, 2, 4, 5, 6, 7, 8, 9, and 13 in Section 4.1. Note that many of these exercises are Calculus I problems so you should be able to do them easily at this point in your mathematical career. Be prepared to offer solutions to these problems at the board.

for Friday, October 20

1. Do exercises 12, 18, 19, and 31 in Section 4.1.
2. Turn in a solution for exercise 19 in Section 4.1.
3. Turn in solutions for exercises 3.4.7, 3.4.10, and 3.6.29. This is the third of our no help assignments; see the syllabus for the guidelines on these types of assignments. For the record, following these guidelines falls under the umbrella of the academic dishonesty policy of Whitman College.

for Monday, October 23

1. Read Section 4.2 up to the paragraph prior to Theorem 4.13.
2. Do exercises 8, 11, 13, 14, 15, and 18 in Section 4.2. Be prepared to discuss these problems in class.
3. Turn in a solution for exercise 18 in Section 4.2. Your proof should be very short and involve referencing a problem that you have already solved in a previous assignment.

for Wednesday, October 25

1. Finish reading Section 4.2.
2. Students will present a solution (try for five minutes for your presentation) for the 4.2 exercise listed by their name: Emilia (7), Olivia (21), Eli (27), Thomas (37), Yidan (38a), Awa (38b), and Jonathan (39). You should also spend some time looking over the exercises that you are not presenting.
3. Turn in solutions for exercises 20 and 24 in Section 4.2.

for Friday, October 27

1. Read Section 4.3 through the proof of Theorem 4.24.
2. Do exercises 4, 7, 11, 12, and 13 in Section 4.3. Be prepared to discuss these exercises in class.
3. Turn in a solution for the following problem:

   Suppose that f is a nonnegative function defined on $(0, 1)$ and that f''' exists on $(0, 1)$. Suppose further that f has at least two zeros in $(0, 1)$. Prove that f''' has a zero in the interval $(0, 1)$.

for Monday, October 30

1. Spend some time reviewing the portions of Chapter 4 that we have discussed.
2. Turn in solutions for exercises 12 and 13 in Section 4.4.
3. Students will present solutions for the Section 4.4 problem listed by their name: Jonathan (4), Thomas (5), Awa (8), Yidan (11), Olivia (20: the example needs to be unbounded as $x \to \infty$), Eli (21), and Emilia (27).

for Wednesday, November 1

1. We have a test focused on the material that we have covered in Chapters 3 and 4 since the first exam. However, you do need to know some of the basic content from earlier sections of the book. If you have been keeping up (doing the reading, working on the problems, thinking about the concepts and notation), you should have a good start for preparing for the exam. You should be able to state the definitions of the common terms we have been using, state the theorems of key results we have covered, and give examples of functions exhibiting various properties.

for Friday, November 3

1. Read the introduction to Chapter 5 and the beginning of Section 5.1 through the definition of a Riemann sum. This reading should take less than 15 minutes, but it will be helpful for you to be familiar with the terminology used in the study of integration.
2. **IMPORTANT:** Make note of the special assignment due next Wednesday.

for Monday, November 6

1. Read Section 5.1.
2. Do exercises 1, 2, 3, 4, and 9 in Section 5.1. Be prepared to discuss these problems in class.
3. Turn in solutions for exercises 8 and 12 portion in Section 5.1. Give two proofs for exercise 12; one directly from the definition and another using the results of parts (a) and (b) of Theorem 5.5.

for Wednesday, November 8

1. Review Section 5.1 to be certain that you understand the definitions and the notation.
2. Do exercises 15, 16, 18, 19, and 21 in Section 5.1. Think carefully about exercise 15 to find a way to solve this problem with very little need of tagged partitions, etc.; just use previous results. You may use Exercise 20 in the solution for Exercise 19. Be prepared to discuss these problems in class.
3. Turn in solutions for Exercise 4.4.44 (you may use, without proof, the result from Exercise 4.4.43) as well as the two problems listed below. This is the fourth of our no help assignments; see the syllabus for the guidelines on these types of assignments. For the record, following these guidelines falls under the umbrella of the academic dishonesty policy of Whitman College.

i. Suppose that the function $f\colon \mathbb{R} \to \mathbb{R}$ is twice differentiable on $\mathbb{R}$ and has at least five distinct real roots. Prove that the function g defined by $g(x) = f(x) + 2f'(x) + f''(x)$ has at least three distinct real roots. You may use the usual properties of the function e^x (which gives you quite a hint for the solution).

ii. Consider the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, where a and b are positive numbers. Each tangent line to this ellipse in the first quadrant meets the coordinate axes at points P and Q with coordinates $(u, 0)$ and $(0, v)$, respectively. Find the minimum length of the line segment PQ. Try to write your solution as clearly and simply as possible.

for Friday, November 10

1. Read Section 5.2 through the proof of Theorem 5.10; this reading may take an hour or so.
2. Do exercises 2 and 4 in Section 5.2.
3. Turn in a solution for exercise 4 in Section 5.2.
4. **IMPORTANT:** Make note of the special assignment due next Wednesday.

for Monday, November 13

1. Finish reading Section 5.2, recording any questions that arise.
2. Do exercises 11 and 12 in Section 5.2.

for Wednesday, November 15

1. Read Section 5.3 through the proof of the Fundamental Theorem of Calculus.
2. Do exercises 10, 11, 12, 13 (you should NOT use the Fundamental Theorem of Calculus for any of these exercises) 15, and 16 in Section 5.3.
3. This is the fifth of our no help assignments; see the syllabus for the guidelines on these types of assignments. For the record, following these guidelines falls under the umbrella of the academic dishonesty policy of Whitman College. Turn in solutions for exercises 7, 10, and 18 in Section 5.2. You may use Theorem 5.15 for your solution to exercise 18.

for Friday, November 17

1. Finishing reading Section 5.3, recording any questions that arise.
2. Do exercises 17, 18, 20, and 21 in Section 5.3.
3. Students will present solutions for the Section 5.3 problem listed by their name: Eli (26), Jonathan (27a), Yidan (27b), Thomas (29), and Olivia (30).
4. Turn in a solution (without using the FTC) for exercise 10 in Section 5.3.

for Monday, November 27

1. Read Section 5.4 through the proof of Theorem 5.24.
2. Do exercises 4, 6, 7, 9, and 12 in Section 5.4.
3. Turn in a solution for exercise 9 in Section 5.4.

for Wednesday, November 29

1. Do exercises 1, 2, 4, 7, 10, 11, and 12 in Section 5.6.
2. Turn in a solution for exercise 1 in Section 5.6.
3. Students will present solutions for the Section 5.6 problem listed by their name: Yidan (2), Jonathan (4), Eli (10a), Olivia (11), and Thomas (12).

for Friday, December 1

1. We have an exam on this day, focused primarily on the material covered since the last exam, that is, the parts of Chapter 5 on integration that we have discussed. However, it is assumed that you know the key concepts from previous chapters that we have continued to use on occasion. Look over the material we have covered in Chapter 5 and review all of the assigned problems; this should be good preparation for the exam.

for Monday, December 4

1. Read Sections 6.1 and 6.2. This material should look familiar from Calculus II, but you should find that you have a much a deeper understanding of it now.
2. Be prepared to discuss (as in go to the board) exercises 1c, 2a, 5, 12, 13, 14, and 20 in Section 6.1 and exercises 1, 3, 5, and 14ac in Section 6.2.
3. Turn in a solution for exercise 3 in Section 6.5 (not a typo).
4. Our final special assignment is due Friday, December 8; see the assignment for that day.

for Wednesday, December 6

1. Read Section 6.3. You can ignore the lim inf and lim sup symbols for now and treat them as ordinary limits. Since the general ideas should be somewhat familiar to you from calculus, focus on the proofs and concepts rather than just the computational details.

2. Be prepared to discuss exercises 3, 5, 6, and 12cf in Section 6.3.

3. Turn in a solution for exercise 4 in Section 6.3.

for Friday, December 8

1. Our sixth and final special assignment is due on this day; see the syllabus for the guidelines on these types of assignments. For the record, following these guidelines falls under the umbrella of the academic dishonesty policy of Whitman College. Turn in solutions for Exercise 18 in Section 5.6, Exercise 9 in Section 6.3, and Exercise 6a in Section 6.5. Exercise 5.6.18 is not too difficult once you recognize what needs to be done. The key issue is that the points s_i and t_i (which technically also depend on n but the notation then becomes very awkward) are different points (tags) in the subinterval. Exercise 6.3.9 is intended to be a simple computational problem using a result that you can accept as a given. For Exercise 6.5.6a, start by reading (and then modifying) the suggestions for Exercise 6.5.5.

2. We will review for the final exam. Information regarding the final exam, which takes place in our usual classroom from 9:00 to noon on Friday, December 15, is given below. As usual, a good place to start is to spend an hour or two looking over the sections that we have covered and reviewing the exercises that have been assigned.

3. You should be able to give the definition of any concept that we have used regularly during the semester, be able to state any major result we have considered, and be able to generate examples that illustrate these ideas and concepts. You should also be prepared to prove (as in know the basic idea so that you can reconstruct a proof) the following results.

   1. convergent sequences are bounded
   2. algebraic (add, subtract, multiply, divide) properties of sequences
   3. bounded monotone sequences converge
   4. Intermediate Value Theorem
   5. continuity implies uniform continuity on $[a, b]$
   6. differentiability implies continuity
   7. the monotonicity theorem for differentiable functions
   8. the product rule and quotient rule for derivatives
   9. continuous implies Riemann integrable
   10. monotone implies Riemann integrable
   11. Fundamental Theorem of Calculus (both parts)
   12. Comparison Test

4. The final exam will consist of some subset of item 3 (statements, examples, and proofs) along with some new problems that involve the ideas we have considered and the problem solving skills you have acquired.

5. Here are some exercises (some of which were assigned during the semester) that you might want to consider after preparing for the exam. More likely than not, it will not be possible for you to do all of these so start by reading through the exercises and focus on those that seem to be at your level of understanding. The parenthetical label is intended to give you a rough idea about the level of difficulty of the problem, but this sort of label is highly subjective.

1. (easy) Let r be a rational number. Prove that there exists a strictly decreasing sequence of irrational numbers that converges to r.

2. (medium) Exercise 2.4.2.

3. (easy) Exercise 3.1.4c.

4. (medium) Exercise 3.2.4.

5. (medium) Exercise 3.4.10.

6. (medium) Let $f\colon [0,4] \to \mathbb{R}$ be continuous on $[0,4]$ and suppose that $f(0) = f(4)$. Prove that there exists a point $c \in [0,1]$ such that $f(3c+1) = f(c)$.

7. (medium) Exercise 3.6.31.

8. (easy) Exercise 4.2.18.

9. (hard) Exercise 5.6.5. The following remnant of scratchwork may be helpful.

$$M - 2\epsilon < (M-\epsilon)\sqrt[n]{d-c} \le v_n \le M\sqrt[n]{b-a} < M + \epsilon \quad \text{for } n \ge N$$

10. (easy) Exercise 5.6.7

11. (medium) Exercise 5.6.18.

12. (hard) Exercise 6.5.6b.

13. (medium) Exercise 6.5.13.

14. (medium) Exercise 6.5.22.