for Wednesday, September 4

- 1. Read the syllabus for the course (available on the website), then read the following parts of the textbook: Appendix A.1, the preface, and the short introduction to Chapter 1.
- 2. Turn in a carefully written solution to the following problem:

Prove that
$$\sum_{i=1}^{n} (-1)^{i+1} i^2 = \frac{1}{2} (-1)^{n+1} n(n+1)$$
 for each positive integer *n*.

You should use mathematical induction for your proof. I ask that you work alone on this problem (getting no help from any source other than the textbook or your Math 260 notes) so that I can get a sense of your level of preparation for solving problems and writing proofs.

for Friday, September 6

- 1. Read Section 1.1. Except for the last part of this section, the material should read rather quickly. (If you have not yet taken abstract algebra, then you may skim the discussion of fields.) Start now to really learn what it means to read and understand a mathematics textbook as you will be doing quite a bit of this over the next four months.
- 2. Do exercises 7, 10, 12 (use Exercise 11), 14, and 15 in Section 1.1. Many of the solutions to these exercises involve proof by contradiction.
- 3. Turn in a carefully written solution for exercise 15. Be certain to include a (perhaps abbreviated) statement of the exercise and to use words and complete sentences in your proof.

for Monday, September 9

- 1. Read Section 1.2 through the discussion of geometric sums.
- 2. Do exercises 6, 7, 9, 12, and 17 in Section 1.2. Note carefully that Exercise 12 can be used to help solve Exercise 17. The fact that previous results can often be used to simplify the proofs of later results is an important observation; keep this in mind as you do later assignments.
- 3. Turn in a solution for exercise 7; think carefully about how you approach this problem.

for Wednesday, September 11

- 1. Read Section 1.3 through the paragraph following the proof of Theorem 1.18. The Completeness Axiom and the Archimedean Property are extremely important so be certain to ask questions if you find anything confusing about them.
- 2. Do exercises 12, 15, 17, 19, and 20 in Section 1.3.
- 3. Turn in a solution for exercise 15; you may use Theorem 1.17 in your solution.

for Friday, September 13

- Read Section 1.4, focusing primarily on the proof of Theorem 1.29 since it gives a different proof that the set of real numbers is uncountable than the one you have most likely seen. Note the use of the Completeness Axiom. Hopefully, much of the rest of the material in this section is familiar to you from Math 260 or previous math courses.
- 2. Do exercises 4, 9, and 17 in Section 1.4.
- 3. Skim Section 1.5. Much of this material should be familiar to you but be certain that you know the vocabulary. Think carefully about and ponder the implications of the rather bizarre functions we discussed in class. It is important to realize that functions can behave in strange and unexpected ways so definitions of properties (such as continuity) must be stated very carefully.
- 4. Do exercises 23, 24, 25, 27, 34, 35, and 43 in Section 1.5. Be prepared to discuss these exercises in class.

for Monday, September 16

- 1. Read Section 2.1 through the proof of Theorem 2.4. Become very familiar with the adjectives for sequences, being able to state their definitions and giving examples of sequences with or without a given property. This will most likely be your first introduction to proofs that involve "Let $\epsilon > 0$ be given" so study the examples in the textbook and your notes carefully.
- 2. Do exercises 5, 7, 9, 10, 11, and 15 in Section 2.1. For exercises that ask for examples, try to find several different sequences with each property. Be prepared to share your examples in class as well as outline your solutions to the exercises that require proofs.
- 3. Turn in a solution to the following problem: use the definition of convergence to prove that the sequence $\left\{\frac{3n+11}{4n+7}\right\}$ converges. Try to keep your computations as simple as possible.

for Wednesday, September 18

- 1. Finish reading Section 2.1. Be certain to make note of any questions that you have.
- 2. Do exercises 17, 19, 20, 21, 33, and 42 in Section 2.1.
- 3. Turn in solutions for exercises 17 (do not use the Squeeze Theorem) and 20c in Section 2.1.

for Friday, September 20

- 1. Read Section 2.2 through the proof of Theorem 2.13. The two main results presented here are very important so read this material carefully and note any questions you have.
- 2. Do exercises 2, 8, 12, 14, 15, 16, and 20 in Section 2.2.
- 3. Turn in solutions for exercises 2 and 15 in Section 2.2.
- 4. **IMPORTANT:** Make note of the assignment for next Wednesday. You should begin thinking about these problems soon.

for Monday, September 23

- 1. Finish reading Section 2.2.
- 2. Do exercises 21, 24, 31, 32, 36, and 37 in Section 2.2.
- 3. Read Section 2.3 through the statement of the Bolzano-Weierstrass Theorem; we will not consider any of the rest of this section at this time.
- 4. Do exercises 3, 5, and 6 in Section 2.3.
- 5. Turn in solutions for exercises 24 and 37 in Section 2.2.

for Wednesday, September 25

- 1. Review Chapter 2, looking over the main results that we have discussed and the exercises that have been assigned.
- 2. Do exercises 5, 7, 16, 17, and 19 in Section 2.4.
- 3. Turn in solutions for exercises 2, 20, and 25 in Section 2.4; for Exercise 25, you may find the result of Exercise 1.3.19 useful. This is the first of our no help assignments. **IMPORTANT:** Please refer to the syllabus for the guidelines on these types of assignments. For the record, following these guidelines falls under the umbrella of the academic dishonesty policy of Whitman College.

for Friday, September 27

- 1. Read the introduction to Chapter 3 and Section 3.1 through the discussion after the proof of Theorem 3.2.
- 2. Do exercises 1, 2, 4abcf, and 8 in Section 3.1.
- 3. Turn in a solution for exercise 4b.

for Monday, September 30

- 1. Finish reading Section 3.1.
- 2. Do exercises 10, 13, 16, 18, 22, and 34 in Section 3.1.
- 3. Turn in solutions for exercises 22 and 34c.
- 4. We will cover part of Section 3.2 as well as review for the exam.

for Wednesday, October 2

- 1. We have a test on the material we have covered thus far. You need to be familiar with the concepts and theorems that are included in Sections 1.1 to 3.1. You should be able to state definitions of commonly used terms (such as supremum, Cauchy sequence, and limit of a function) and provide examples to illustrate these terms. By flipping through the pages of these sections, you should be able to identify the main results we have considered. You should be able to prove the Archimedean property of the real numbers, the algebraic properties of sequences and limits of functions, that convergent sequences are bounded, and that bounded monotone sequences converge.
- 2. On the test, you may be asked to state some definitions or theorems, give some examples, prove some of the results listed in item (1), solve some familiar problems (very similar to homework problems), and solve one or two problems that you have not seen. The best preparation for the test is to have been keeping up with the material during these first few weeks of the semester; pondering concepts, learning various ways to approach problems, and knowing why certain results are true.

for Friday, October 4

- 1. Read Section 3.2.
- 2. Do exercises 1c, 2, 4, 7, 8, and 10 in Section 3.2.
- 3. Turn in solutions for exercises 7 and 10.

for Monday, October 7

- 1. Read Section 3.3 through the paragraph following Theorem 3.18.
- 2. Do exercises 1 (add $S \neq \emptyset$), 5, 6, 7, 9, 12, 13, and 14 in Section 3.3.
- 3. Turn in solutions for exercises 6 and 9.

for Wednesday, October 9

- 1. Finish reading Section 3.3.
- 2. Do exercises 20, 27, 36, 37, 38, 39, and 44b in Section 3.3.
- 3. Turn in solutions for exercise 38 and the portion of the solution for exercise 44b that is omitted below.

Let $S = \{x \in [a, b] : f \text{ is bounded on } [a, x]\}$. Since f is locally bounded at a, there exist positive numbers M_a and $\delta_a < b - a$ such that $|f(x)| \le M_a$ for all $x \in [a, a + \delta_a)$. It follows that $[a, a + \delta_a) \subseteq S$. Since the set S is nonempty and bounded above, the Completeness Axiom asserts that S has a supremum, call it β .

Suppose that $\beta \notin S$. (obtain a contradiction)

Now suppose that $\beta < b$. Then there exist positive numbers M_2 and δ_2 such that $|f(x)| \leq M_2$ for all $x \in [\beta, \beta + \delta_2]$. Since f is bounded on both of the intervals $[a, \beta]$ and $[\beta, \beta + \delta_2]$, we find that $\beta + \delta_2 \in S$, a contradiction to the fact that $\beta = \sup S$. It follows that $\beta = b$. We conclude that $b \in S$, that is, the function f is bounded on [a, b].

- 4. Students will present solutions at the board for the Section 3.3 problem listed by their name: Warren (7), Jenner (36), Sam (1), Eli (27), Jacob (12), Peter (17), Ronnie (11), Uli (5), Chris (30), and Nikita (39).
- 5. There is a special assignment due next Wednesday (10/16); see the assignment for that date.

for Friday, October 11

1. No class today due to the October Break.

for Monday, October 14

- 1. Read Section 3.4; pay careful attention to the proofs that appear here.
- 2. Do exercises 1, 2, 3, 4, and 5 in Section 3.4.
- 3. Turn in a solution for exercise 5; do not use the fact that Cauchy sequences converge.

for Wednesday, October 16

- 1. Read Section 3.5 through page 117; we will not be discussing any more of this section. However, if you are curious, then you should read the rest of the section to get an introduction to the concept of bounded variation. This idea, along with that of absolute continuity (see the last set of exercises in Section 3.6), is extremely important in higher level analysis courses.
- 2. Do exercises 7, 8, and 9 in Section 3.5. Be prepared to present solutions to the class for the requested functions in Exercise 9.
- 3. Turn in solutions for exercises 3.2.31, 3.3.43, and 3.6.29. This is the second of our no help assignments; see the syllabus for the guidelines on these types of assignments. For the record, following these guidelines falls under the umbrella of the academic dishonesty policy of Whitman College.
- 4. There is a special assignment due next Wednesday (10/23); see the assignment for that date.

for Friday, October 18

- 1. Review Chapter 3, making sure you know the definitions of key concepts and the statements of important theorems, along with the basic ideas behind their proofs.
- 2. Do exercises 1, 4, 8, and 34–38 in Section 3.6. Be prepared to discuss these exercises in class.
- 3. Turn in a solution for Exercise 3.6.10.

for Monday, October 21

- 1. Read the introduction to Chapter 4 and Section 4.1.
- 2. Do exercises 1, 2, 4, 5, 6, 7, 8, 9, and 13 in Section 4.1. Note that many of these exercises are Calculus I problems so you should be able to do them easily at this point in your mathematical career.
- 3. Students will present solutions at the board for the Section 4.1 problem listed by their name: Warren (6c), Jenner (6a), Sam (13), Eli (1c), Jacob (5), Peter (6b), Ronnie (1a), Uli (2), Chris (4), and Nikita (7). Try to gear your presentations to students taking calculus.

for Wednesday, October 23

- 1. Do exercises 12, 18, 19, and 31 in Section 4.1.
- Turn in solutions for exercises 3.4.10, 3.6.22 (change the last R to [0,∞)), and 3.6.31. This is the third of our no help assignments; see the syllabus for the guidelines on these types of assignments. For the record, following these guidelines falls under the umbrella of the academic dishonesty policy of Whitman College.
 Important Note: For this problem set, I request that all of the solutions be formatted using IAT_EX.

for Friday, October 25

- 1. Read Section 4.2 up to the paragraph prior to Theorem 4.13.
- 2. Do exercises 8, 11, 13, 14, 15, and 18 in Section 4.2. Be prepared to discuss these problems in class.
- 3. Turn in a solution for exercise 18 in Section 4.2. Your proof should be very short and involve referencing a problem that you have already solved in a previous assignment.

for Monday, October 28

- 1. Finish reading Section 4.2.
- Students will present solutions at the board for the Section 4.2 problem listed by their name: Warren (37), Jenner (7), Sam (27), Eli (38b), Jacob (21), Peter (28), Ronnie (39), Uli (38a), Chris (3c), and Nikita (32). You should also spend some time looking over the exercises that you are not presenting.
- 3. Turn in a solution for exercise 20 in Section 4.2.
- 4. IMPORTANT: Make note of the special assignment due next Monday; see that date for the problems.

for Wednesday, October 30

- 1. Read Section 4.3 through the proof of Theorem 4.24.
- 2. Do exercises 4, 7, 11, 12, and 13 in Section 4.3. Be prepared to discuss these exercises in class.
- 3. Turn in a solution for the following problem:

Suppose that f is a nonnegative function defined on (0,1) and that f''' exists on (0,1). Suppose further that f has two zeros in (0,1). Prove that f''' has a zero in the interval (0,1).

for Friday, November 1

- 1. Spend some time reviewing the portions of Chapter 4 that we have discussed.
- 2. Turn in solutions for exercises 12 and 13 in Section 4.4.
- 3. Students will present solutions at the board for the Section 4.4 problem listed by their name: Warren (20, the example function should be unbounded as x → ∞), Jenner (27), Sam (21), Eli (4), Jacob (7), Peter (22), Ronnie (11), Uli (3), Chris (8), and Nikita (5). You should also spend some time looking over the exercises that you are not presenting.

for Monday, November 4

- 1. Spend some time reviewing Chapters 3 and 4, looking over the results and problems that we have covered.
- 2. Turn in solutions for Exercise 4.4.44 (you may use, without proof, the result from Exercise 4.4.43) as well as the two problems listed below. This is the fourth of our no help assignments; see the syllabus for the guidelines on these types of assignments. For the record, following these guidelines falls under the umbrella of the academic dishonesty policy of Whitman College.
- i. Suppose that the function $f: \mathbb{R} \to \mathbb{R}$ is twice differentiable on \mathbb{R} and has at least five distinct real roots. Prove that the function g defined by g(x) = f(x) - 2f'(x) + f''(x) has at least three distinct real roots. You may use the usual properties of the function e^x (which gives you quite a hint for the solution).
- ii. Consider the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a and b are positive numbers. Each tangent line to this ellipse in the first quadrant meets the coordinate axes at points P and Q with coordinates (u, 0) and (0, v), respectively. Find the minimum length of the line segment PQ. Try to write your solution as clearly and simply as possible.
- 3. We will spend some time reviewing for the Wednesday exam.

for Wednesday, November 6

1. We have a test focused on the material that we have covered in Chapters 3 and 4 since the first exam. However, you do need to know some of the basic content from earlier sections of the book. If you have been keeping up (doing the reading, working on the problems, thinking about the concepts and notation), you should have a good start for preparing for the exam. You should be able to state the definitions of the common terms we have been using, state the theorems of key results we have covered, give examples of functions exhibiting various properties, and solve problems involving the concepts we have discussed of late.

for Friday, November 8

- 1. Read the introduction to Chapter 5 and the beginning of Section 5.1 through the definition of a Riemann sum. This reading should take less than 15 minutes, but it will be helpful for you to be familiar with the terminology used in the study of integration.
- 2. Look over exercises 1, 2, 3, and 4 in Section 5.1. Be prepared to discuss these problems in class.

for Monday, November 11

- 1. Read Section 5.1.
- 2. Do exercises 8, 9, 12, 15, 16, 18, 19, and 21 in Section 5.1. Think carefully about exercise 15 to find a way to solve this problem with very little need of tagged partitions, etc.; just use previous results. You may use Exercise 20 in the solution for Exercise 19.
- 3. Turn in solutions for exercises 8 and 12 in Section 5.1. Give two proofs for exercise 12; one directly from the definition and another using the results of parts (a) and (b) of Theorem 5.5.

for Wednesday, November 13

- 1. Read Section 5.2 through the proof of Theorem 5.10; this reading may take an hour or so.
- 2. Do exercises 2 and 4 in Section 5.2.
- 3. Turn in a solution for exercise 4 in Section 5.2.
- 4. IMPORTANT: Make note of the special assignment due next Wednesday.

for Friday, November 15

- 1. Finish reading Section 5.2, recording any questions that arise.
- 2. Do exercises 11 and 12 in Section 5.2; I will not be collecting either of these solutions.

for Monday, November 18

- 1. Read Section 5.3 through the proof of the Fundamental Theorem of Calculus.
- 2. Do exercises 10, 11, 12, 13 (you should NOT use the Fundamental Theorem of Calculus for any of these exercises) 15, and 16 in Section 5.3.
- 3. Turn in a solution for exercise 12 in Section 5.3.
- 4. Students will present solutions for the Section 5.3 problem listed by their name: Warren (29), Jenner (20), Sam (30), Eli (27b), Jacob (16), Peter (26), Ronnie (15), Uli (27a), Chris (21), and Nikita (18). You should also spend some time looking over the exercises that you are not presenting.

for Wednesday, November 20

- 1. Finishing reading Section 5.3, recording any questions that arise.
- 2. Do exercises 17, 18, 20, and 21 in Section 5.3.
- 3. Turn in solutions for exercises 7, 10, and 18 in Section 5.2. You may use Theorem 5.15 for your solution to exercise 18. This is the fifth of our no help assignments; see the syllabus for the guidelines on these types of assignments. For the record, following these guidelines falls under the umbrella of the academic dishonesty policy of Whitman College. **Important Note:** For this problem set, I request that all of the solutions be formatted using LAT_EX.

for Friday, November 22

- 1. Read Section 5.4 through the proof of Theorem 5.24.
- 2. Do exercises 4, 6, 7, 9, and 12 in Section 5.4.
- 3. Turn in a solution for exercise 9 in Section 5.4.

Enjoy the Thanksgiving break.

for Monday, December 2

- 1. Do exercises 1, 2, 4, 7, 10, 11, and 12 in Section 5.6.
- 2. Turn in a solution for exercise 1 in Section 5.6.
- 3. **IMPORTANT:** Make note of the special assignment due next Monday.

for Wednesday, December 4

- 1. Read Sections 6.1 and 6.2. This material should look familiar from Calculus II, but you should develop a much a deeper understanding of it now.
- 2. Work on exercises 1c, 2a, 5, 12, 13, 14, and 20 in Section 6.1 and exercises 1, 3, 5, and 14ac in Section 6.2.
- 3. Turn in a solution for exercise 3 in Section 6.5, yes, I do mean Section 6.5.
- Students will present solutions for the problem listed (in the form Section, Exercise) by their name: Warren (6.2, 5), Jenner (6.2, 19), Sam (6.1, 2b), Eli (6.2, 14b), Jacob (6.1, 5), Peter (6.1, 20), Ronnie (6.2, 14e), Uli (6.2,3), Chris (6.2, 17), and Nikita (6.1, 13).

for Friday, December 6

- 1. Read Section 6.3. You can ignore the lim inf and lim sup symbols for now and treat them as ordinary limits. Since the general ideas should be somewhat familiar to you from calculus, focus on the proofs and concepts rather than just the computational details.
- 2. Work on exercises 3, 5, 6, 9, and 12cf in Section 6.3.
- 3. Turn in a solution for exercise 4 in Section 6.3.

for Monday, December 9

- 1. Review the material that we have covered in Chapters 5 and 6. Record any questions you have on proofs in the textbook or on exercises that have been assigned thus far. We will discuss these questions and related ideas in preparation for the test on Wednesday.
- 2. Our sixth and final special assignment is due on this day; see the syllabus for the guidelines on these types of assignments. For the record, following these guidelines falls under the umbrella of the academic dishonesty policy of Whitman College. Turn in solutions for Exercise 18 in Section 5.6, Exercise 17 in Section 6.1, and Exercise 6a in Section 6.5. Exercise 5.6.18 is not too difficult once you recognize what needs to be done. The key issue is that the points s_i and t_i (which technically also depend on n but the notation then becomes very awkward) are different points (tags) in the subinterval. For Exercise 6.1.17, I request that you first find a concise formula for the sequence of partial sums. For Exercise 6.5.6a, start by reading (and then modifying) the suggestions for Exercise 6.5.5. Important Note: For this problem set, I request that all of the solutions be formatted using IAT_EX.

for Wednesday, December 11

1. We have an exam on this day, focused primarily on the material covered since the last exam. However, it is assumed that you know the key concepts from previous chapters that we have continued to use on occasion.

for Friday, December 13

1. We will review for the final exam, scheduled for next Friday. Information to follow.