Math 260 Assignments for Spring 2012

for Tuesday, January 17

1. Read the syllabus carefully.
2. Read Section 1.1. Although some of the terms here may be new to you, they are not difficult and we will use them quite a bit in the next few weeks. There is no need to memorize many of these terms (especially those in Theorem 1.3) but you certainly need to be familiar with the connectives, truth tables, and the language associated with conditional statements (such as converse and contrapositive).
3. This is the first of many reading assignments. You need to learn how to read a mathematics textbook (or a journal article); this is an important aspect of being prepared for higher mathematics. This type of reading is an acquired skill and takes plenty of patience to learn. You cannot skim, you cannot gloss over details, you may need to pause to think or write, you may need to read a sentence or paragraph multiple times before it is clear, or you may get completely stumped. If you do get stuck at some point, do not just quit reading; put a question in the margin and keep going, trying to make as much sense as you can of what follows. Sometimes your questions will be answered later (either in the text or with a flash of insight). However, if they are not, it is important that you ask for help. Also, you need to be certain that you are not fooling yourself that you actually understand when you really don't.
4. Do problems 1a, 1d, 2h, 2k (the letters on problem 2 refer to the parts of Theorem 1.3), 3, and 4 in Section 1.1. We should have time to discuss these in class if necessary.
5. Turn in solutions for problems 4a, 4b, and 4c from Section 1.1. (Note that the instructions to consider different universes is relevant for problem 4a.) Write your solutions very carefully and provide reasons for your answers of true or false. Your solution should include the original statement, along with the converse and the contrapositive, all written clearly. This is your first opportunity to show me the quality of your work so it would be in your best interest to get off to a good start. The written assignment is due at the beginning of class on Tuesday.

for Thursday, January 19

1. Read Section 1.2. Remember to read carefully and to make certain you understand each and every statement. For this section, you should probably pause to reflect at times and create further examples. If you have any questions, write them down and ask them in class or during office hours.
2. You should work all of the problems in this section; each problem should only take a few minutes.
3. Turn in solutions to problems 2, 5, and 6d from Section 1.2. For problems 2 and 5, you should include the verbal statement along with the symbolic statement. This means that you need to copy some version of the problem before writing the solution. For problem 6d, you will need a complete sentence or two, along with an algebraic equation, for your explanation. Once again, think about how you are writing and presenting mathematics.

for Friday, January 20

1. Read Section 1.3 carefully and ponder the implications of quantifiers and their negations. Also read the biography of De Morgan; you should be "nerdy" enough (in a good way :)) to determine the year in which he was born. Do you have friends or relatives who can make a similar claim?
2. Be prepared to discuss all the problems in Section 1.3. This means that you should do the problems and be able to explain your solution to someone in the class, possibly at the board.
3. Turn in solutions for problems 7b, 7c, and 7e from Section 1.3. You do not need to use symbols and quantifiers to negate these definitions but use them if you find them helpful. Really work on the wording of the negations to make sure they are clear and be certain that only simple sentences are negated. Since the terms in 7c are probably unfamiliar, you do not need to give any examples, but you should provide examples for the other two definitions. This means to give an example that illustrates the definition and one that illustrates the negation of the definition. The purpose of these definitions is not so much that you grasp them fully but that you are exposed to some common terms from higher mathematics and that you can work with them symbolically even if you are unclear as to their meaning.

for Tuesday, January 24

1. Read Section 1.4. You may need to read very slowly in a few places and think about the order of the quantifiers.
2. Be prepared to discuss problems 1, 2, and 3 from Section 1.4. Make sure you have clear reasons to support your answers for these problems.
3. Turn in solutions for problems 4a, 4b, and 4d from Section 1.4. As before, word your negated definitions carefully. You do not need to provide examples this time. Rather use your negated definitions to "prove" each of the following:

   4a: the set $[0, 1)$ is not open

   4b: the function f defined by $f(x) = \sin(1/x)$ for $x \neq 0$ and $f(0) = 0$ is not continuous at 0

   4d: the sequence $\{(-1)^n\}$ does not converge to 1

for Thursday, January 26

1. Read Section 1.5. Since sets appear in almost all areas of mathematics, it is important to acquire a firm understanding of them and their associated notation.
2. Ponder the problems in Section 1.5. Most of these should be go quickly.
3. Turn in solutions to problems 1f, 2b, and 5. Your proof for 5 should look something like

$$
\begin{aligned}
(A \setminus B) \cup (B \setminus A) &= (A \cap B^c) \cup (B \cap A^c) && \text{(Problem 3)} \\
&= && \text{(Theorem 1.10e)} \\
&\ \ \vdots && \text{(3 steps, give reasons)} \\
&= && \text{(De Morgan's Laws)} \\
&= (A \cup B) \setminus (A \cap B) && \text{(Problem 3)}
\end{aligned}
$$

for Friday, January 27

1. Read Section 1.6. The concepts of pairwise disjoint, partition, and power set are important.
2. Be prepared to discuss the problems in Section 1.6.
3. Turn in solutions for problems 2 and 6 in Section 1.6. Problem 6 requires a proof—three proofs actually—so give it your full attention. To prove that the collection is a partition, you need to prove that two sets are equal. This involves two "chasing points" proofs. Then you need to show that the sets are pairwise disjoint and this also requires proof, hopefully one that involves properties of sets. This result is not very deep (in fact, it is quite easy) but the point is to write a careful proof, not just believe that the result is true or obvious. If you cannot write proofs for simple statements, then you will have a great deal of trouble writing proofs for more complicated statements.

for Tuesday, January 31

1. Read Section 1.7 and work to understand this new concept. This is the first section in which proofs of results begin to appear so study them carefully.
2. Be prepared to discuss problems 1–5 from Section 1.7; this means that I may call on you to tell me how you solved the problem. Even if a result seems "obvious", make certain you can write out the details to show how it works.
3. Turn in solutions to problems 6 and 8 from Section 1.7 and the extra problem added below. Be certain to include all of the necessary details. For problem 6, you need to think carefully about the meaning of equivalence classes and use proper set notation. For problem 8, be extremely thoughtful so that fractions and division do not appear in your solution; there are easy ways to prove the transitive property that do not involve fractions. For the third problem, let A be the set of all continuous functions defined on $\mathbb{R}$ and define $\sim$ on A by $f \sim g$ if $f - g$ is differentiable at 0. Prove that $\sim$ is an equivalence relation on A. Find two elements that belong to the set $[|x|]$.

At this stage of the course, you might consider reading the syllabus again to remind yourself that frustration is not unusual at the beginning of this course. You are being immersed in a new culture and it takes time to overcome the initial shock. You have to trust me (and many other people in the math and sciences) that the world you are being introduced to is worth entering. As an analogy, how would you respond to a 13 year-old kid taking algebra who asked you what the point was of using letters to represent numbers and why anyone would ever need to solve an equation? Granted, it is true that for many fascinating areas of life, it is not necessary but if you are going on in a number of other fields, it is important. Is it possible to explain to this middle school kid how interesting a study of trigonometry and calculus can be? (Note that you could not even tell them what differential equations are, another common course in undergraduate mathematics.) How would they even be able to understand the terms you are using? With regards to higher mathematics, you are in a similar situation as the algebra student is with calculus.

The syllabus contains some guidelines for how to approach this material. You need to read and reread the text, you need to close your eyes and ponder the concepts, you need to do problems (this includes doing problems over again several days later if you were stuck on them the first time and then got some help—see if you can recreate the solution), and you need to keep working even when it feels like you are beating your head against a wall. The payoff is huge down the road but only if you invest the time and energy now.

for Thursday, February 2

1. Read the introduction to Chapter 2 and Section 2.1. Since we are beginning proofs in earnest now, read very carefully and make note of any questions or confusions that arise.
2. Be prepared to discuss problems 1–11 from Section 2.1.
3. Turn in solutions for problems 4, 7, and 11 from Section 2.1. For problem 4, write your solution in the style of the example done in class or those in the text. Be very clear (and patient) and include every step. For problem 7, you can closely imitate the examples in the text; do include the parenthetical remarks this one time. You may find problem 11 a bit of a challenge since you will need to use other results proved in this section or given in the earlier exercises; when you use these results, just refer to them by theorem number or exercise number. It is probably a good idea to do some of the other proofs first before tackling this one. By the way, you will have to rewrite your proof for this problem at least once if you want it to read well. Remember that for all of these turned-in problems, you should start by copying the problem (or some variation) before you give your solution.

for Friday, February 3

1. Read Section 2.2 carefully. Although $a|b$ is a simple notion and one that is familiar to you, it is very important that you learn how to work with this concept at this level. By the way, definitions are like formulas in the sense that you need to know them inside and out. Just as you can immediately tell someone the derivative of x^2, you should be able to readily quote (essentially verbatim) the definition for $a|b$.
2. Be prepared to discuss the problems in Section 2.2. For problem 2, use the definition of $n|(2n+3)$ and then some factoring to learn something about n. The proofs requested in problem 3 should follow easily from the definitions. For problem 4, consider using part (f) of Theorem 2.7, which, as I mentioned in class, is a very powerful result. Finding a has nothing to do with the proof; it is just an interesting aspect of the numbers. For problem 5, remember to avoid fractions. It is also preferred that you avoid "cancelling"; use the zero product property of the integers explicitly. For problem 8b, use the standard method for proving that two sets are equal.
3. Turn in solutions for problems 3d (the intent should be clear), 4 (note that this is a proof even though the answer is a number), and 5 (this is a biconditional so two proofs are needed; note also that it is implicitly assumed that n is nonzero).

for Tuesday, February 7

1. Read Section 2.3. This should be an easier section since it is about finding examples rather than writing proofs. However, it is important to note that it can be difficult to find examples in certain situations.
2. Be prepared to discuss all of the problems from Section 2.3. I may be asking people to go to the board to present their solutions. Read the heading of the exercises before proceeding. I will not be collecting any of these problems.
3. You should take some time to review the topics we have covered thus far in preparation for the test on Friday.

for Thursday, February 9

1. Go over all of the topics and problems that have been discussed or assigned thus far and come in with any remaining questions that you have. After our review, we will start discussing mathematical induction (which will not be on the test).

for Friday, February 10

1. We have a test covering Chapter 1 and Sections 2.1 through 2.3.

for Tuesday, February 14

1. Read Section 2.4. The PMI is extremely important and, once you grasp the essential idea and learn how to write it well, is a powerful method for proving certain types of results. However, this technique of proof is notorious for causing confusion for many people so study it carefully.
2. Do as many of the problems in Section 2.4 as you can. Once you get the basic format down, the inductive step should become your main focus. To help with the proper wording of these proofs, see the induction link on the website. It presents the proofs that we did in class, along with three poorly written proofs of the same result, all taken from common student errors. If you do not spot the errors, please let me know as it is important that you become critical readers of proofs. You should probably try problem 5 on your own (after studying Theorem 2.13) before reading the solutions online. For the record, it is possible to prove most of the results in this section without induction (essentially by appealing to other results that are already known and require induction to prove). If you have time, it is good practice to ponder these other solution methods.
3. Turn in three different induction proofs for the following result, using the three examples on the last page of the induction link as a model. That is, give an S proof, a P_n proof, and an informal proof.

   For each positive integer n, the number $2^{5n-4} + 5^{2n-1}$ is a multiple of 7.

for Thursday, February 16

1. Read Section 2.5. It is important that you know what each of these theorems says AND that you truly understand the proofs. Recall that the purpose of this course is to prepare you for higher mathematics and the ability to read and follow proofs is one of the skills necessary for success in upper level mathematics classes. Do not "fake" yourself into believing you understand a proof when you really don't; ask questions about anything that seems unclear. If one step does not make sense, do not give up on the proof. Just believe the result that you do not understand and see if the rest of the proof makes sense from that point on.
2. Be prepared to discuss problems 1, 2, 4, 7, 9, and 11 from Section 2.5.
3. Turn in solutions for problem 11 in Section 2.4 and problem 10 in Section 2.5; for the latter, you need to use the AM/GM inequality.

for Friday, February 17

1. Read Section 2.6. Be certain you understand the distinction between the two forms of induction and ALWAYS use the form that is most appropriate for the given situation. The FTA is very important so make sure you know what it says and how to prove it. The boring form of induction is certainly boring but it is important to know how it works and to notice how often it implicitly appears.
2. Work on the problems in Section 2.6; the more the better but start with problems 1, 3, and the first few parts of 5. You should be focusing on the induction step as the basic idea behind induction proofs should be clear by now. For problem 3, you might want to write out the cases $n = 8$ to $n = 16$ to see what is going on and why strong induction is needed.
3. Turn in a solution for problem 5d from Section 2.6. Be very careful with the equation found in this problem. In particular, note that the "long" sum always has an odd number of terms and always ends with a Fibonacci number with an even subscript. Checking the cases $n = 1$, $n = 2$, and $n = 3$ would be a good idea. Note that strong induction is not needed here.

for Tuesday, February 21

1. Read Section 2.7. I suggest spending about 45 minutes on this, learning as much as you can in that amount of time. If something needs to receive less attention, let it be the proof of Theorem 2.25. The Division Algorithm is rather important so read the proof carefully and fill in the missing details; this includes solving problem 3 at the end of the section.
2. Work on problems 2, 3, 4, 5, 6, 8, 9, and 11 in Section 2.7. You have an example to imitate for problem 2. For problem 3, follow the directions and use the first part of the proof (note that $-a$ is positive if a is negative). Do problem 4 without a calculator. Problems 5 and 6 seem trivial but do write out the proofs carefully. For the uniqueness part, start with "Suppose that y and z both satisfy the equation" then prove that $y = z$. Problem 8 comes with directions as well; follow them. Rather than doing random guessing for problem 9, think about a patterned approach. For problem 11, assume there are two such points and use a standard calculus result to obtain a contradiction.
3. Turn in solutions for problems 6 and 11 in Section 2.7.
4. For practice making conjectures, look for a formula for $\sum_{i=1}^{n} f_{4i}$. Can you prove it?

for Thursday, February 23

1. Read Section 2.8. Indirect proof is a good way to go in some cases, but try not to overdo it. You should know the proof (as in be able to write it out on an exam) that there are an infinite number of primes.
2. Be prepared to discuss the problems in Section 2.8. I have not been doing well at getting students to the board this semester so we will begin to remedy that this class period. When presenting a solution at the board, you need to clearly state the problem and provide sufficient details for people to follow your reasoning. You do not need to write out a proof like a homework assignment; abbreviations and the like are fine. If you can proceed without notes, that is preferable. Try to keep your presentations short and to the point. For this day, I ask the following students to present the corresponding problem listed after their name: Michael (problem 1), Luke (problem 2 directly), Jon (problem 2 indirectly), and Sophie (problem 3). These should only take 3 to 5 minutes each and we will do them in order at the beginning of class. For those of you in the audience, pay attention and ask questions on anything that is not clear. Also make note of what aspects of the presentation you like (and try to emulate these when it is your turn) and do not like (avoid making the same errors).
3. Turn in solutions for problems 6 and 9 in Section 2.8.

for Friday, February 24

1. Look back over Chapter 2 and come in with any remaining questions you have about the reading or exercises.
2. You have a special assignment due at the beginning of class.

for Tuesday, February 28

1. Read Section 3.1. Congruence is an elementary concept once you become familiar with it, but you do need to think carefully about its meaning and properties.
2. Be prepared to discuss the problems in Section 3.1. You will find that some of these are quite easy while others take a bit more time. For problems 4, 5, and 6, be certain to use congruence properties even though there are other ways to proceed. For student solutions at the board, we will have Abby (problem 2, the (5) part), Jamie (problem 6), Geneva (problem 7, the $Q(101)$ part), Paivand (problem 8d), and Sara (problem 8e).
3. Turn in solutions for problems 2 (a proof of part (7) only), 9, and 10. Write your solutions carefully even for problems that involve computations. Your solution to problem 2 should not be very involved; if it is, look for a better way. For problem 9, note that x must have the form $17k + 4$ to solve the first congruence. Now substitute this into the second congruence and determine k. Problem 10 is an if and only if statement so two proofs are required. Do avoid division here.

for Thursday, March 1

1. Read Section 3.2. Since we are beginning a study of an abstract space, you really need to work hard to understand what is going on.
2. Be prepared to discuss the problems in Section 3.2. For student solutions at the board, we will have Tiffani (parts d, e, and f of problem 2, giving two different approaches for each part), Paul (problem 5, without reproducing the entire $\mathbb{Z}_6$ table), and Josh (problem 7, there is more going on here than might first appear).
3. Turn in solutions for problems 3e (that is, give a proof of part (e) of the theorem) and 8 from Section 3.2. You need to be extremely careful on these proofs. Pretend that you are trying to defuse a bomb and a wrong move will blow up the classroom. Make certain that every single step has a reason, one that you can verify from the text, not just because "it has to be true." For 3e, you should start with the left-hand side of the equality and perform five (5) steps, each one with a clear reason, the last one being the right-hand side of the equality. You can line up the equations and provide the reason off to the side. You also need to be careful as you write the solution to problem 8; avoid division and numbers that may not be integers. After collecting some data, you will most likely discover that you need to consider two cases.

for Friday, March 2

1. Read Section 3.3 carefully. Computationally, the Euclidean algorithm is quite elementary. However, the results given in Theorems 3.11 and 3.13 are extremely (as in EXTREMELY) important. Keep track of any questions that arise during the reading, especially any that occur while reading the second proof of Theorem 3.11.
2. Be prepared to discuss the problems in Section 3.3. Try doing the computations for 1 as efficiently as possible, preferably without technology. You should find that problems 2 through 9 go pretty quickly. If they do not, ask about them in class and hopefully someone other than me can explain their solutions; learning to discuss your ideas in a group setting is a useful skill to acquire.
3. Turn in solutions for problems 10 and 11 from Section 3.3. Problem 10 is a biconditional so two proofs are required. Both proofs should be of the "follow your nose" variety if you take advantage of results in this section. Problem 11 is also a biconditional. Half of the proof is elementary whereas the other half only becomes elementary after making a "convenient" observation.

for Tuesday, March 6

1. Read Section 3.4 carefully.
2. Be prepared to discuss (as in be willing to go to the board) the problems in Section 3.4.
3. Turn in solutions for problems 4, 6e, and 10 in Section 3.4.

for Thursday, March 8

1. This will be a review day for the test on Friday (3/9). Look over Chapter 2 and Sections 3.1 through 3.4. I will say more about the test and what you should know soon.

for Friday, March 9

1. As has been mentioned, we have an exam on Chapter 2 and Sections 3.1 through 3.4. You need to be familiar with (and be able to use) the definitions and proof techniques we have discussed thus far, you need to understand the concepts that have been introduced, and you need to be able to solve problems similar to those assigned in the exercises. In addition, you need to know (as in present them if requested) the proofs of the following theorems:

   a. part (f) of Theorem 2.7 (surprise, surprise)
   b. Theorem 2.22 (the Fundamental Theorem of Arithmetic)
   c. Theorem 2.27 (the Division Algorithm, the $a > b > 1$ case only)
   d. Theorem 2.33 (there are an infinite number of primes)
   e. Theorem 3.2 (congruences and remainders)
   f. part (7) of Theorem 3.3 (congruences and products)
   g. part (f) of Theorem 3.10 (gcd and congruences)
   h. Theorem 3.11 (the second version of the proof about gcd and linear combinations)
   i. Theorem 3.14 (relatively prime numbers and the concept of divides)
   j. Theorem 3.19 (relatively prime and existence of inverses)

   This is not intended to be an exercise in memorization; if you interpret it in that way you are missing the point. For each of these results, you should know the basic idea behind the proof and use this knowledge (along with your improving ability to write mathematics) to write out the details of the proof.

   So reread the sections, thinking about examples and concepts, look over the exercises, trying to solve them without looking at your notes, and practice some congruence computations to brush up on your arithmetic skills. Bring any questions you have to class on the day of the review or stop by office hours any day before the exam.

for Tuesday, March 27

1. Spend some time reviewing the portions of Chapter 3 we have discussed thus far to make sure it does not fade from your memory.
2. Go to the website

   http://www.ams.org/mathscinet/msc/msc2010.html

   and spend some time navigating around the subject classification. By clicking on the box on the left, you can see the various subject headings and then continue going into more depth by clicking on a given two digit number. Alternatively, you can load the entire file in PDF form and look through it; be aware that it is a rather large file. The purpose for having you spend some time looking over these areas is to give you a sense of the scope of mathematics.

   Next, you can try the link

   http://www.whitman.edu/penrose/

   Click on E-Resources via Expert Guides (under the Quick Links on the right)

   Click on Mathematics

   Click on MathSciNet

   Try typing in the name of a person in the mathematics faculty to see what we have published of late. For example, you can type 'Gordon, R*' in the author box and see what comes up. You can also look for various topics. You can type 'Euclidean algorithm' in the review text box to see a list of papers that make use of this term. This is a good resource when you are looking for reviews of papers published in the last 50 years or so.

   Finally, go to the site

   http://www.ams.org/mathweb/mi-journals5.html

   and scroll down the list of journals, just skimming the titles. If you become intrigued by a title, click on the link and check out the table of contents of a recent issue of the journal.
3. When you are finished with item (2) (I am assuming that you spend at least 30 minutes doing this but you do not need to do much more unless you become curious), write two or more paragraphs (take the writing seriously) on your impressions (personal or otherwise) concerning mathematics after surfing these sites and developing a sense of the scope of the field of mathematics.

for Thursday, March 29

1. Read Section 3.5. Since we discussed much of this section in class, the reading should not take too long.
2. Be prepared to discuss the problems in Section 3.5. Do not neglect the computational problems and think carefully about efficient ways to solve them. Take advantage of previous results when solving problem 8; refer to exercises in this section and in previous sections.
3. Turn in solutions for problems 6 and 10b from Section 3.5. For problem 6, you should use Theorem 3.25 for the first part of the problem and Theorem 3.29 for the second part. For problem 10b, you may (and should) use the result in part (a).
4. Spend 10 to 15 minutes previewing Section 3.6.

for Friday, March 30

1. Read Section 3.6. Be certain that you understand the proof of the FTA and the results that lie behind it. Read Theorems 3.31 through 3.33 enough times so that the notation does not disguise the easy ideas that they express.
2. Look over the problems in Section 3.6; hopefully you can solve some of them rather quickly. Since many of these problems are computational in nature, be prepared to go to the board and present a quick solution for the problem listed next to your name. Geneva (1a, with an explanation as to how to proceed in an organized way), Sophie (1b, with a focus on the question), Paivand (2, with a careful writing of the problem), Jamie (3, note that a proof is required), Josh (5, with a focus on the key observation), Abby (7, each of the parts should go very quickly), Luke (8, there are several proof options but focus on the FTA approach), Sara (9, clearly identify a patterned way to do this), Paul (10–11, recall our notation for products), Jon (12, do prove your conjecture), Tiffani (13, discuss your thought process), and Michael (16a, just the proof that $m|x$ implies $[x]$ is in the radical). Gear your presentation for two minutes; a couple of the problems might take a bit longer. We will do the problems in order and move along as quickly as we can. Do focus on clarity and efficiency in your solutions.
3. Turn in solutions to problems 14b and the following modification of problem 15: find the exponent on 11 in the canonical factorization of 2012! (note the factorial) as a product of primes. For problem 14b, you need to express your conjecture clearly and then prove it carefully. You need to find a condition xxxx so that the following result holds:

   The positive integer n satisfies xxxx if and only if for each integer a, $n|a^2$ implies $n|a$.

   This is a biconditional so two proofs are needed and one of them might best be done using contraposition. For problem 15, don't just give a numerical answer; explain how you arrived at your answer. A general method for this sort of problem would be nice; see if you can formulate one.

for Tuesday, April 3

1. Read Section 3.7. Expect to spend at least an hour reading through the details of the proofs and results in this section. Keep track of any questions that you have.
2. Since there are many problems in this section, focus on problems 1–3, 5–7, 10–12, and 14–16. You will notice that many of these are computational in nature; try to be as efficient as possible, taking advantage of modular arithmetic and results in this section.
3. Turn in solutions for problems 7 and 14 in Section 3.7. For problem 7, you simply need to give an example and show that it works. However, you must provide an example for each integer $e > 2$; you cannot simply give an example for $e = 3$ or $e = 4$ and be done. You want a formula (in terms of e) that gives an example for each such e. Examples can point the way, then write down your candidate AND prove that it works. You should be able to write a very short proof for problem 14, perhaps using Theorem 2.7(f) yet again.

for Thursday, April 5

1. Spend about two hours studying Section 3.8 (this does not include the time spent on the exercises), probably in at least two different time periods. You should devote at least 30 minutes to the proof of Theorem 3.47 but don't go beyond an hour; write down questions that arise as you ponder this nontrivial proof. You should at least understand all of the results in this section and how to apply them even if the proofs are hard to follow. Try the computational problems 1–9 in the exercises.
2. Turn in solutions for problems 6b, 7, and 8c. Give efficient solutions that a person can follow without a calculator; this may require some trial and error. The answer for 8c is one and this means that the equation $x^2 \equiv 283 \pmod{577}$ has a solution. Use technology (such as a short program in Maple) to solve this equation.

for Friday, April 6

1. Spend about an hour studying Section 3.9. If you take your time, you should find that the proof of Theorem 3.49 is really not that bad. Note the use of the Well-Ordering Property. Be certain to do the computational problems (1–4).
2. Turn in solutions for problems 5 and 8 in Section 3.9. For problem 8, consider a mod 4 argument.

for Tuesday, April 10

1. There is no class meeting today because of the undergraduate conference. Do attend some of the talks presented by your peers, trying at least one talk outside of your comfort zone.

for Thursday, April 12

1. Special Assignment 2 is due at the beginning of class.
2. If time permits, consider the following problems from Chapter 3: problem 10 in Section 3.8 and extend it to include -3 as well, problem 10 in Section 3.9, and a proof that $\sqrt{n}$ is irrational if n is a positive integer that is not a perfect square (perhaps taking advantage of the FTA).

for Friday, April 13

1. Read Section 4.1. These ideas may seem simple (and they are probably quite familiar) but do give them careful thought, especially when the sets A and B are not sets of numbers.
2. Be prepared (as in willing to voice your solution or present it at the board) to discuss all of the problems in Section 4.1. I will not be collecting any problems from this section.

for Tuesday, April 17

1. Read Section 4.2.
2. Be prepared to discuss all of the problems in Section 4.2. I will most likely be calling on students to give their solutions. Try to do problem 4 without technology.
3. Turn in solutions for problems 4b, 4d, and 7 from Section 4.2. For problems 4b and 4d, provide careful reasoning (and perhaps a graph) to justify your answers. As mentioned above, you should be able to do this problem without technology based upon the mathematics you have learned in the past three years. For problem 7, I ask that you give a careful (read *very* careful) "chasing points" proof for each of the two inclusions you need to verify.

for Thursday, April 19

1. Read Section 4.3 and think carefully about these new terms.
2. Be prepared to discuss the problems from Section 4.3. Many of these problems request examples or ask simple questions so be prepared to present your ideas on the board. For problem 10, give a direct proof that uses the standard definition of surjective functions; avoid using induced set functions. This should be a "follow your nose" type proof but do write the details clearly, probably in five short sentences.
3. Turn in solutions for problems 5, 7, and 8 from Section 4.3. Treat this as a no-help assignment in the same category as the special assignments; I am sure you remember the guidelines for these types of assignments so I will not repeat them here. For problem 5 give clear examples and some indication as to why your examples have the desired properties. I request that you do a modified version of problem 7; use the rule of correspondence
$$f(a,b) = 2^{a-1}(2b-1)$$
and carefully prove that this function mapping $\mathbb{N} \times \mathbb{N}$ into $\mathbb{N}$ is both injective and surjective. Problem 8 should be self-explanatory. I will most likely count this short assignment as 20 points.

for Friday, April 20

1. Read Section 4.4 (it is very short).
2. Be prepared to discuss the problems in Section 4.4; this means that you should spend some time thinking about each of the problems.
3. Turn in solutions for problems 5 and 7 from Section 4.4. Hopefully, I discussed the general idea behind problem 5 in class. Your table should look a bit like the multiplication table for $\mathbb{U}_9$ on page 77. In this case, you should end up with a matrix of letters. The product $a * b$ means the left side column entry a times the upper row entry b. The result for problem 7 should be similar to the two parts of problem 6. Your solution requires the correct statements along with careful proofs.
4. As a reminder, we have a test next Friday focusing primarily on Sections 3.5 through 4.6. Look ahead to next Thursday's assignment to find some details.

for Tuesday, April 24

1. Read Sections 4.5 and 4.6. Once again, these sections are rather short.
2. Be prepared to discuss problems 1, 4, 5, and 6 from Section 4.5 and problems 2, 3, 6, 9, 11, and 12 from Section 4.6.
3. Turn in solutions for problems 11, 12b, and 12c in Section 4.6. For problem 11, you should use Definition 4.24. You may find problems 12b and 12c a bit challenging but working on them provides a good review for some of the material in Chapter 3. You may find that your solution provides a general method for solving congruence systems like the one in special assignment 2.

for Thursday, April 26

1. Spend some time reviewing the material that we have covered since the last exam. (You should start with Section 3.5 but do be aware that some of the concepts we use have appeared in earlier sections.) For this exam, I will not be requesting proofs of facts that you were expected to learn. Rather, the test questions will involve the theorems, concepts, and techniques that we have developed over the past few weeks. You thus need to know what these results and concepts are and be able to apply them in various situations. You can flip through the pages of the text, you can go over previous homework problems, and you can spend some time meditating on the concepts and how they can be used to solve problems.

for Friday, April 27

1. We have a test focusing primarily on Sections 3.5 through 4.6. Here is a sampling of problems that could be asked on this exam. Do not assume that the actual test will be a modified version of this set of problems. You can work with each other to check your solutions. It would be in your best interest to tackle these problems as if it were a testing situation.

Math 260 **Third Exam Prep** **Spring 2012**

1. Determine the number of positive divisors of 90^{12}.

2. Find $\phi(2727)$. Show your work clearly and put in intermediate steps so that your reasoning is clear. State briefly what this number represents.

3. Solve the congruence $3^{100}x \equiv 100! \pmod{103}$. Use set notation to list all of the solutions. Show your work clearly (and as simply as possible) and explicitly mention results you are using.

4. Determine $\left(\frac{73}{101}\right)$. Show your work clearly.

5. Express 7373 as a sum of two squares in two distinctly different ways.

6. Suppose that $a^2 + b^2 = c^2$, where a, b, and c are relatively prime positive integers with the further property that a and b are not multiples of 11. Prove that 11 does not divide $a^2 - b^2$.

7. Give an example of (a) a four digit positive integer with an odd number of divisors and (b) (a separate problem) a four digit positive integer that is the product of its proper positive divisors.

8. Let $n \geq 2$ be a positive integer and suppose that p is a prime that divides $\binom{2n}{n}$. Prove that $p < 2n$.

9. Let $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{r, s, t, u\}$, then consider the function $f\colon A \to B$ defined by

$$f(1) = t; \qquad f(3) = u; \qquad f(5) = t;$$
$$f(2) = s; \qquad f(4) = s; \qquad f(6) = s.$$

Find the image of 3 under f, the pre-image(s) of t under f, a pseudo-inverse g of f, and the number of distinct pseudo-inverses of f.

10. Let $f\colon \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 2x^2$. Find each of the following:

a) $f((-2, 3])$ b) $f^{-1}([-1, 8])$ c) $f^{-1}\Big(f\big((1, \sqrt{3})\big)\Big)$

11. Consider the function $f\colon \big(\mathbb{Z}^+ \times \mathbb{Z}^+ \times \mathbb{Z}^+\big) \to \mathbb{Z}^+$ defined by $f(a, b, c) = 2^a + 3^b + 7^c$. Prove that f is not injective.

12. Give an example of a function $f\colon \mathbb{Z}^+ \to \mathbb{Z}^+$ that is surjective but not injective. You must include a proof that your function has the desired properties.

for Tuesday, May 1

1. Read Section 4.7 carefully. This is a longer section and it contains some rather strange ideas for the novice. We are going to consider different sizes of infinity.
2. Be prepared to discuss the problems in Section 4.7. There are some simple computational problems, some basic logic problems, and some challenging problems. I will not be collecting any of these problems but we will discuss them in class at length.

for Thursday, May 3

1. Your rewrite of the third exam is due at the beginning of class. Pay particular attention to the heading of the exam, both in terms of the proper guidelines for taking it and the care in which the writing should be done.
2. If time permits, spend 20–30 minutes going over Section 4.7 again.

for Friday, May 4

1. Read Section 4.8.
2. Read through the problems at the end of this section to get a sense for what they are like and which ones interest you, then solve some of the problems. As time allows, we will discuss the problems that create the greatest interest or difficulties.

for Tuesday, May 8

1. Look over the textbook to recall the topics we have covered this semester.

for Saturday, May 12

1. The comprehensive final exam is scheduled for 9:00 AM in our usual classroom. You will have three hours for the exam if you so desire.