Math 260 Assignments for Spring 2016 (due on given date)

for Tuesday, January 19

1. Read the syllabus carefully.

2. Read Section 1.1. Although some of the terms here may be new to you, they are not difficult and we will use them quite a bit in the next few weeks. There is no need to memorize many of these terms (especially those in Theorem 1.3) but you certainly need to be familiar with the connectives, truth tables, and the language associated with conditional statements (such as converse and contrapositive).

3. This is the first of many reading assignments. You need to learn how to read a mathematics textbook (or a journal article); this is an important aspect of being prepared for higher mathematics. This type of reading is an acquired skill and takes plenty of patience to learn. You cannot skim, you cannot gloss over details, you may need to pause to think or write, you may need to read a sentence or paragraph multiple times before it is clear, or you may get completely stumped. If you do get stuck at some point, do not just quit reading; put a question in the margin and keep going, trying to make as much sense as you can of what follows. Sometimes your questions will be answered later (either in the text or with a flash of insight). However, if your questions are not resolved, it is important that you ask for help. Also, you need to be certain that you are not fooling yourself into thinking that you actually understand when you really do not.

4. Do exercises 1, 2a, 2d, 3h, 3k (the letters on exercise 3 refer to the parts of Theorem 1.3), 4, and 5 in Section 1.1. We should have time to discuss these in class if necessary.

5. Turn in solutions for exercises 5a, 5b, and 5c from Section 1.1. (Note that the instructions to consider different universes are relevant for exercise 5a.) Write your solutions very carefully and provide reasons for your answers of true or false. Your solution should include the original statement, along with the converse and the contrapositive, all written clearly. This is your first opportunity to show me the quality of your written work so it would be in your best interest to get off to a good start. The written assignment is due at the beginning of class on Tuesday.

for Thursday, January 21

1. Read Section 1.2. Remember to read carefully and to make certain you understand each and every statement. For this section, you should probably pause to reflect at times and create further examples. If you have any questions, write them down and ask them in class or during office hours.

2. You should work all of the exercises in this section; each exercise should only take a few minutes.

3. Turn in solutions for exercises 2, 5, and 6d from Section 1.2. For exercises 2 and 5, you should include the verbal statement along with the symbolic statement. This means that you need to copy some version of the exercise before writing the solution. For exercise 6d, you will need a complete sentence or two, along with an algebraic equation, for your explanation. Once again, think about how you are writing and presenting mathematics.

for Monday, January 25

1. Read Section 1.3 carefully and ponder the implications of quantifiers and their negations. Also read the biography of De Morgan; you should be "nerdy" enough (in a good way :)) to determine the year in which he was born. Do you have friends or relatives who can make a similar claim?

2. Be prepared to discuss exercises 1, 2, 3, 4, and 7 in Section 1.3. This means that you should do the exercises and be able to explain your solution to someone in the class, possibly at the board.

3. Turn in solutions for exercises 7b, 7c, and 7e from Section 1.3. You do not need to use symbols and quantifiers to negate these definitions but use them if you find them helpful. Really work on the wording of the negations to make sure they are clear and be certain that only simple sentences are negated. Since the terms in 7c are probably unfamiliar, you do not need to give any examples, but you should provide examples for the other two definitions. This means to give an example that illustrates the definition and one that illustrates the negation of the definition. In fact, it is a good idea to come up with a variety of examples to help clarify the concept. The purpose of these definitions is not so much that you grasp them fully but that you are exposed to some common terms from higher mathematics and that you can work with them symbolically even if you are unclear as to their meaning.

for Tuesday, January 26

1. Read Section 1.4. You may need to read very slowly in a few places and think carefully about the implications of the order of the quantifiers.

2. Be prepared to discuss exercises 1, 2, and 3 from Section 1.4. Make sure you have clear reasons to support your answers for these exercises.

3. Turn in solutions for exercises 4b and 4c from Section 1.4. As before, word your negated definitions carefully. You do not need to provide examples this time. Rather use your negated definitions to "prove" each of the following:

   i) the sequence $\{\sqrt[3]{n}\}$ is not bounded

   ii) the sequence $\{(-1)^n\}$ does not converge to 1

for Thursday, January 28

1. Be prepared to discuss exercises 4 and 5 from Section 1.4.

2. Turn in solutions to exercises 4e and 4f. For these exercises, you only need to negate the definition; it is not necessary to provide examples. Be certain that your wording is clear. In addition, provide solutions to the following two problems:

   i) Prove that the sequence $\left\{\sum_{k=1}^{n} \frac{1}{k}\right\}$ is not a Cauchy sequence.

   ii) Prove that the function f defined by $f(x) = \sin(1/x)$ for $x \neq 0$ and $f(0) = 0$ is not continuous at 0.

   For the first of these extra problems, consider letting $m = N$ and $n = 2N$.

for Monday, February 1

1. Read Section 1.5. Since sets appear in almost all areas of mathematics, it is important to acquire a firm understanding of them and their associated notation.

2. Do exercises 1–8 in Section 1.5. Most of these should be go quickly.

3. Turn in solutions for exercises 1f, 2b (meaning to use the sets in 1b), and 5 (you might consider using the sets in 1b to illustrate this result). Your proof for exercise 5 should look something like (as in really look like this: format, aligned = signs, reasons, etc.)

$$\begin{aligned} (A \setminus B) \cup (B \setminus A) &= (A \cap B^c) \cup (B \cap A^c) && \text{(Exercise 3)} \\ &= && \text{(Theorem 1.10 e)} \\ &\ \ \vdots && \text{(3 more steps, give reasons)} \\ &= && \text{(De Morgan's Laws)} \\ &= (A \cup B) \setminus (A \cap B) && \text{(Exercise 3)} \end{aligned}$$

for Tuesday, February 2

1. Read Section 1.6. The concepts of pairwise disjoint, partition, and power set are important.

2. Do exercises 1, 2, 3, 6, and 10 in Section 1.6.

3. Turn in solutions for exercises 2 and 6 in Section 1.6.

for Thursday, February 4

1. Read Section 1.7 and work to understand this new concept. This is the first section in which proofs of results play a more prominent role so study them carefully.

2. Be prepared to discuss exercises 1, 2, 3, 6, and 8 from Section 1.7. Even if a result seems "obvious", make certain you can write out the details to show how it works. For exercise 6, you need to think carefully about the meaning of equivalence classes and use correct set notation.

3. Turn in solutions for exercise 8 in Section 1.6 and exercise 8 from Section 1.7. Exercise 1.6.8 requires a proof—three proofs actually—so give it your full attention. To prove that the collection is a partition, you need to prove that two sets are equal. This involves two "chasing points" proofs. Then you need to show that the sets are pairwise disjoint and this also requires proof, hopefully one that involves properties of sets. This result is not very deep (in fact, it is easy to believe) but the point is to write a careful proof, not just believe that the result is true or obvious. If you cannot write proofs for simple statements, then you will have a great deal of trouble writing proofs for more complicated statements. For exercise 1.7.8, be extremely thoughtful so that fractions and division do not appear in your solution; there are easy ways to prove the transitive property that do not involve fractions.

for Monday, February 8

1. Read the introduction to Chapter 2 and Section 2.1. Since we are beginning proofs in earnest now, read very carefully and make note of any questions or confusions that arise.

2. Be prepared to discuss exercises 1–7 from Section 2.1.

3. Turn in solutions for exercises 4 and 7 from Section 2.1. For exercise 4, write your solution in the style of the example done in class or those in the text. Be very clear (and patient) and include every step. For exercise 7, you can closely imitate the examples in the text; do include the parenthetical remarks this one (and probably only) time.

4. You should take some time over the weekend to review the topics we have covered thus far in preparation for the test on Thursday.

for Tuesday, February 9

1. Review Chapter 1 as needed.

2. Turn in solutions for the problems on the handout. Write your solutions in the space provided on the handout as neatly, clearly, and concisely as is possible for you.

for Thursday, February 11

1. We have a test covering Chapter 1 and Section 2.1.

for Monday, February 15

1. No class on this date due to President's Day.

for Tuesday, February 16

1. Read Section 2.2 carefully. Although $a|b$ is a simple notion and one that is familiar to you, it is very important that you learn how to work with this concept at this level. Note that this concept does not involve fractions or rational numbers or division in the usual sense. Recall that definitions are like formulas in the sense that you need to know them inside and out. Just as you can immediately tell someone the derivative of x^2, you should be able to readily quote (essentially verbatim) the definition for $a|b$.

2. Be prepared to discuss exercises 1–4 in Section 2.2. For Exercise 2, use the definition of $n|(2n+3)$ and then some factoring to learn something about n. The proofs requested in exercise 3 should follow easily from the definitions. For exercise 4, consider using part (f) of Theorem 2.7, which is a very powerful result. Finding a has nothing to do with the proof; it is just an interesting aspect of the numbers.

3. Turn in a solution to exercise 2.1.11. You may find this exercise a bit of a challenge since you will need to use other results proved in this section or given in the earlier exercises; when you use these results, just refer to them by theorem number or exercise number. Most likely, you will have to rewrite your proof for this exercise at least once if you want it to read well.

4. Turn in a two column logic proof for the following:

Prove $P \Rightarrow Q$;   given R, $(P \wedge R) \Rightarrow S$, $\neg Q \Rightarrow \neg T$, $S \Leftrightarrow T$.

for Thursday, February 18

1. No class on this date due to the Power and Privilege Symposium.

for Monday, February 22

1. Review Sections 2.1 and 2.2 if necessary.

2. Be prepared to discuss exercises 5–11 in Section 2.2.

3. Turn in solutions for exercises 8c and 8d. Note that 8d is a biconditional so two proofs are needed.

4. Special assignment #1 is due at the beginning of class.

for Tuesday, February 23

1. Read Section 2.3. This should be an easier section since it is about finding examples rather than writing proofs. However, it is important to note that it can be difficult to find examples in certain situations.

2. Be prepared to discuss all of the exercises from Section 2.3. I may be asking people to go to the board to present their solutions. Read the heading of the exercises before proceeding.

3. Turn in solutions for exercises 9b and 12.

As you begin to read and write proofs on a regular basis, you might consider reading the syllabus again. It contains some guidelines for how to approach this material. You need to read and reread the text, you need to close your eyes and ponder the concepts, you need to do lots of exercises (this includes redoing exercises several days later if you were stuck on them the first time and then got some help—see if you can recreate the solution), and you need to keep working even when it feels like you are beating your head against a wall. Remember that frustration is not unusual as you seek to understand mathematics in a different way.

You are being immersed in a new culture and it takes time to overcome the initial shock. You have to trust me (and many other people in the math and sciences) that the world you are being introduced to is worth entering. As an analogy, how would you respond to a 13 year-old kid taking algebra who asked you what the point was of using letters to represent numbers and why anyone would ever need to solve an equation? Granted, it is true that for many fascinating areas of life, it is not necessary to do much math, but if you are going on in a number of other fields, mathematics is important. Is it possible to explain to this middle school kid how interesting a study of calculus or differential equations can be? How would they even be able to understand the terms you are using? With regards to higher mathematics, you are in a similar situation as the middle school algebra student is with differential equations.

for Thursday, February 25

1. Read Section 2.4. The PMI is extremely important and, once you grasp the essential idea and learn how to write it well, is a powerful method for proving certain types of results. However, this technique of proof is notorious for causing confusion for many people so study it carefully.

2. Do exercises 2, 3, 4, 5, 6, 7, 8, 9, and 11 in Section 2.4. Once you get the basic format down, the inductive step should become your main focus. To help with the proper wording of these proofs, see the induction link on the website. It presents a proof of a result similar to the one that we did in class, along with three poorly written proofs of the same result, all taken from common student errors. If you do not spot the errors, please let me know as it is important that you become critical readers of proofs. You should probably try exercise 5 on your own (after studying Theorem 2.13) before reading the solutions online. For the record, it is possible to prove most of the results in this section without induction (essentially by appealing to other results that are already known and require induction to prove). If you have time, it is good practice to ponder these other solution methods.

3. Turn in solutions for exercises 2 and 7 in Section 2.4 as well as a solution for the following result:

    For each positive integer n, the integer $2^{5n-4} + 5^{2n-1}$ is divisible by 7.

    However, I want you to use a different style for each of these proofs. Use the set S method for exercise 2, use the statement P_n method for the extra exercise, and use the informal method for exercise 7. I want you to be familiar with each of these styles but you should soon learn to use the informal method since it is much more common. You can see the link 'Model Induction Proofs' at the class website for examples that illustrate each of these three styles.

for Monday, February 29

1. Read Section 2.5 very carefully. Be certain that you truly understand the proofs of these two major results. Recall that the purpose of this course is to prepare you for higher mathematics and the ability to read and follow proofs is one of the skills necessary for success in upper level mathematics classes. Do not "fake" yourself into believing you understand a proof when you really don't; ask questions about anything that seems unclear. If one step does not make sense, do not give up on the proof. Just believe the result that you do not understand and see if the rest of the proof makes sense from that point on..

2. Be prepared to discuss exercises 1, 2, 5, 9, 10, and 11 in Section 2.5.

3. Turn in solutions for the following two problems:

    i. Prove that for each positive integer $n \geq 2$, the inequality $\dfrac{4^n}{2n} < \dbinom{2n}{n}$ is satisfied.

    ii. Find the maximum value of $x^2 y$ subject to the conditions $x > 0$, $y > 0$, and $4x + 3y = 30$. (Do not use calculus.)

for Tuesday, March 1

1. Read Section 2.6. Be certain you understand the distinction between the two forms of induction and try to use the form that is most appropriate for the given situation. The FTA is very important so make sure you know what it says and how to prove it. The boring form of induction is certainly boring but it is important to know how it works and to notice how often it implicitly appears.

2. Work on exercises 1, 2, 3, 4, and 7 in Section 2.6. You should be focusing on the induction step as the basic idea behind induction proofs should be clear by now. For exercise 3, you might want to write out the cases $n = 8$ to $n = 16$ to see what is going on and why strong induction is needed.

3. Turn in solutions for exercises 1 and 3 from Section 2.6.

for Thursday, March 5

1. Work on exercises 5 and 6 in Section 2.6. Be certain to devote sufficient time to all of the parts of exercise 5.

2. Turn in solutions for exercises 5d and 5f from Section 2.6. Be very careful with the equation found in exercise 5d. In particular, note that the "long" sum always has an odd number of terms and always ends with a Fibonacci number with an even subscript. Checking the cases $n = 1$, $n = 2$, and $n = 3$ would be a good idea. Note that strong induction is not needed here.

for Monday, March 7

1. Read Section 2.7. The Division Algorithm is rather important so read the proof carefully and fill in the missing details; this includes solving exercise 3 at the end of the section.

2. Work on exercises 2, 3, 4, 5, 6, 8, and 11 in Section 2.7. You have an example to imitate for exercise 2. For exercise 3, follow the directions and use the first part of the proof (note that $-a$ is positive if a is negative). Do exercise 4 without a calculator. Exercises 5 and 6 seem trivial but do write out the proofs carefully. For the uniqueness part, start with "Suppose that y and z both satisfy the equation" then prove that $y = z$. Exercise 8 comes with directions as well; follow them. For exercise 11, assume there are two such points and use a standard calculus result to obtain a contradiction.

3. Turn in solutions for exercises 6 and 11 in Section 2.7.

4. Special assignment #2 is due at the beginning of class.

for Tuesday, March 8

1. Read Section 2.8. Indirect proof is a good way to go in some cases, but try not to overdo it.

2. Be prepared to discuss exercises 1 through 9 in Section 2.8.

3. This will be a review day for the test on Thursday (3/10). Look over the material and exercises we have discussed so far this semester. You need to be familiar with (and be able to use) the definitions and proof techniques we have discussed thus far, you need to understand the concepts that have been introduced, and you need to be able to solve problems similar to those assigned in the exercises. In addition, you need to know (as in present them if requested) the proofs of the following theorems:

    a. Theorem 1.17 (properties of equivalence classes)

    b. Theorem 2.7 (f) (divisibility and linear combinations)

    c. Theorem 2.22 (the Fundamental Theorem of Arithmetic)

    d. Theorem 2.27 (the Division Algorithm, the $a > b > 1$ case only)

    e. Theorem 2.33 (there are an infinite number of primes)

for Thursday, March 10

1. We have a test covering the material in Chapters 1 and 2.

for Monday, March 28

1. Go to the website

   `http://www.ams.org/mathscinet/msc/msc2010.html`

   and spend some time navigating around the subject classification. By clicking on the box on the lower left, you can see the various subject headings and then continue going into more depth by clicking on a given two digit number. Alternatively, you can load the entire file in PDF form and look through it; be aware that it is a rather large file. Spend some time reading all of the headings and then go into more depth for several of those that intrigue you. The purpose for having you spend some time looking over these areas is to give you a sense of the scope of mathematics.

   Next, you can try the link

   `http://www.whitman.edu/penrose/`

   Click on Databases and More

   Click on Subject Guides

   Click on Mathematics

   Click on MathSciNet

   Try typing in the name of a person in the mathematics faculty to see what we have published of late. For example, you can type 'Gordon, R*' in the author box and see what comes up. You can also look for various topics. You can type 'Binomial Theorem' in the review text box to see a list of papers that make use of this term. This is a good resource when you are looking for reviews of papers published in the last 60 years or so.

   Finally, go to the site

   `http://www.ams.org/mathweb/mi-journals5.html`

   and scroll down the list of journals, just skimming the titles. If you become intrigued by a title, click on the link and check out the table of contents of a recent issue of the journal.

2. When you are finished with item (1) (I am assuming that you spend at least 30–45 minutes doing this but you do not need to do much more unless you become curious), write two or more paragraphs (take the writing seriously) on your impressions (personal or otherwise) concerning mathematics after surfing these sites and developing a sense for the scope of the field of mathematics.

for Tuesday, March 29

1. Read Section 3.1. Congruence is an elementary concept once you become familiar with it, but you do need to think carefully about its meaning and properties.

2. Be prepared to discuss the exercises in Section 3.1. You will find that some of these are quite easy while others take a bit more time. For exercises 4, 5, and 6, be certain to use congruence properties even though there are other ways to proceed.

3. Turn in solutions for exercises 2 (a proof of part (7) only) and 9. Write your solutions carefully even for exercises that involve computations. Your solution to exercise 2 should not be very involved; if it is, look for a better way. For exercise 9, note that x must have the form $17k + 4$ to solve the first congruence. Now substitute this into the second congruence and determine k.

for Thursday, March 31

1. Read Section 3.2. Since we are beginning a study of an abstract space, you really need to work hard to understand what is going on.

2. Be prepared to discuss the exercises in Section 3.2.

3. Turn in solutions for exercises 3e (that is, give a proof of part (e) of the theorem) and a modified version of 6 from Section 3.2. You need to be extremely careful for the proof requested in exercise 3e. Pretend that you are trying to defuse a bomb and a wrong move will blow up the classroom. Make certain that every single step has a reason, one that you can verify from the text, not just because "it has to be true." You should start with the left-hand side of the equality and perform five (5) steps, each one with a clear reason, the last one being the right-hand side of the equality. You can line up the equations and provide the reason off to the side. The modification for exercise 6 is to consider the space $\mathbb{Z}_{65}$ rather than the space $\mathbb{Z}_{14}$.

for Monday, April 4

1. Read Section 3.3 carefully. Computationally, the Euclidean algorithm is quite elementary. However, the results given in Theorems 3.11 and 3.13 are extremely (as in EXTREMELY) important. Keep track of any questions that arise during the reading, especially any that occur while reading the second proof of Theorem 3.11. Please ask questions in class about anything you do not understand and hopefully someone other than me can provide an answer; learning to discuss your ideas in a group setting is a useful skill to acquire.

2. Be prepared to discuss exercises 1–12 in Section 3.3. Try doing the computations for exercise 1 as efficiently as possible, preferably without technology.

3. Turn in solutions for exercises 7, 10, and 11 in Section 3.3. Note that two of these exercises involve biconditional statements so two proofs are required. Most of these proofs should be of the "follow your nose" variety if you take advantage of results in this section.

for Tuesday, April 5

1. Read Section 3.4 carefully.

2. Be prepared to discuss exercises 1–10 in Section 3.4.

3. Turn in solutions for exercises 5 and 8 in Section 3.4.

for Thursday, April 7

1. Read Section 3.5.

2. Be prepared to discuss exercises 11–15 in Section 3.4 and exercises 1–8 in Section 3.5.

3. Turn in solutions for exercises 3.4.13 and 3.5.6.

4. Special assignment #3 is due at the beginning of class.

for Monday, April 11

1. Read Section 3.6. Be certain that you understand the proof of the FTA and the results that lie behind it. Read Theorems 3.31 through 3.33 enough times so that the notation does not disguise the easy ideas that they express.

2. Look over the exercises in Section 3.6; hopefully you can solve some of them rather quickly. Since many of these exercises are computational in nature, I will most likely assign some students to present solutions at the board. For exercise 15, don't just give a numerical answer; explain how you arrived at your answer. A general method for this sort of problem would be nice; see if you can formulate one.

3. Turn in solutions for exercises 15 and 16 in Section 3.6.

for Tuesday, April 12

1. There is no class meeting today because of the undergraduate conference. Do attend some of the talks presented by your peers, trying at least one talk outside of your comfort zone.

for Thursday, April 14

1. Read Section 4.1. These ideas may seem simple (and they are probably quite familiar) but do give them careful thought, especially when the sets A and B are not sets of numbers.

2. Be prepared to discuss exercises 1–6 in Section 4.1; at the beginning of class, I will be assigning students to write their solutions on the board. I will not be collecting any solutions to exercises from this section.

3. Read the proof on the handout very carefully and be prepared to discuss the ideas therein.

for Monday, April 18

1. Read Section 4.2. Be prepared to discuss all of the exercises in Section 4.2. Once again, I will be assigning students (at the beginning of class) to write their solutions on the board. You should be able to do exercise 4 without technology; note that parts (b) and (d) require careful reasoning (and perhaps a graph) to justify your answers. For exercise 7, you should give a careful (read *very* careful) "chasing points" proof for each of the two inclusions you need to verify.

2. Turn in a solution for exercise 7 in Section 4.2.

for Tuesday, April 19

1. Read Section 4.3 and think carefully about these new terms.

2. Be prepared to discuss the exercises in Section 4.3. Many of these exercises request examples or ask simple questions so be prepared to present your ideas at the board. For exercise 10, give a direct proof that uses the standard definition of surjective functions; avoid using induced set functions. This should be a "follow your nose" type proof but do write the details clearly, probably in five short sentences.

3. Turn in solutions for exercises 8 and 10 from Section 4.3.

for Thursday, April 21

1. Read Section 4.4 (it is very short).

2. Be prepared to discuss the exercises in Section 4.4; this means that you should spend some time thinking about each of the exercises.

3. Turn in solutions for exercises 5 and 7 from Section 4.4. Hopefully, I discussed the general idea behind problem 5 in class. Your table should look a bit like the multiplication table for $\mathbb{U}_9$ on page 77. In this case, you should end up with a matrix of letters. The product $a * b$ means the left side column entry a times the upper row entry b. The result for problem 7 should be similar to the two parts of problem 6. Your solution requires the correct statements along with careful proofs.

4. Special assignment #4 is due at the beginning of class.

for Monday, April 25

1. Read Section 4.6. (We are omitting Section 4.5.)

2. Be prepared to discuss exercises 1, 2, 3, 6, 7, 9, 10, and 11 from Section 4.6. I will not be collecting any of these solutions.

3. Spend some time over the weekend reviewing the material that we have covered since the last exam, namely Sections 3.1–7 and Sections 4.1–4,6, and looking over the review problems. See the April 26 assignment for more detail about the upcoming exam.

for Tuesday, April 26

1. Continue reviewing the material that we have covered since the last exam. There are quite a few definitions and theorems that you need to know; not in the sense of quoting them verbatim but by being able to work with them and apply them. You should also have a rough idea about how to prove some of the more complicated results, even without knowing the full details of the proof. The following is a list of explicit proofs that I may ask you to give on the exam.

    a. Theorem 3.2 (congruences and remainders)

    b. part (7) of Theorem 3.3 (congruences and products)

    c. part (f) of Theorem 3.10 (gcd and congruences)

    d. Theorem 3.11 (the second version of the proof about gcd and linear combinations)

    e. Theorem 3.14 (relatively prime numbers and the concept of divides)

    f. Theorem 3.19 (relatively prime and existence of inverses)

    g. Theorem 3.39 ([1] and [−1] are the only elements in $\mathbb{U}_p$ that are their own inverses)

    h. Theorem 4.9 (properties of the induced set function f^{-1})

    i. Theorem 4.13 (composition of injective functions is injective)

This is not intended to be an exercise in memorization; if you interpret it in that way you are missing the point. For each of these results, you should know the basic idea behind the proof and use this knowledge (along with your improving ability to write mathematics) to write out the details of the proof.

In summary, reread the sections, thinking about examples and concepts, look over the exercises, trying to solve them anew without looking at your notes, practice some congruence computations to brush up on your arithmetic skills, and carefully ponder the third exam prep handout. Bring any questions you have to class on the day of the review or stop by office hours any day before the exam.

for Thursday, April 28

1. We have a test covering Sections 1–7 of Chapter 3 and Sections 1–4,6 of Chapter 4.

for Monday, May 2

1. Read Section 4.7 very carefully and think deeply about the ideas that are discussed. This is a longer section and it contains some rather strange ideas concerning different sizes of infinity. If necessary, you can skim the proofs of Theorems 4.32 and 4.33.

2. Be prepared to discuss exercises 1, 4, 7, 8, and 10 in Section 4.7. There are some simple computational exercises, some basic logic exercises, and some challenging exercises. I will not be collecting any of these solutions but we will discuss them in class at length, probably with students at the board.

for Tuesday, May 3

1. Read Section 4.8 through the proof of Corollary 4.37.
2. Be prepared to discuss exercises 2 and 9 in Section 4.8.

for Thursday, May 5

1. To Be Determined
1. Special assignment 5 is due at the beginning of class.

for Monday, May 9

1. Finish reading Section 4.8.
2. Do exercise 8 and spend some time pondering exercise 10 in Section 4.8.
3. Write down any lingering questions you have concerning Sections 4.7 and 4.8.
4. Flip through the textbook to remind yourself of the topics we have covered this semester. Hopefully, you will be surprised how much of it is familiar to you. The final exam is comprehensive so look over all of the sections except for 3.8, 3.9, and 4.5.

for Friday, May 13

1. The comprehensive final exam is scheduled for 9:00 AM in our usual classroom (for the 11:00 AM section) and 2:00 PM in our usual classroom (for the 9:00 AM section). You will have three hours for the exam if you so desire.
2. I have decided not to have you learn specific proofs for the final exam. I do expect that you can write proofs of simpler facts (such as Theorem 2.7(f)) that we have used often, but I do not expect you to remember how to recreate the more involved proofs (such as the proof of Division Algorithm).
3. A list of the main topics (perhaps not exhaustive but it is close) we have discussed this semester appears on the next page. To prepare for the exam, after making sure you are familiar with the ideas, go over the exercises, special assignments, and previous exams that we have worked on during the semester. This seems like a lot (and it probably is), but the idea is to be able to use your newly acquired skills to solve problems and write proofs clearly.

a list of topics for Math 260

1. the five logical connectives

2. tautologies, especially contraposition and modus ponens

3. converse and contrapositive

4. existential and universal quantifiers

5. DeMorgan's Laws and negations of statements, including definitions

6. sets and set operations

7. Cartesian product of sets

8. partitions of sets and power sets

9. equivalence relations and their properties

10. definition and properties of $n|a$

11. the Principle of Mathematical Induction (both versions)

12. the Well-Ordering Property of the positive integers

13. the Binomial Theorem

14. the Arithmetic Mean / Geometric Mean Inequality

15. Fibonacci numbers

16. the Fundamental Theorem of Arithmetic

17. the Division Algorithm

18. the set of primes is infinite

19. notion of congruence and its properties

20. greatest common divisors and least common multiples

21. the Euclidean Algorithm

22. the spaces $\mathbb{Z}_n$ and $\mathbb{U}_n$

23. relatively prime integers

24. the Euler ϕ function

25. Wilson's Theorem

26. Euler's Theorem and Fermat's Little Theorem

27. definition of a function, including domain and codomain

28. induced set functions and their properties

29. injective, surjective and bijective functions

30. cardinality of a set

31. finite and infinite sets, countable and uncountable sets