Math 260 Assignments for Spring 2018 (due on given date)

for Wednesday, January 17

1. Read the syllabus carefully.
2. Read Section 1.1. Although some of the terms here may be new to you, they are not difficult and we will use them quite a bit in the next few weeks. There is no need to memorize all of these terms (especially those in Theorem 1.3) but you certainly need to be familiar with the connectives, truth tables, and the language associated with conditional statements (such as converse and contrapositive).
3. This is the first of many reading assignments. You need to learn how to read a mathematics textbook (or a journal article); this is an important aspect of being prepared for higher mathematics. This type of reading is an acquired skill and takes plenty of patience to learn. You cannot skim, you cannot gloss over details, you may need to pause to think or write out some details, you may need to read a sentence or paragraph multiple times before it is clear, or you may get completely stumped. If you do get stuck at some point, do not just quit reading; put a question in the margin and keep going, trying to make as much sense as you can of what follows. Sometimes your questions will be answered later (either in the text or with a flash of insight). However, if your questions are not resolved, it is important that you ask for help. Also, you need to be certain that you are not fooling yourself into thinking that you actually understand when you really do not.
4. Do exercises 1, 2a, 2d, 3h, 3k (the letters on exercise 3 refer to the parts of Theorem 1.3), 4, and 5 in Section 1.1. We should have time to discuss these in class if necessary.
5. Turn in solutions for exercises 5a, 5b, and 5c from Section 1.1. (Note that the instructions to consider different universes are relevant for exercise 5a.) Write your solutions very carefully and provide reasons for your answers of true or false. Your solution should include the original statement, along with the converse and the contrapositive, all written clearly. This is your first opportunity to show me the quality of your written work so it would be in your best interest to get off to a good start. The written assignment is due at the beginning of class on Wednesday.

for Friday, January 19

1. Read Section 1.2. Remember to read carefully and to make certain you understand each and every statement. For this section, you should probably pause to reflect at times and create further examples. If you have any questions, write them down and ask them in class or during office hours.
2. You should work all of the exercises in this section; each exercise should only take a few minutes.
3. Turn in solutions for exercises 2, 5, and 6d from Section 1.2. For exercises 2 and 5, you should include the verbal statement along with the symbolic statement. This means that you need to copy some version of the exercise before writing the solution. For exercise 6d, you will need a complete sentence or two, along with an algebraic equation, for your explanation. Once again, think about how you are writing and presenting mathematics.

for Monday, January 22

1. Read Section 1.3 carefully and ponder the implications of quantifiers and their negations. Also read the biography of De Morgan; you should be “nerdy” enough (in a good way :)) to determine the year in which he was born. Do you have friends or relatives who can make a similar claim?
2. Be prepared to discuss exercises 1, 2, 3, 4, and 7 in Section 1.3. This means that you should do the exercises and be able to explain your solution to someone in the class, possibly at the board.
3. Turn in solutions for exercises 7b, 7c, and 7e from Section 1.3. You do not need to use symbols and quantifiers to negate these definitions but use them if you find them helpful. Really work on the wording of the negations to make sure they are clear and be certain that only simple sentences are negated. Since the terms in 7c are probably unfamiliar, you do not need to give any examples, but you should provide examples for the other two definitions. This means to give an example that illustrates the definition and one that illustrates the negation of the definition. In fact, it is a good idea to come up with a variety of examples to help clarify the concept. The purpose of these definitions is not so much that you grasp them fully but that you are exposed to some common terms from higher mathematics and that you can work with them symbolically even if you are unclear as to their meaning.

for Wednesday, January 24

1. Read Section 1.4. You may need to read very slowly in a few places and think carefully about the implications of the order of the quantifiers.
2. Be prepared to discuss exercises 1, 2, 3, and 4a–e from Section 1.4. Make sure you have clear reasons to support your answers for these exercises.
3. Turn in solutions for exercises 4b and 4c from Section 1.4. As before, word your negated definitions carefully. You do not need to provide examples this time. Rather use your negated definitions to carefully prove each of the following:
   - i) the sequence $\{\sqrt[3]{n}\}$ is not bounded
   - ii) the sequence $\{(-1)^n\}$ does not converge to 1

for Friday, January 26

1. Read Sections 1.5 and 1.6. Since sets appear in almost all areas of mathematics, it is important to acquire a firm understanding of them and their associated notation. The concepts of pairwise disjoint, partition, and power set (defined in Section 1.6) are important.
2. Do exercises 1abcfg, 2, 4ef, 5, and 8 in Section 1.5 and exercises 1, 2, 3, 6, and 10 in Section 1.6. Most of these should be go quickly.
3. Turn in solutions for exercises 1.5.5, 1.6.2, and 1.6.6. Your proof for exercise 1.5.5 should look something like (as in really look like this: format, aligned = signs, reasons, etc.)

$$\begin{aligned} (A \setminus B) \cup (B \setminus A) &= (A \cap B^c) \cup (B \cap A^c) && \text{(Exercise 3)} \\ &= && \text{(Theorem 1.10 e)} \\ &\quad \vdots && \text{(3 more steps, give reasons)} \\ &= && \text{(De Morgan's Laws)} \\ &= (A \cup B) \setminus (A \cap B) && \text{(Exercise 3)} \end{aligned}$$

for Monday, January 29

1. Read Section 1.7 and work to understand this new concept. This is the first section in which proofs of results play a more prominent role so study them carefully.
2. Be prepared to discuss exercises 1, 2, 3, 5, 6, 8, and 9 from Section 1.7. Even if a result seems "obvious", make certain you can write out the details to show how it works. For exercise 6, you need to think carefully about the meaning of equivalence classes and use correct set notation.
3. Turn in solutions for exercises 8 and 9a in Section 1.7. For exercise 1.7.8, be extremely thoughtful so that fractions and division do not appear in your solution; there are easy ways to prove the transitive property that do not involve fractions. The reason that no fractions should appear is due to the fact that the universe involves integers. In fact, this equivalence relation is used to **define** the rational numbers.

for Wednesday, January 31

1. Read the introduction to Chapter 2 and Section 2.1. Since we are beginning proofs in earnest now, read very carefully and make note of any questions or confusions that arise.
2. Be prepared to discuss exercises 1–11 from Section 2.1.
3. Turn in a carefully written (as in the text) two column logic proof for the following:

   Prove $P \Rightarrow Q$;   given R, $(P \wedge R) \Rightarrow S$, $\neg Q \Rightarrow \neg T$, $S \Leftrightarrow T$.
4. Turn in solutions for exercises 7 and 11 from Section 2.1. For exercise 7, you can closely imitate the examples in the text; do include the parenthetical remarks this one (and probably only) time. There are several ways to solve exercise 11; feel free to use previous results and/or exercises in this section by referring to them by number. Most likely, you will have to rewrite your proof for this exercise at least once if you want it to read well.

for Friday, February 2

1. Read Section 2.2 carefully. Although $a|b$ is a simple notion and one that is familiar to you, it is very important that you learn how to work with this concept at a deeper level. Note that this concept does not involve fractions or rational numbers or division in the usual sense. Recall that definitions are like formulas; you need to know them inside and out. Just as you can immediately tell someone the derivative of x^2, you should be able to readily quote (essentially verbatim) the definition for $a|b$.
2. Be prepared to discuss exercises 1–8 in Section 2.2. For Exercise 2, use the definition of $n|(2n+3)$ and then some factoring to learn something about n. The proofs requested in exercise 3 should follow easily from the definitions. For exercise 4, consider using part (f) of Theorem 2.7, which is a very powerful result. Finding a has nothing to do with the proof; it is just an interesting aspect of the numbers.
3. Turn in solutions for exercises 4 and 8d in Section 2.2. Since the statement that you are proving in exercise 8d is a biconditional, two proofs are required.

for Monday, February 5

1. Read Section 2.3. This should be an easier section since it is about finding examples rather than writing proofs. However, it is important to note that it can be difficult to find examples in certain situations.
2. Be prepared to discuss all of the exercises from Section 2.3. I may be asking people to go to the board to present their solutions. Read the heading of the exercises before proceeding.
3. Turn in a solution for exercise 12 in Section 2.3, including the thought process you used to approach the problem.
4. Turn in a solution for exercise 9 in Section 2.2. The statement of the problem and your solution should be typeset in LaTeX.
5. You should take some time over the weekend to review the topics we have covered thus far in preparation for the test coming up on Friday.

for Wednesday, February 7

1. Review the material that we have covered thus far.
2. Turn in solutions for the problems on the handout. Write your solutions in the space provided on the handout as neatly, clearly, and concisely as is possible for you.

for Friday, February 9

1. We have a test covering Chapter 1 and Sections 2.1 to 2.3.

for Monday, February 12

1. Read Section 2.4. The PMI is extremely important and, once you grasp the essential idea and learn how to write it well, is a powerful method for proving certain types of results. However, this technique of proof is notorious for causing confusion for many people so study it carefully.
2. Do exercises 2, 3, 4, and 6 in Section 2.4. Use induction for exercises 2, 3, and 6, but do not use induction for exercise 4. Rather, find a way to use the formula from exercise 2 to quickly find and establish a formula for the sum given in exercise 6.
3. Turn in a carefully written solution for exercise 6 using the informal style for induction proofs. You might find the simple fact $\dfrac{1}{(k+1)!} \leq \dfrac{1}{k(k+1)}$ helpful in your proof.

for Wednesday, February 14

1. Read Section 2.4 again if necessary and look over the proofs found at the link 'Model Induction Proofs' on the course website. There you will find a proof of a simple result along with three poorly written proofs of the same result, all taken from common student errors. If you do not spot the errors, please let me know as it is important that you become critical readers of proofs. You should probably try exercise 5 on your own (after studying Theorem 2.13) before reading the solutions online. For the record, it is possible to prove most of the results in this section without induction (essentially by appealing to other results that are already known and require induction to prove). If you have time, it is good practice to ponder these other solution methods.
2. Do exercises 5, 7, 8, 9, 11, and 12 in Section 2.4.
3. Turn in solutions for exercises 11 and 12 in Section 2.4 as well as a solution for the following result:

   For each positive integer n, the integer $2^{5n-4} + 5^{2n-1}$ is divisible by 7.

   However, I want you to use a different style for each of these proofs. Use the set S method for exercise 11, use the statement P_n method for the extra exercise, and use the informal method for exercise 12. I want you to be familiar with each of these styles but you should soon learn to use the informal method since it is much more common. You can find examples of each type at the aforementioned link.

for Friday, February 16

1. Read Section 2.5 very carefully. Be certain that you truly understand the proofs of these two major results. Recall that the purpose of this course is to prepare you for higher mathematics and the ability to read and follow proofs is one of the skills necessary for success in upper level mathematics classes. Do not "fake" yourself into believing you understand a proof when you really don't; ask questions about anything that seems unclear. If one step does not make sense, do not give up on the proof. Just believe the result that you do not understand and see if the rest of the proof makes sense from that point on.
2. Be prepared to discuss exercises 1, 2, 5, 9, 10, and 11 in Section 2.5.
3. Turn in solutions for the following two problems:
   i. Prove that for each positive integer $n \geq 2$, the inequality $\dfrac{4^n}{2n} < \dbinom{2n}{n}$ is satisfied.
   ii. Find the maximum value of x^2y subject to the conditions $x > 0$, $y > 0$, and $4x + 3y = 30$. (Do not use calculus.)

for Monday, February 19

1. No class on this date due to President's Day.

for Wednesday, February 21

1. Read Section 2.6. Be certain you understand the distinction between the two forms of induction and try to use the form that is most appropriate for the given situation. The FTA is very important so make sure you know what it says and how to prove it. The boring form of induction is certainly boring but it is important to know how it works and to notice how often it implicitly appears.
2. Work on exercises 1, 3, 4, and 5abcd in Section 2.6. At this stage of the learning process, you should be focusing on the induction step as the basic idea behind induction proofs should be clear by now. For exercise 3, you might want to write out the cases $n = 8$ to $n = 16$ to see what is going on and why strong induction is needed. Be very careful with the equation found in exercise 5d. In particular, note that the "long" sum always has an odd number of terms and always ends with a Fibonacci number with an even subscript. Checking the cases $n = 1$, $n = 2$, and $n = 3$ would be a good idea. Note that strong induction is not needed here.
3. Turn in a solution for exercise 3 from Section 2.6.
4. Special assignment #1 is due at the beginning of class.

for Friday, February 23

1. Work on exercises 5efghi, 6, and 7 in Section 2.6. The formula in exercise 5i should be easy to spot; the challenge is giving a proof of the conjecture. See how much progress that you can make.
2. Turn in solutions for exercises 5f and 5h from Section 2.6. Then use the result from 5h to compute the value of $\lim_{n\to\infty} \dfrac{f_{n+1}}{f_n}$.

for Monday, February 26

1. Read Section 2.7. The Division Algorithm is rather important so read the proof carefully and fill in the missing details; this includes solving exercise 3 at the end of the section.
2. Work on exercises 2, 3, 4, 5, 6, 8, and 11 in Section 2.7. You have an example to imitate for exercise 2. For exercise 3, follow the directions and use the first part of the proof (note that $-a$ is positive if a is negative). Do exercise 4 without a calculator. Exercises 5 and 6 seem trivial but do write out the proofs carefully. For the uniqueness part, start with "Suppose that y and z both satisfy the equation" then prove that $y = z$. Exercise 8 comes with directions as well; follow them. For exercise 11, assume there are two such points and use a standard calculus result to obtain a contradiction.
3. Turn in solutions for exercises 8 and 11 in Section 2.7.
4. Declan will present a solution for Exercise 2.7.3 and Henry will present a solution for Exercise 2.7.6.

for Wednesday, February 28

1. Read Section 2.8. Indirect proof is a good way to go in some cases, but try not to overdo it.
2. Be prepared to discuss exercises 1, 2, 5, 7, 10, and 11 in Section 2.8. For some of the exercises that request examples, you might find the computer algebra system Maple to be helpful. If you have an account in the math lab, you have probably already used Maple. If not, you can use the username "maple" with the password "cauchy" to log in to any of the computers in Olin 207. Open a command window and type "xmaple". A Maple window will open and you can try commands such as `ifactor(2^(10)+1);` or `ifactor(2*3*5*7+1);` and see what happens. To exit Maple, press Alt F4.
3. Turn in solutions for exercises 6 and 9 in Section 2.8. As a hint for exercise 6, let $n = ab$ with $a \geq 3$ odd and use the Binomial Theorem on the quantity $\big((2^b + 1) - 1\big)^a$.
4. Sarah will present a solution for Exercise 2.8.4, Sean will present a solution for Exercise 2.8.8, and Jimmy will present a solution for Exercise 2.8.13.

for Friday, March 2

1. Read Section 3.1. Congruence is an elementary concept once you become familiar with it, but you do need to think carefully about its meaning and properties.
2. Be prepared to discuss the exercises 1–9 in Section 3.1. You will find that some of these are quite easy while others take a bit more time. For exercises 4, 5, and 6, be certain to use congruence properties even though there are other ways to proceed.
3. Turn in solutions for exercises 2 (a proof of part (7) only) and 9. Write your solutions carefully even for exercises that involve computations. Your solution to exercise 2 should not be very involved; if it is, look for a better way. For exercise 9, note that x must have the form $17k + 4$ to solve the first congruence. Now substitute this into the second congruence and determine k.
4. Yiwen will present a solution for Exercise 3.1.6 and Jack will present a solution for Exercise 3.1.7.

for Monday, March 5

1. Read Section 3.2. Since we are beginning a study of an abstract space, you really need to work hard to understand what is going on.
2. Be prepared to discuss exercises 1–6 in Section 3.2.
3. Turn in solutions for exercises 3e (that is, give a proof of part (e) of the theorem) and a modified version of 6 from Section 3.2. You need to be extremely careful for the proof requested in exercise 3e. Pretend that you are trying to defuse a bomb and a wrong move will blow up the classroom. Make certain that every single step has a reason, one that you can verify from the text, not just because "it has to be true." You should start with the left-hand side of the equality and perform five (5) steps, each one with a clear reason, the last one being the right-hand side of the equality. You can line up the equations and provide the reason off to the side. The modification for exercise 6 is to consider the space $\mathbb{Z}_{65}$ rather than the space $\mathbb{Z}_{14}$.
4. Special assignment #2 is due at the beginning of class.

for Wednesday, March 7

1. Read Section 3.3 carefully. Computationally, the Euclidean algorithm is quite elementary. However, the results given in Theorems 3.11 and 3.13 are extremely (as in EXTREMELY) important. Keep track of any questions that arise during the reading, especially any that occur while reading the second proof of Theorem 3.11. Please ask questions in class about anything you do not understand and hopefully someone other than me can provide an answer; learning to discuss your ideas in a group setting is a useful skill to acquire.
2. Be prepared to discuss exercises 1, 2, and 6 in Section 3.3. Try doing the computations for exercise 1 as efficiently as possible, preferably without technology.
3. Turn in a solution for exercise 11 in Section 3.3.
4. This will be a review day for the test on Friday (3/9). Look over the material and exercises we have discussed so far this semester. You need to be familiar with (and be able to use) the definitions and proof techniques we have discussed thus far, you need to understand the concepts that have been introduced, and you need to be able to solve problems similar to those assigned in the exercises. In addition, you need to know (as in present them if requested) the proofs of the following theorems:

    a. Theorem 1.17 (properties of equivalence classes)
    b. Theorem 2.7 (f) (divisibility and linear combinations)
    c. Theorem 2.22 (the Fundamental Theorem of Arithmetic)
    d. Theorem 2.27 (the Division Algorithm, the $a > b > 1$ case only)
    e. Theorem 2.33 (there are an infinite number of primes)
    f. Theorem 3.2 (congruence and equal remainders)
    g. Theorem 3.3 (basic properties of congruence)

    This is not intended to be an exercise in memorization. You should know the basic idea behind each proof and then be able to write your own version of it without needing too much time for thought.

for Friday, March 9

1. We have a test this day covering the material that we have discussed thus far.

for Monday, March 26

1. Go to the website

   `http://www.ams.org/mathscinet/msc/msc2010.html`

   and spend some time navigating around the subject classification. By clicking on the "Browse Classification" box, you can see the various subject headings and then continue going into more depth by clicking on a given two digit number. Spend some time reading all of the headings and then go into more depth for several of those that intrigue you. The purpose for having you spend some time looking over these areas is to give you a sense of the scope of mathematics.

   Next, you can try the link

   `http://www.whitman.edu/penrose/`

   Click on Databases and More

   Click on Subject Guides

   Click on Mathematics and Computer Science

   Click on MathSciNet

   Try typing in the name of a person in the mathematics faculty to see what we have published of late. For example, you can type 'Gordon, R*' in the author box and see what comes up. You can also look for various topics. You can type 'Binomial Theorem' in the review text box to see a list of papers that make use of this term. This is a good resource when you are looking for reviews of papers published in the last 60 years or so.

   Finally, go to the site

   `http://www.ams.org/mathweb/mi-journals5.html`

   and scroll down the list of journals, just skimming the titles. If you become intrigued by a title, click on the link and check out the table of contents of a recent issue of the journal.

2. When you are finished with item (1) (I am assuming that you spend at least 30–45 minutes doing this but you do not need to do much more unless you become curious), write two or more paragraphs (take the writing seriously) on the links you browsed and your impressions (personal or otherwise) concerning mathematics after surfing these sites and developing a sense for the scope of the field of mathematics. It is preferred that you typeset your paragraphs using LaTeX.

for Wednesday, March 28

1. Read Section 3.4 carefully. Since we discussed much of this section in class, the reading should not take too long.
2. Work on exercises 5–15 in Section 3.4.
3. Students will present solutions at the board as follows: Declan (Exercise 3.4.7ef), Sarah (Exercise 3.4.8), Henry (Exercise 3.4.11), and Jimmy (Exercise 3.4.13).
4. Turn in solutions for exercises 5 and 15 in Section 3.4. Present two proofs for Exercise 15; one using induction (start with a linear combination of f_n and f_{n+2} that equals 1 and then generate a linear combination of f_{n+1} and f_{n+3} that equals 1) and another using the result of Exercise 2.6.5e.

for Friday, March 30

1. Read Section 3.5, paying careful attention to the details in the proof of Theorem 3.29.
2. Be prepared to discuss exercises 1–7 in Section 3.5.
3. Students will present solutions at the board as follows: Jack (Exercise 3.5.1), Sean (Exercise 3.5.5).
4. Turn in solutions for exercises 3.4.14 and 3.5.6. For Exercise 3.4.14, use the hint and carefully prove that the two sets are equal. For Exercise 3.5.6, use Theorem 3.25 to prove the first equality, then use this result and Theorem 3.29 to obtain the second equality.

for Monday, April 2

1. Read the first part of Section 3.6, stopping at the end of the proof of the FTA. Be certain that you understand the proof of the FTA and the results that lie behind it. Also, think carefully about the examples presented at the beginning of the section to better appreciate the need for a proof of the FTA. In keeping with this last sentence, do the first exercise in Section 3.6.
2. Turn in solutions for exercises 3.5.8 and 3.5.10. For Exercise 3.5.8, use the result in Exercise 3.5.6 along with other previous results we have proved (these should be properly referenced in your proof). For Exercise 3.5.10a, define two sets of divisors and show that the sets are equal.

for Wednesday, April 4

1. Finish reading Section 3.6. Read Theorems 3.31 through 3.33 enough times so that the notation does not disguise the simple ideas that they express.
2. Look over the exercises in Section 3.6; hopefully you can solve some of them rather quickly. Since many of these exercises are computational in nature, I will most likely assign some students to present solutions at the board. For Exercise 15, don't just give a numerical answer; explain how you arrived at your answer. Formulate a general method for this sort of problem, then apply your method to determine the exponents on the primes 7 and 11 in the canonical factorization of 2015!.
3. Turn in solutions for exercises 8 and 16 in Section 3.6.

for Friday, April 6

1. Read Section 3.7. Expect to spend at least an hour reading through the details of the proofs and results in this section. Keep track of any questions that you have.
2. Since there are many exercises in this section, focus on exercises 3, 5, 10, and 11. You will notice that many of these are computational in nature and provide practice with the ideas used or developed in this section. Try to be as efficient as possible in your computations, taking advantage of modular arithmetic and results in this section.
3. Turn in a solution for exercise 6 in Section 3.7. For exercise 6, you simply need to give an example and prove that it works. However, you must provide an example for each integer $e > 2$; you cannot simply give an example for $e = 3$ or $e = 4$ and be done. You want a formula (in terms of e) that gives an example for each such e.
4. Special assignment #3 is due at the beginning of class.

for Monday, April 9

1. Reread Section 3.7 if necessary, making certain that you understand and can use the results of these named theorems and that you can follow the steps in the proofs.
2. Work on exercises 4, 6, 7, 8, 9, 12, 14, and 15.
3. Turn in solutions for exercises 7 and 12 in Section 3.7 as well as the following extra exercise: find an integer x such that $0 < x < 101$ and 101 divides the integer $11^{98}x - 96!$.

for Wednesday, April 11

1. Spend at least two hours reading and studying Section 3.8 (this does not include the time spent on the exercises), probably in at least two different time periods. You should devote at least 30 minutes to the proof of Theorem 3.47 but don't go beyond an hour; write down questions that arise as you ponder this nontrivial proof. You should at least understand all of the results in this section and how to apply them even if the proofs are hard to follow.
2. Work on exercises 1 through 9. Be prepared to present efficient solutions to the computational problems on the board so that other people can follow your work without a calculator. Coming up with such solutions may require some trial and error.
3. Turn in solutions for exercise 4 and the extra exercise 6 problem of determining $\left(\frac{13}{41}\right)$.

for Friday, April 13

1. Spend about an hour carefully reading Section 3.9. If you take your time, you should find that the proof of Theorem 3.49 is really not that difficult to follow. Note the use of the Well-Ordering Property in the second part of the proof. Be certain to do the computational exercises 1, 2, 3, and 4.
2. Turn in solutions for exercises 5 and 6 in Section 3.9.

for Monday, April 16

1. Carefully read the handout concerning the sum of the reciprocals of the primes.
2. Turn in a solution for exercise 11 in Section 3.9.
3. Special assignment #4 is due by 4:00 pm on this day.
4. There will be no class on Monday. Turn in your homework (both the 3.9 problem and the special assignment) in my mailbox that can be found in the Olin Hall divisional office near the Sheehan Gallery. If you are curious, my absence is due to the fact that my son has a dance performance in Seattle over the weekend: see `https://www.olympicballet.org/obthome/performance-season/`

for Wednesday, April 18

1. Read Section 4.1. These ideas may seem simple (and they are probably quite familiar) but do give them careful thought, especially when the sets A and B are not sets of numbers.
2. Be prepared to discuss exercises 1–6 in Section 4.1; at the beginning of class, I will be assigning students to write their solutions on the board. I will not be collecting any solutions to exercises from this section.

for Friday, April 20

1. Read Section 4.2. Be prepared to discuss all of the exercises in Section 4.2. Once again, I will be assigning students (at the beginning of class) to write their solutions on the board. You should be able to do exercise 4 without technology; note that parts (b) and (d) require careful reasoning (and perhaps a graph) to justify your answers. For exercise 7, you should give a careful (read *very* careful) "chasing points" proof for each of the two inclusions you need to verify.
2. Turn in a solution for exercise 7 in Section 4.2.

for Monday, April 23

1. Read Section 4.3 and think carefully about these new terms.
2. Be prepared to discuss the exercises in Section 4.3. Many of these exercises request examples or ask simple questions so be prepared to present your ideas at the board. For exercise 10, give a direct proof that uses the standard definition of surjective functions; avoid using induced set functions. This should be a "follow your nose" type proof but do write the details clearly, probably in five short sentences.
3. Turn in solutions for exercises 8 and 10 from Section 4.3.

for Wednesday, April 25

1. Read Section 4.4 (it is very short).
2. Be prepared to discuss the exercises in Section 4.4; this means that you should spend some time thinking about each of the exercises.
3. Turn in a solution for exercise 5 from Section 4.4. Hopefully, I discussed the general idea behind this problem in class. Your table should look a bit like the multiplication table for $\mathbb{U}_9$ on page 77. In this case, you should end up with a matrix of letters. The product $a * b$ means the left side column entry a times the upper row entry b.
4. Special assignment #5 is due at the beginning of class.

for Friday, April 27

1. Read Section 4.6. (We are omitting Section 4.5.)
2. Be prepared to discuss exercises 1, 2, 3, 9, and 10 from Section 4.6.
3. Turn in a solution for part (b) of exercise 12. A proof that f is injective should follow easily from previous results. There are several ways to prove that f is surjective but I request that you attempt the following approach. Since a and b are relatively prime, there exist integers u and v such that $au+bv = 1$. Show that a preimage for $([r],[s])$ is $[aus + bvr]$.

for Monday, April 30

1. Read Section 4.7 very carefully and think deeply about the ideas that are discussed. This is a longer section and it contains some rather strange ideas concerning different sizes of infinity. If necessary, you can skim the proofs of Theorems 4.32 and 4.33.
2. Be prepared to discuss exercises 1, 4, 7, 8, and 10 in Section 4.7. There are some simple computational exercises, some basic logic exercises, and some challenging exercises. I will not be collecting any of these solutions but we will discuss them in class at length, probably with students at the board.
3. Spend some time over the weekend reviewing the material that we have covered since the last exam, namely Sections 3.4–9 and Sections 4.1–4, 6, and looking over the review problems. There are quite a few definitions and theorems that you need to know; not in the sense of quoting them verbatim but by being able to work with them and apply them. You should also have a rough idea about how to prove some of the more complicated results, even without knowing the full details of the proof. The following is a list of explicit proofs that I may ask you to give on the exam.

   a. Theorem 3.14 (relatively prime numbers and the concept of divides)
   b. Theorem 3.19 (relatively prime and existence of inverses)
   c. Theorem 3.35 (value of $\phi(p^e)$)
   d. Theorem 3.39 ($[1]$ and $[-1]$ are the only elements in $\mathbb{U}_p$ that are their own inverses)
   e. Theorem 4.9 (properties of the induced set function f^{-1})
   f. Theorem 4.13 (composition of injective functions is injective)

   This is not intended to be an exercise in memorization; if you interpret it in that way you are missing the point. For each of these results, you should know the basic idea behind the proof and use this knowledge (along with your improving ability to write mathematics) to write out the details of the proof.

   In summary, reread the sections, thinking about examples and concepts, look over the exercises, trying to solve them anew without looking at your notes, practice some congruence computations to brush up on your arithmetic skills, and carefully ponder the third exam prep handout. Bring any questions you have to class on the day of the review or stop by office hours any day before the exam.

for Wednesday, May 2

1. We have a test covering Sections 4–9 of Chapter 3 and Sections 1–4, 6 of Chapter 4.

for Friday, May 4

1. Do items 1 and 2 from the Monday assignment.

for Monday, May 7

1. Read Section 4.8.
2. Be prepared to discuss exercises 2, 5, 8, 9, and 10 in Section 4.8.

for Monday, May 14

1. The comprehensive final exam is scheduled for 9:00 AM in our usual classroom. You will have three hours for the exam if you so desire. A few details appear below.
2. Begin your review by flipping through the textbook to remind yourself of the topics we have covered this semester. Hopefully, you will be surprised how much of it is familiar to you. The final exam is comprehensive so look over all of the sections except for 4.5.
3. I have decided not to have you learn specific proofs for the final exam. I do expect that you can write proofs of simpler facts (such as Theorem 2.7(f)) that we have used often, but I do not expect you to remember how to recreate the more involved proofs (such as the proof of Division Algorithm).
4. A list of the main topics (perhaps not exhaustive but it is close) we have discussed this semester appears on the next page. To prepare for the exam, after making sure you are familiar with the ideas, go over the exercises, special assignments, and previous exams that we have worked on during the semester. This seems like a lot (and it probably is), but the idea is to be able to use your newly acquired skills to solve problems and write proofs clearly.
5. At the site `http://people.whitman.edu/~gordon/mathwrittens.html`, you can click on the Intro to Higher Math link for some further information and some problems to try. Our final exam will be more involved than the information presented here for math writtens, but it is one place to start.

1. the five logical connectives
2. tautologies, especially contraposition and modus ponens
3. converse and contrapositive
4. existential and universal quantifiers
5. DeMorgan's Laws and negations of statements, including definitions
6. sets and set operations
7. Cartesian product of sets
8. partitions of sets and power sets
9. equivalence relations and their properties
10. definition and properties of $n|a$
11. the Principle of Mathematical Induction (both versions)
12. the Well-Ordering Property of the positive integers
13. the Binomial Theorem
14. the Arithmetic Mean / Geometric Mean Inequality
15. Fibonacci numbers
16. the Division Algorithm
17. numbers such as $\sqrt{2}$ are irrational
18. the set of primes is infinite
19. notion of congruence and its properties
20. greatest common divisors and least common multiples
21. the Euclidean Algorithm
22. the spaces $\mathbb{Z}_n$ and $\mathbb{U}_n$
23. relatively prime integers
24. the Fundamental Theorem of Arithmetic
25. the Euler ϕ function
26. Wilson's Theorem
27. Euler's Theorem and Fermat's Little Theorem
28. quadratic residues
29. Euler's criterion
30. the Quadratic Reciprocity Theorem
31. conditions for an integer to be a sum of two squares
32. definition of a function, including domain and codomain
33. induced set functions and their properties
34. injective, surjective and bijective functions
35. cardinality of a set
36. finite and infinite sets, countable and uncountable sets
37. $\mathbb{Q}$ is a countable set and $\mathbb{R}$ is an uncountable set