Math 456 Assignments for Spring 2025 (due on given date)

for Tuesday, January 21

1. We will spend most of our time working through Chapters 7 and 8 in the text (R. Gordon, *Real Analysis, A First Course*, 2nd ed.). We will treat this as a Tu/Th class in terms of due dates for assignments and we can meet at a convenient time on those days (to be specified soon) to discuss the material. Homework will be assigned every class period. These assignments will include a fair amount of reading and some problems; a few of these will be specified to be written up carefully and submitted for grading. For these assignments, you can contact me if you need some help. There will be several assignments during the semester, sometimes assigned a week in advance, for which you will be required to work alone, that is, no help from other students, books, or the Internet.

   There will be two exams during the semester (the week beginning Mar. 26 and the week beginning Apr. 28, with specific dates and timing to be determined) and a comprehensive final exam (scheduled at a suitable time for all of you during finals week). Each of the hour exams represents 20% of your course grade and the final exam represents another 25% of your course grade. The regular graded homework counts as 20% and the special homework (the problems you must do on your own) as 15% of your course grade. I will make an attempt to keep you posted on your grade for the course as the semester progresses.

for Thursday, January 23

1. Flip through Chapter 6 of the textbook, noting all of the theorems and working through some of the proofs. Read the proofs of Theorems 6.9, 6.12, and 6.18 carefully, recording any questions that arise.
2. Work through Exercises 8 and 10 in Section 6.4.
3. Turn in a solution for Exercise 6.4.10 for the special case $p = 3$ and $q = 2$. You may refer to Exercise 7 without giving a proof of that exercise.

for Tuesday, January 28

1. Read Section 7.1. Study the examples very carefully and do your best to understand this new concept.
2. Look over all of the exercises in Section 7.1 to get a sense for these problems then do some of the ones that look interesting to you.
3. Turn in solutions for Exercises 5 and 14 in Section 7.1.

for Thursday, January 30

1. Read Section 7.2. Uniform convergence is a very important topic in real analysis.
2. Look over all of the exercises in Section 7.2 to get a sense for these problems then focus on exercises 3, 4, 5, 6, 7, 9, and 14. Note that Exercise 14 provides a counterexample for the converse of the Weierstrass M-test.
3. Turn in solutions for exercises 4c and 5 in Section 7.2.

for Tuesday, February 4

1. Read Section 7.3.
2. Do exercises 1, 2, 3, 5, 9, 13, 14, and 18 in Section 7.3.
3. Turn in solutions for exercises 5b and 5c in Section 7.3. For part (b), the hypothesis (in case it is not clear) is that all of the functions f_n are continuous and that the sequence $\{f_n\}$ is equicontinuous. For part (c), assume that $I = [a, b]$.
4. (IMPORTANT) There is a special assignment due next Tuesday; see that date for the assignment.

for Thursday, February 6

1. Read Section 7.4. Hopefully, much of this material is familiar to you from calculus.
2. Do exercises 1, 2, 3, 4, 5, 12, 14, 15, and 16 in Section 7.4.
3. Turn in solutions for exercises 2c and 20 in Section 7.4; see the bottom of page 263 for a suggestion for 20.

for Tuesday, February 11

1. Read Section 7.5, keeping track of any questions that arise.
2. Do exercise 7 in Section 7.5.
3. A special assignment is due on this date. The three problems are listed below:

i. Exercise 6 in Section 7.3. An outline for this proof is there to guide you, but you should write out the proof without reference to the various steps.

ii. Consider the function f defined by $\displaystyle\sum_{k=0}^{\infty} \frac{k+2}{3^k}(x-4)^k$. Find the domain of f and the value of $f^{(79)}(4)$. In addition, represent the function f as a rational function (that is, express the sum of the power series as a more familiar function).

iii. Exercise 12 in Section 7.7.

for Thursday, February 13

1. Read Section 7.5 again, noting any questions that arise.
2. Do exercises 1, 12, 14, and 18 in Section 7.5.
3. Turn in solutions for exercises 1 and 14 in Section 7.5.

for Tuesday, February 18

1. Read Section 7.6 through the statement of Theorem 7.29.
2. Do exercises 1, 3, 4, 5a, 6, and 8 in Section 7.6.
3. Turn in solutions for Exercises 1b and 3 in Section 7.6.
4. There is a special assignment due next Tuesday; see that date for the assignment.

for Thursday, February 20

0. Technically, there is no class today due to the Power and Privilege Symposium. If this creates any difficulty, please let me know.
1. Finish reading Section 7.6. Do your best to make sense of the proofs for these two important results.
2. Do exercises 19, 20, 21, and 22 in Section 7.6.
3. Turn in solutions for exercises 20b and 22 in Section 7.6. For exercise 22, you may use the result of Exercise 5.3.10.

for Tuesday, February 25

1. Look over Chapter 7 to review the key ideas and results.
2. Do exercises 1, 7, 11, 27, 28, and 33 in Section 7.7. If the ideas in Exercise 7.7.6 intrigue you, spend some time pondering this problem.
3. The second special assignment is due this day. The three problems are listed below:

i. Suppose that the sequence $\{f_n\}$ converges uniformly to a continuous function f on $[a, b]$ and that the equation $f_n(x) = 0$ has exactly one solution in $[a, b]$ for each positive integer n. Prove that the equation $f(x) = 0$ has a solution in $[a, b]$.

ii. Prove that the Cauchy product of the series $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}$ with itself converges. Letting h_k represent the kth partial sum of the harmonic series, recall that $h_k = \gamma_k + \ln k$ and $\{\gamma_k\}$ converges (see page 177).

iii. Exercise 7.6.24, parts (a) and (b) only.

for Thursday, February 27

1. Read the introduction to Chapter 8 and the portion of Section 8.1 through the proof of Theorem 8.5.
2. Do exercises 1 through 10, excluding 7, in Section 8.1.
3. Turn in solutions for exercises 6 and 8 in Section 8.1. These two exercises are intended to have short and simple solutions.