Total: Name:

Math 125a **Final Exam** **Fall 2016**

Write neat, concise, and accurate solutions to each of the problems below—I will not give partial credit for steps I cannot follow. In fact, partial credit will be rather minimal for this exam so pay careful attention to details. Include all relevant details, use correct notation, and finish problems with a complete sentence when appropriate. No electronic devices are allowed for this exam.The first twenty problems are worth three points each and the remaining four problems are worth five points each.

Find and simplify the derivative of each function given in problems 1–5.

1. $f(x) = \dfrac{x}{4x+3}$

2. $g(x) = \sqrt{x^4 + 3x^2 + 7}$

3. $h(\theta) = \cos^3(2\theta)$

4. $F(t) = t^2 \sin(3t)$

5. $w(x) = \ln(\sec x + \tan x)$

6. State the Extreme Value Theorem.

7. State the definition for $\lim_{x \to c} f(x) = L$.

8. Find the cubic polynomial that best approximates the function $f(x) = x^{1/5}$ at the point $a = 1$.

9. Evaluate $\lim_{x\to 0}\big(1+2\sin(3x)\big)^{1/x}$.

10. Find the maximum and minimum outputs of the function $f(x) = 4x - x^{4/3}$ on the interval $[1, 64]$.

11. Find a function $g(t)$ such that $g'(t) = 4g(t)$ and $g(0) = 200$.

12. Determine the largest interval on which the function f defined by $f(x) = x^4 e^{-x}$ is increasing. (Be certain to pay careful attention to the endpoints of your interval.)

13. For the curve defined by $x^3 + 3xy - 2y^5 - x + 5y = 6$, find $\dfrac{dy}{dx}$.

14. Find the (x, y) coordinates of the inflection point of the cubic curve $y = x^3 + 6x^2 + 3x + 2$. Explain how you know it is an inflection point.

15. Find an equation for the line tangent to the graph of $f(x) = 7x + \frac{12}{x}$ when $x = 2$.

16. Use Newton's method to approximate $\sqrt[3]{61}$. After determining the proper equation, start with an appropriate integer as your initial guess and compute the first iteration only. Give your answer as a simplified rational number.

17. Evaluate $\lim_{x \to 0} \frac{\arctan(3x)}{e^{2x} - 1}$.

18. Use the definition of the derivative to find the derivative of the function $f(x) = \sqrt{x}$.

19. The height $h(t)$ in feet of a beanstalk at time t hours is given by $h(t) = 3t + 28\sqrt{5t+9} - 84$. What is the instantaneous growth rate of the beanstalk at $t = 8$ hours? Include appropriate units for your answer.

20. Sketch one (careful and clear) graph of a function f that satisfies all of the following properties.

a) the domain of f is all real numbers (be careful with this part!)

b) f is continuous except when $x = -1$ and $x = 3$

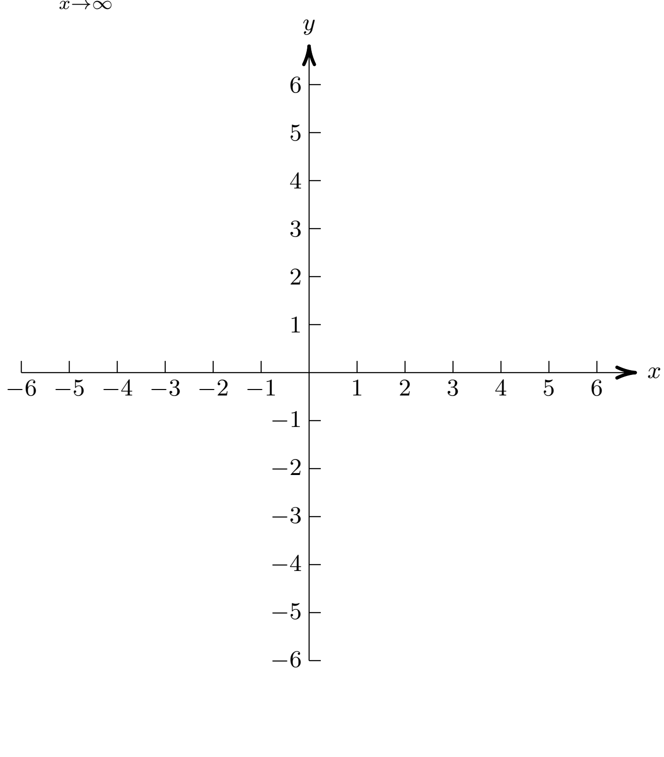
c) f is differentiable except when $x = -1$, $x = 1$, and $x = 3$

d) $\lim_{x \to -1} f(x) = \infty$

e) f has different one-sided limits at 3

f) f does not have a vertical asymptote at $x = 3$

g) $\lim_{x \to \infty} f(x) = 2$



21. Suppose that a particle is moving in a straight line with a velocity of 120 m/sec. At time $t = 0$, the particle begins to decelerate at the variable rate of $6\,t$ m/sec^2 and does so for 4 seconds. After this time, it then continues to decelerate at the constant rate of 24 m/sec^2. What is the total distance traveled by the particle from the instant the deceleration begins to the instant that the particle stops?

22. An open top rectangular box with a square base is to be constructed using materials that cost \$2 per square foot for the sides and \$5 per square foot for the base. Find the minimum cost for the box if the volume must be 80 cubic feet. (Remember to include all aspects of a solution to an optimization problem such as this.)

23. A large tank contains 400 gallons of brine with a concentration of 0.25 pounds of salt per gallon of water. A brine containing one pound of salt per gallon of water runs into the tank at the rate of four gallons per minute, and the well-stirred mixture runs out of the tank at the same rate. When will the concentration of the brine in the tank be 0.75 pounds of salt per gallon of water? (You may use $\ln 2 \approx 0.7$ and $\ln 3 \approx 1.1$ in your calculations.)

24. An inverted cone (vertex down) with a radius of five feet and height of fifteen feet is being filled with water at a rate of three cubic feet per minute. How fast is the water level rising when the water inside the cone is three feet deep?