0. Find all values of x that satisfy the equation $2x^4 - 7x^2 = 20$.

Let $y = x^2$. Then $y \geq 0$ and $2y^2 - 7y - 20 = 0$. By the quadratic formula, we have

$$y = \frac{7 \pm \sqrt{49 - 4(2)(-20)}}{4} = \frac{1}{4}\left(7 \pm \sqrt{209}\right).$$

Since $y \geq 0$, we can only use the plus sign value. It follows that

$$x^2 = y = \frac{1}{4}\left(7 + \sqrt{209}\right) \quad \text{and thus} \quad x = \pm\frac{1}{2}\sqrt{7 + \sqrt{209}}.$$

The solutions to the equation are $\frac{1}{2}\sqrt{7 + \sqrt{209}}$ and $-\frac{1}{2}\sqrt{7 + \sqrt{209}}$.

1. Find all values of x that satisfy the equation $2x = 1 + \dfrac{4}{x}$.

$2x = 1 + \dfrac{4}{x}$ use quadratic formula

$2x^2 = x + 4$ $x = \dfrac{1 \pm \sqrt{1 + 32}}{4}$

$2x^2 - x - 4 = 0$

The solutions are $x = \dfrac{1}{4}\left(1 \pm \sqrt{33}\right)$.

2. Find all values of x that satisfy the equation $x^4 = 2x^2 + 1$.

Let $y = x^2$, then $y \geq 0$ and $y^2 = 2y + 1$.

$y^2 - 2y + 1 = 1 + 1$ complete the square

$(y - 1)^2 = 2$

$y - 1 = \pm\sqrt{2}$

$y = 1 \pm \sqrt{2}$ $1 - \sqrt{2} < 0$ so must reject

$x^2 = y = 1 + \sqrt{2}$

The solutions are $x = \pm\sqrt{1 + \sqrt{2}}$.

3. Find all values of x that satisfy the equation $x - 3\sqrt{x} = 10$.

I. Let $y = \sqrt{x}$. Then $y \geq 0$ and

$y^2 - 3y - 10 = 0$

$(y - 5)(y + 2) = 0$

$y = 5$ since $y = -2$ fails

$\sqrt{x} = 5 \Rightarrow x = 25$

II. $x - 10 = 3\sqrt{x}$, need $x \geq 10$ (since $3\sqrt{x} \geq 0$)

$(x - 10)^2 = 9x$

$x^2 - 29x + 100 = 0$

$(x - 25)(x - 4) = 0$

$x = 25$ only valid solution

The only solution to the equation is $x = 25$.

4. Find all values of x that satisfy the equation $25 - 8e^{-x/3} = 23$. use properties of $\ln$

$25 - 8e^{-x/3} = 23$

$2 = 8e^{-x/3}$

$e^{-x/3} = \frac{1}{4}$

$-\frac{x}{3} = \ln \frac{1}{4}$

$x = -3\ln \frac{1}{4}$

$x = 6\ln 2$

OR

$\frac{1}{e^{x/3}} = \frac{1}{4}$

$e^{x/3} = 4$

$\frac{x}{3} = \ln 4 = \ln 2^2 = 2\ln 2$

$x = 6\ln 2$

$\longrightarrow = 3\ln\left(\frac{1}{4}\right)^{-1} = 3\ln 4$

The solution is $x = 6\ln 2$.

5. Find all values of x that satisfy the equation $e^x = \dfrac{8e^x - 12}{e^x}$.

Let $y = e^x$. Then $y > 0$ and

$y = \frac{8y - 12}{y} \Rightarrow y^2 - 8y + 12 = 0$

$(y - 2)(y - 6) = 0$

$y = 2$ or $y = 6$

$x = \ln 2$ or $x = \ln 6$

The solutions are $\ln 2$ and $\ln 6$.