Name: due August 26 by 3:00 pm Pacific

Math 126 **Homework Assignment 1** **Fall 2020**

1. Find an equation for the line tangent to the graph of $y = 3x^2 + \frac{4}{x}$ when $x = 1$.

$y = 3x^2 + \frac{4}{x}$

when $x = 1$

$y = 7$

$\frac{dy}{dx} = 6x - \frac{4}{x^2}$

$\frac{dy}{dx} = 2$

point $(1, 7)$, slope 2

$y - 7 = 2(x-1)$

An equation for the tangent line is $y = 2x + 5$.

2. Evaluate $\lim_{x\to\infty} \frac{10x^2 + 2x + 9}{\sqrt{5x^4 + 3x^2 + 7}}$.

highest power in denominator is x^2

$$\lim_{x\to\infty} \frac{10x^2 + 2x + 9}{\sqrt{5x^4+3x^2+7}} = \lim_{x\to\infty} \frac{10x^2+2x+9}{\sqrt{5x^4+3x^2+7}} \cdot \frac{1/x^2}{1/x^2}$$

$$= \lim_{x\to\infty} \frac{10 + \frac{2}{x} + 9/x^2}{\sqrt{5 + \frac{3}{x^2} + \frac{7}{x^4}}}$$

$$= \frac{10}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$$

$$= 2\sqrt{5}.$$

(Note that the equation is a sentence.)

3. Use the definition of the derivative to find the derivative of the function $f(x) = \dfrac{1}{x}$.

$$f'(x) = \lim_{v \to x} \frac{f(v) - f(x)}{v - x}$$
$$= \lim_{v \to x} \frac{\frac{1}{v} - \frac{1}{x}}{v - x}$$
$$= \lim_{v \to x} \frac{x - v}{vx(v - x)}$$
$$= \lim_{v \to x} \frac{-1}{vx}$$
$$= -\frac{1}{x^2}.$$

or

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$
$$= \lim_{h \to 0} \frac{1}{h} \cdot \frac{x - (x+h)}{(x+h)x}$$
$$= \lim_{h \to 0} \frac{-1}{(x+h)x}$$
$$= -\frac{1}{x^2}.$$

4. Find and simplify the derivative of the function f defined by $f(x) = \dfrac{\sqrt{x^2+6}}{x}$.

Using the quotient rule and chain rule, we have

$$f'(x) = \frac{x \cdot \frac{1}{2}(x^2+6)^{-1/2} \cdot 2x - (x^2+6)^{1/2} \cdot 1}{x^2}$$
$$= \frac{(x^2+6)^{-1/2}\left[x^2 - (x^2+6)\right]}{x^2}$$
$$= \frac{-6}{x^2\sqrt{x^2+6}}.$$