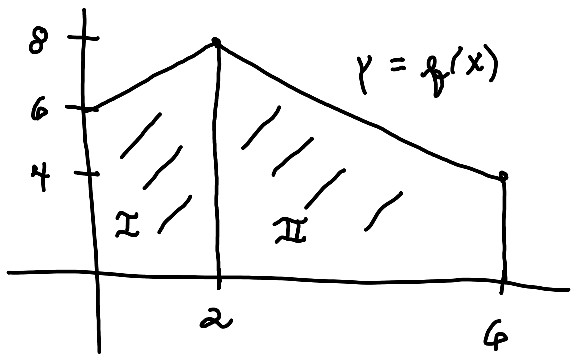
Name: due August 28 by 3:00 pm Pacific

Math 126 **Homework Assignment 2** **Fall 2020**

1. Use simple facts from geometry to find the area under the graph of each function and above the x-axis on the given interval. Include a sketch of the region whose area is being computed.

   a) $f(x) = 8 - |x - 2|$ on $[0, 6]$



two trapezoids

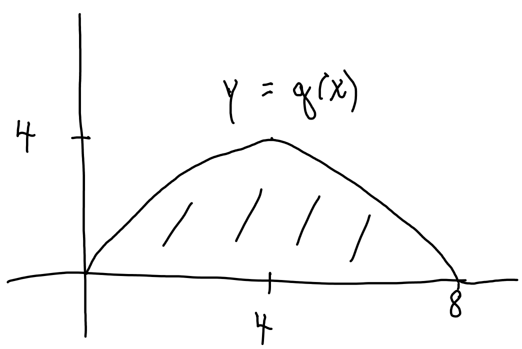
$A_{I} = \frac{1}{2}(6+8)\cdot 2 = 14$

$A_{II} = \frac{1}{2}(8+4)\cdot 4 = 24$

total area 38

The area of this region is 38 square units.

   b) $g(x) = \sqrt{8x - x^2}$ on $[0, 8]$



$y = \sqrt{8x - x^2}$

$y^2 = 8x - x^2$

$x^2 - 8x + 16 + y^2 = 16$ (complete the square)

$(x-4)^2 + y^2 = 4^2$

circle with center $(4, 0)$, radius 4

region is a semicircle so $A = \frac{1}{2}\cdot \pi \cdot 4^2 = 8\pi$

The area of the region is 8π square units.

2. Use the definition of the integral to express the given integral as a limit of a sum.

a) $\int_1^3 (x^2+6x)\,dx$ $= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(1+\frac{2i}{n}\right)^2 + 6\left(1+\frac{2i}{n}\right)\right)\frac{2}{n}$

$a = 1$

$b = 3$

$\frac{b-a}{n} = \frac{2}{n}$

$a + i \cdot \frac{b-a}{n} = 1 + \frac{2i}{n}$

this becomes x

b) $\int_0^{\pi/2} \cos x\,dx$ $= \lim_{n\to\infty} \sum_{i=1}^{n} \cos\left(\frac{\pi i}{2n}\right)\frac{\pi}{2n}$

$a = 0$

$b = \frac{\pi}{2}$

$\frac{b-a}{n} = \frac{\pi}{2n}$

$a + i \cdot \frac{b-a}{n} = \frac{\pi i}{2n}$

this replaces x in the function

3. Use the definition of the integral to express the given limit as an integral. For part (a), give two different integral representations for the limit. Start with a little bit of factoring for part (b).

a) $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{4+\frac{3i}{n}} \cdot \frac{3}{n}$

I $b - a = 3$

$a = 4,\ b = 7$

$a + i\,\frac{b-a}{n} = 4 + \frac{3i}{n}$

this is the x value

$f(x) = \sqrt{x}$

II $b - a = 3$

$a = 0,\ b = 3$

$a + i \cdot \frac{b-a}{n} = \frac{3i}{n}$

with this x value

$f(x) = \sqrt{4+x}$

The limit represents $\int_4^7 \sqrt{x}\,dx$ and also $\int_0^3 \sqrt{4+x}\,dx$.

b) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{n}{n^2+i^2}$

$\frac{n}{n^2+i^2} = \frac{n}{n^2\left(1+\frac{i^2}{n^2}\right)} = \frac{1}{1+\left(\frac{i}{n}\right)^2} \cdot \frac{1}{n}$

$b - a = 1$

$a = 0,\ b = 1$

$a + i \cdot \frac{b-a}{n} = \frac{i}{n}$ (x value)

$f(x) = \frac{1}{1+x^2}$

It follows that

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{n}{n^2+i^2} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{1+\left(\frac{i}{n}\right)^2} \cdot \frac{1}{n} = \int_0^1 \frac{1}{1+x^2}\,dx.$$