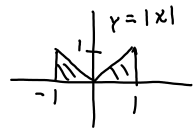
Name: due August 31 by 3:00 pm Pacific

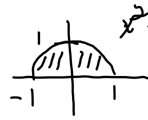
Math 126 **Homework Assignment 3** **Fall 2020**

1. Evaluate $\int_{-1}^{1} \left(4|x| + 6\sqrt{1-x^2}\right) dx$.

$$\int_{-1}^{1}\left(4|x| + 6\sqrt{1-x^2}\right)dx = 4\int_{-1}^{1}|x|\,dx + 6\int_{-1}^{1}\sqrt{1-x^2}\,dx$$

$\int_{-1}^{1}|x|\,dx$ is the area 

which is 1

$\int_{-1}^{1}\sqrt{1-x^2}\,dx$ is the area 

$x^2+y^2=1$ which is $\frac{1}{2}\pi$

so we have $4\cdot 1 + 6\cdot\frac{\pi}{2} = 4 + 3\pi$

The value of the integral is $4 + 3\pi$.

2. Referring to Exercise 4 in Section 2.5, find the value of $\int_{1}^{2}\big(3f(x) - g(x)\big)\,dx$.

$\int_{1}^{3} f(x)\,dx = 6$

$\int_{2}^{3} f(x)\,dx = 2$

$\Longrightarrow \int_{1}^{2} f(x)\,dx = \int_{1}^{3} f(x)\,dx - \int_{2}^{3} f(x)\,dx = 4$

$\int_{1}^{3} g(x)\,dx = 10$

$\int_{2}^{3} g(x)\,dx = -3$

$\Longrightarrow \int_{1}^{2} g(x)\,dx = \int_{1}^{3} g(x)\,dx - \int_{2}^{3} g(x)\,dx = 13$

Therefore $\int_{1}^{2}\big(3f(x) - g(x)\big)\,dx = 3\int_{1}^{2} f(x)\,dx - \int_{1}^{2} g(x)\,dx$

$= 3\cdot 4 - 13$

$= -1$

3. Without evaluating either of the integrals, determine which integral is larger and explain why.

$$\int_1^6 \sqrt[3]{x^6 + x + 1}\, dx, \quad \int_1^6 x^2\, dx.$$

since $\sqrt[3]{x^6 + x + 1} > \sqrt[3]{x^6} = x^2$ for all x in $[1, 6]$,

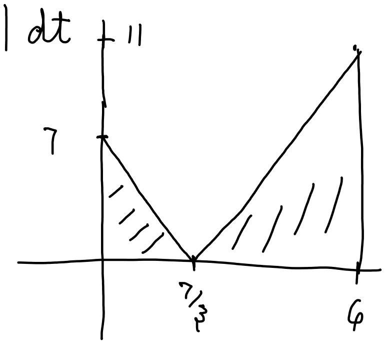
we know that $\int_1^6 \sqrt[3]{x^4 + x + 1}\, dx > \int_1^6 x^2\, dx$.

(see property (6) of integrals in section 2.5.)

4. Suppose that $v(t) = 7 - 3t$ gives the velocity in meters per second of a particle at time t seconds. Find the total distance traveled by the particle during the time interval $0 \leq t \leq 6$.

distance $= \int_0^6 |v(t)|\, dt = \int_0^6 |7 - 3t|\, dt$

interpret integral as area



area $= \frac{1}{2} \cdot 7 \cdot \frac{7}{3} + \frac{1}{2} \cdot \frac{11}{3} \cdot 11$

$= \frac{49 + 121}{6} = \frac{170}{6} = \frac{85}{3}$

$\int_0^6 |7 - 3t|\, dt = \frac{85}{3}$

The particle travels $\frac{85}{3}$ meters during the time interval $0 \leq t \leq 6$ seconds.