Name: due September 2 by 3:00 pm Pacific

Math 126 **Homework Assignment 4** **Fall 2020**

1. Find the derivative of the function f defined by $f(x) = \int_0^{3x^2} \ln(1+t^2)\,dt$.

Using the FTC and the chain rule, we find that

$$f'(x) = \ln(1+(3x^2)^2)\cdot 6x$$
$$= 6x\ln(1+9x^4).$$

2. Determine $F''(\pi/18)$ given that $F(x) = \int_x^{2\pi} f(t)\,dt$ and $f(x) = \int_1^{3x} \frac{\sin t}{t}\,dt$.

$$F(x) = -\int_{2\pi}^{x} f(t)\,dt \qquad f'(x) = \frac{\sin(3x)}{3x}\cdot 3 \text{ by FTC}$$
$$= \frac{\sin(3x)}{x}$$

$F'(x) = -f(x)$ by FTC

$F''(x) = -f'(x)$

It follows that $F''(x) = -\frac{\sin(3x)}{x}$ and thus

$$F''\left(\frac{\pi}{18}\right) = -\frac{\sin(\pi/6)}{\pi/18} = -\frac{18}{\pi}\cdot\frac{1}{2} = -\frac{9}{\pi}.$$

3. Evaluate $\lim_{x\to 0} \frac{1}{x^4} \int_0^x \sin(6t^3)\, dt$.

$$\lim_{x\to 0} \frac{1}{x^4} \int_0^x \sin(6t^3)\,dt = \lim_{x\to 0} \frac{\int_0^x \sin(6t^3)\,dt}{x^4} \qquad \frac{0}{0} \text{ form}$$

$$= \lim_{x\to 0} \frac{\sin(6x^3)}{4x^3} \qquad \text{L'Hopital's Rule, FTC}$$

$$= \lim_{x\to 0} \frac{18x^2\cos(6x^3)}{12x^2} \qquad \text{L'Hopital's Rule}$$

$$= \lim_{x\to 0} \frac{3}{2}\cos(6x^3) \qquad \cos 0 = 1$$

$$= \frac{3}{2}.$$

4. Find an integral expression for a function f such that $f(4) = 0$ and $f'(x) = e^{-2x^2}$.

By the FTC, the function $f(x) = \int_4^x e^{-2t^2}\,dt$ has the desired properties.