Name: due September 7 by 3:00 pm Pacific

Math 126 **Homework Assignment 6** **Fall 2020**

1. Evaluate $\int \frac{24}{(3x+1)^3}\,dx$.

guess: $(3x+1)^{-2}$

check: $\frac{d}{dx}(3x+1)^{-2} = -2(3x+1)^{-3} \cdot 3 = \frac{-6}{(3x+1)^3}$

need $\cdot(-4)$

$$\int \frac{24}{(3x+1)^3}\,dx = \frac{-4}{(3x+1)^2} + C.$$

2. Evaluate $\int \frac{6}{\sqrt{4x+7}}\,dx$.

guess: $(4x+7)^{1/2}$

check: $\frac{d}{dx}(4x+7)^{1/2} = \frac{1}{2}(4x+7)^{-1/2} \cdot 4 = \frac{2}{\sqrt{4x+7}}$

need $\cdot 3$

$$\int \frac{6}{\sqrt{4x+7}}\,dx = 3\sqrt{4x+7} + C$$

3. Evaluate $\int_0^1 \frac{8x+4}{x^2+x+1}\,dx$.

Using mental guess and check to find an antiderivative,

$$\int_0^1 \frac{8x+4}{x^2+x+1}\,dx = 4\ln|x^2+x+1|\Big|_0^1 = 4\ln 3. \qquad [\ln 1 = 0]$$

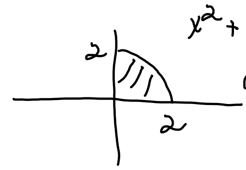
4. Evaluate $\int_0^2 (2x-3)\sqrt{4-x^2}\,dx$. (Carefully use the distributive property to split the integral into two integrals, then think clearly about the best way to evaluate each of the integrals.)

$$\int_0^2 (2x-3)\sqrt{4-x^2}\,dx = \int_0^2 2x\sqrt{4-x^2}\,dx - 3\int_0^2 \sqrt{4-x^2}\,dx$$

do each integral separately

$$\int_0^2 2x(4-x^2)^{1/2}\,dx = -\frac{2}{3}(4-x^2)^{3/2}\Big|_0^2 = 0 - \left(-\frac{2}{3}\cdot 4^{3/2}\right) = \frac{16}{3}$$

$\left[-\frac{2}{3}\cdot\frac{3}{2}\cdot(4-x^2)^{1/2}(-2x) \text{ for mental guess and check}\right]$

$\int_0^2 \sqrt{4-x^2}\,dx$ represents area 

$x^2+y^2=4$ circle $\quad \frac{1}{4}\cdot\pi\cdot 2^2 = \pi$

It follows that

$$\int_0^2 (2x-3)\sqrt{4-x^2}\,dx = \frac{16}{3} - 3\pi.$$