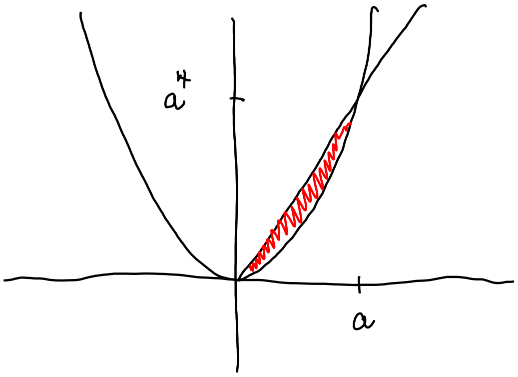
Name: due September 18 by 3:00 pm Pacific

Math 126 **Homework Assignment 9** **Fall 2020**

1. Find the area of the region bounded by the curves $y = x^4$ and $y = a^3x$, where a is a positive constant.



$x^4 = a^3x$

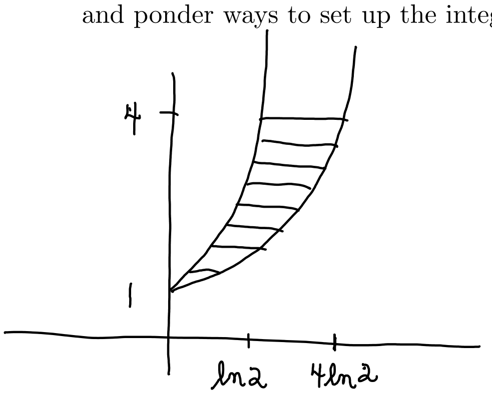
$x(x^3 - a^3) = 0$

$x = 0, \ x = a$ when the curves meet

$$A = \int_0^a (a^3x - x^4)\,dx = \left(\tfrac{1}{2}a^3x^2 - \tfrac{1}{5}x^5\right)\Big|_0^a = \left(\tfrac{1}{2} - \tfrac{1}{5}\right)a^5 = \tfrac{3}{10}a^5$$

The area of the bounded region is $\frac{3}{10}a^5$ square units.

2. Find the area of the region bounded by the curves $y = e^{2x}$, $y = e^{x/2}$, and $y = 4$. Sketch a careful graph and ponder ways to set up the integral before proceeding to evaluate the integral.



$y = e^{2x}$ upper

$x = \frac{1}{2}\ln y$ left

$y = e^{x/2}$ lower

$x = 2\ln y$ right

$$A = \int_1^4 \left(2\ln y - \tfrac{1}{2}\ln y\right)dy = \tfrac{3}{2}\int_1^4 \ln y\,dy$$

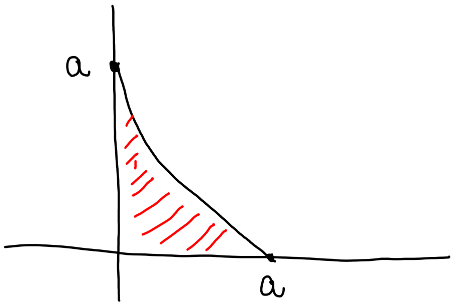
$$= \tfrac{3}{2}\left(y\ln y - y\right)\Big|_1^4$$ [from last assig]

$$= \tfrac{3}{2}\Big((4\ln 4 - 4) - (0 - 1)\Big)$$

$$= \tfrac{3}{2}(8\ln 2 - 3) = 12\ln 2 - \tfrac{9}{2} \approx 3.82$$

The area of the region is $12\ln 2 - \frac{9}{2}$ square units.

3. Let a be a positive constant. Find the area of the region bounded by the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ and the coordinate axes. If necessary, use an electronic device to obtain a graph of the curve; include a sketch of the region as part of your solution.



$$\sqrt{x} + \sqrt{y} = \sqrt{a}$$

$$\sqrt{y} = \sqrt{a} - \sqrt{x}$$

$$y = a - 2\sqrt{a}\sqrt{x} + x$$

$$A = \int_0^a \left(a - 2\sqrt{a}\sqrt{x} + x\right)dx = \left(ax - \frac{4}{3}\sqrt{a}\,x^{3/2} + \frac{1}{2}x^2\right)\Big|_0^a$$

$$= a^2 - \frac{4}{3}a^2 + \frac{1}{2}a^2 = \frac{1}{6}a^2$$

The area of the region is $\frac{a^2}{6}$ square units.