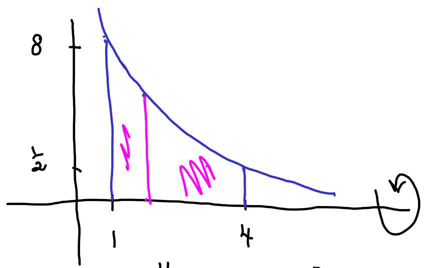
Name: due September 21 by 3:00 pm Pacific

Math 126 **Homework Assignment 10** **Fall 2020**

1. Let R be the region under the graph of $y = 8/x^2$ and above the x-axis on the interval $[1, 4]$. Find the volume of the solid that is generated when R is revolved around the x-axis.



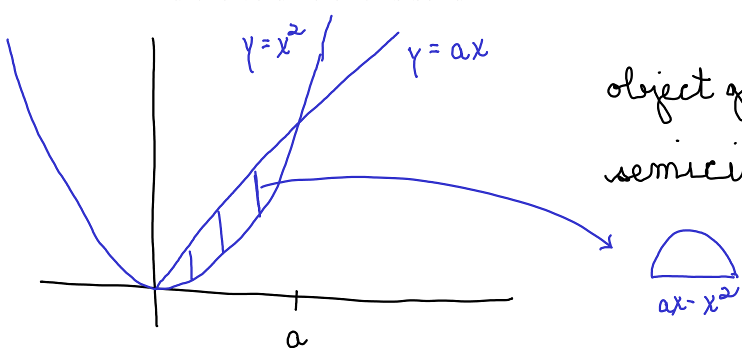
cross sections are disks, radius is $\frac{8}{x^2}$

$$V = \int_1^4 \pi \left(\frac{8}{x^2}\right)^2 dx = 64\pi \int_1^4 \frac{1}{x^4}\, dx$$

$$= 64\pi \left(-\frac{1}{3}\right)\left(\frac{1}{x^3}\Big|_1^4\right) = 64\pi\left(-\frac{1}{3}\right)\left(\frac{1}{64} - 1\right) = -\frac{1}{3}\pi(1-64) = \frac{63}{3}\pi$$

The volume of the generated solid is 21π cubic units.

2. Let a be a positive constant. Suppose that the base of a solid is the region bounded by the curves $y = x^2$ and $y = ax$ and that each cross section of the solid taken perpendicular to the x-axis is a semicircle. Find the volume of this solid.



object glued to the tablet

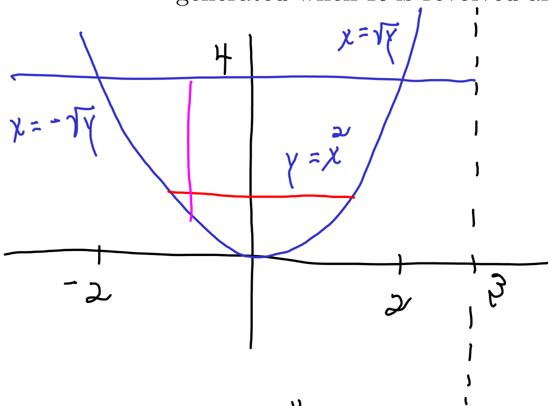
semicircle has diameter $ax - x^2$

$$V = \int_0^a \frac{1}{2}\pi\left(\frac{ax-x^2}{2}\right)^2 dx = \frac{\pi}{8}\int_0^a \left(a^2x^2 - 2ax^3 + x^4\right) dx$$

$$= \frac{\pi}{8}\left(\frac{1}{3}a^2x^3 - \frac{1}{2}ax^4 + \frac{1}{5}x^5\right)\Big|_0^a = \frac{\pi}{8}a^5\left(\frac{1}{3} - \frac{1}{2} + \frac{1}{5}\right) = \frac{\pi}{8}\cdot\frac{1}{30}a^5$$

The volume of the solid is $\frac{\pi a^5}{240}$ cubic units.

3. Let R be the region bounded by the curves $y = x^2$ and $y = 4$. Find the volume of the solid that is generated when R is revolved around (a) the line $x = 3$, (b) the line $y = 4$.



around $x = 3$, get washers

$3 + \sqrt{y}$ outer radius

$3 - \sqrt{y}$ inner radius

around $y = 4$, get disks

radius is $y = 4 - x^2$

$$V_{x=3} = \int_0^4 \left(\pi\left(3+\sqrt{y}\right)^2 - \pi\left(3-\sqrt{y}\right)^2\right) dy$$

$$= \pi \int_0^4 12\sqrt{y}\, dx = \pi \cdot 8\, y^{3/2} \Big|_0^4 = 64\pi$$

The volume of the solid when R is revolved around $x = 3$ is 64π cubic units.

$$V_{y=4} = \int_{-2}^{2} \pi\left(4 - x^2\right)^2 dx = 2\pi \int_0^2 \left(16 - 8x^2 + x^4\right) dx \qquad [\text{note symmetry}]$$

$$= 2\pi\left(16x - \frac{8}{3}x^3 + \frac{1}{5}x^5\right)\Big|_0^2 = 2\pi \cdot 32\left(1 - \frac{2}{3} + \frac{1}{5}\right) = 64\pi \cdot \frac{8}{15} = \frac{512\pi}{15}$$

The volume of the solid when R is revolved around $y = 4$ is $\frac{512\pi}{15}$ cubic units.