Name: due October 5 by 3:00 pm Pacific

Math 126 **Homework Assignment 15** **Fall 2020**

1. Evaluate $\int \frac{6x+7}{\sqrt{9-x^2}}\,dx$.

$$\int \frac{6x+7}{\sqrt{9-x^2}}\,dx = \int \left(\frac{6x}{\sqrt{9-x^2}} + \frac{7}{\sqrt{9-x^2}} \right) dx$$ split up

$$= -6\sqrt{9-x^2} + 7 \arcsin\left(\tfrac{x}{3}\right) + C.$$

[mental guess and check, basic formula]

2. Evaluate $\int \frac{2x-1}{x^2+6x+13}\,dx$.

$$\int \frac{2x-1}{x^2+6x+13}\,dx = \int \frac{2x-1}{(x+3)^2+4}\,dx$$ complete the square

let $u = x+3$
then $du = dx$
$x = u-3$

$$= \int \frac{2(u-3)-1}{u^2+4}\,du$$

$$= \int \left(\frac{2u}{u^2+4} - \frac{7}{u^2+4} \right) du$$

$$= \ln(u^2+4) - \frac{7}{2}\arctan\left(\frac{u}{2}\right) + C$$

$$= \ln(x^2+6x+13) - \frac{7}{2}\arctan\left(\frac{x+3}{2}\right) + C.$$

3. Evaluate $\int \frac{4x-3}{\sqrt{33+8x-x^2}}\,dx$.

$$\left\{ \begin{array}{l} -(x^2-8x+16)+16+33 \\ = -(x-4)^2+49 \end{array} \right\}$$

$$\int \frac{4x-3}{\sqrt{33+8x-x^2}}\,dx = \int \frac{4x-3}{\sqrt{49-(x-4)^2}}\,dx$$ complete the square

$$= \int \frac{4(u+4)-3}{\sqrt{49-u^2}}\,du$$ let $u = x-4$, then $x = u+4$, $dx = du$

$$= \int \left(\frac{4u}{\sqrt{49-u^2}} + \frac{13}{\sqrt{49-u^2}} \right) du$$

$$= -4\sqrt{49-u^2} + 13 \arcsin\left(\frac{u}{7}\right) + C$$

$$= -4\sqrt{33+8x-x^2} + 13 \arcsin\left(\frac{x-4}{7}\right) + C.$$

4. Evaluate $\int \frac{6x^3+14x+7}{x^2+9}\,dx$. use long division

$$\left[6x + \frac{-40x+7}{x^2+9} \right]$$

$$\begin{array}{r} 6x \\ x^2+9 \;\overline{)\; 6x^3+0x^2+14x+7} \\ 6x^3+\quad 54x \\ \hline -40x+7 \end{array}$$

$$\int \frac{6x^3+14x+7}{x^2+9}\,dx = \int \left(6x - \frac{40x}{x^2+9} + \frac{7}{x^2+9} \right) dx$$

$$= 3x^2 - 20\ln(x^2+9) + \frac{7}{3}\arctan\left(\frac{x}{3}\right) + C.$$