Name: due October 16 by 3:00 pm Pacific

Math 126 **Homework Assignment 18** **Fall 2020**

1. Find the limit of the sequence $\{\sqrt{k^2+k}-k\}$. There is no need to switch to the variable x in this case since algebra should be sufficient to find the limit.

$$\lim_{k\to\infty}\left(\sqrt{k^2+k}-k\right) = \lim_{k\to\infty}\frac{\left(\sqrt{k^2+k}-k\right)\left(\sqrt{k^2+k}+k\right)}{\sqrt{k^2+k}+k}$$

$$= \lim_{k\to\infty}\frac{k}{\sqrt{k^2+k}+k}\cdot\frac{1/k}{1/k}$$

$$= \lim_{k\to\infty}\frac{1}{\sqrt{1+1/k}+1}$$

$$= \frac{1}{2}$$

The limit of the sequence $\{\sqrt{k^2+k}-k\}$ is $\frac{1}{2}$.

2. Find the limit of the sequence $\{n\sin(\pi/n)\}$. If you choose to use L'Hôpital's Rule to find the limit, be certain that you switch to the variable x.

$$\lim_{n\to\infty} n\sin\left(\frac{\pi}{n}\right) = \lim_{x\to\infty} x\sin\left(\frac{\pi}{x}\right)$$

$$= \lim_{x\to\infty}\frac{\sin(\pi/x)}{1/x} \qquad \frac{0}{0} \text{ form}$$

$$= \lim_{x\to\infty}\frac{\cos(\pi/x)\left(-\frac{\pi}{x^2}\right)}{-1/x^2}$$

$$= \lim_{x\to\infty}\pi\cos(\pi/x)$$

$$= \pi\cos 0 = \pi$$

The limit of the sequence $\{n\sin(\pi/n)\}$ is π.

or use $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$, $\lim_{n\to\infty} n\sin\left(\frac{\pi}{n}\right) = \lim_{n\to\infty}\pi\cdot\frac{\sin(\pi/n)}{\pi/n}$

$= \pi$ since $\frac{\pi}{n}\to 0$

3. Turn in a solution to problem 5c in Section 3.2. Be careful with the algebra here; I recommend that you write out the first three terms of the sequence. We will be doing lots of work with factorials.

$$x_n = \frac{(2n)!}{(n!)^2} \qquad x_{n+1} = \frac{(2n+2)!}{((n+1)!)^2}$$

$$\frac{x_{n+1}}{x_n} = \frac{(2n+2)!}{((n+1)!)^2} \cdot \frac{(n!)^2}{(2n)!} = \frac{(2n+2)!}{(2n)!} \cdot \left(\frac{n!}{(n+1)!}\right)^2$$

$$= \frac{(2n+2)(2n+1)}{(n+1)\,(n+1)} = \frac{4n+2}{n+1} = 2 + \frac{2n}{n+1}$$

Since $\frac{x_{n+1}}{x_n} \geq 1$ for all n, the sequence $\{x_n\}$ is increasing.

4. Turn in a solution to problem 6 in Section 3.2. Begin by carefully writing out the first four terms of the sequence, then ask yourself how many terms are being added to get the nth term of the sequence and then consider which of the added terms is the smallest.

Let $b_n = \sum_{k=1}^{n} \frac{1}{\sqrt{k}}$

$$\begin{cases} b_1 = 1 \\ b_2 = 1 + \frac{1}{\sqrt{2}} \\ b_3 = 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} \\ b_4 = 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} \end{cases}$$

b_n has n terms and $\frac{1}{\sqrt{n}}$ is the smallest one

$$b_n = 1 + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} \geq \frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n}} + \cdots + \frac{1}{\sqrt{n}} = \frac{n}{\sqrt{n}} = \sqrt{n}$$

Since $\{\sqrt{n}\}$ is unbounded and $b_n \geq \sqrt{n}$ for all n, the sequence $\{b_n\}$ is also unbounded.