Name: due October 19 by 3:00 pm Pacific

Math 126 **Homework Assignment 19** **Fall 2020**

1. Turn in a solution to problem 1b in Section 3.3. This solution should be formatted almost exactly like the first example in the last paragraph just prior to the exercises. For the record, there should be four clear steps and thus four references to theorems.

Find the limit, with proof, of the sequence $\left\{ 3\sqrt[k]{8} + \frac{k}{2^k} \right\}$.

$\left\{ \sqrt[k]{8} \right\}$ converges to 1 by Theorem 3.8 part (5).

$\left\{ 3\sqrt[k]{8} \right\}$ converges to 3 by Theorem 3.6 part (a).

$\left\{ \frac{k}{2^k} \right\} = \left\{ k\left(\frac{1}{2}\right)^k \right\}$ converges to 0 by Theorem 3.8 part (3).

$\left\{ 3\sqrt[k]{8} + \frac{k}{2^k} \right\}$ converges to 3 by Theorem 3.6 part (b).

We have thus proved that $\left\{ 3\sqrt[k]{8} + \frac{k}{2^k} \right\}$ converges to 3.

2. Find the limit of the sequence $\left\{\left(1 - \frac{1}{3n}\right)^n\right\}$. Explain your reasoning.

Since $\left\{ \left(1 - \frac{1}{3n}\right)^n \right\} = \left\{ \left(1 + \frac{-1/3}{n}\right)^n \right\}$, the sequence converges to $e^{-1/3}$ by part (7) of Theorem 3.8.

3. Turn in a solution to problem 2f in Section 3.3. Use the squeeze theorem and use estimates for sums like some of the examples you have seen, either in video lectures or in the extra notes.

Find the limit of the sequence $\left\{\sqrt[n]{2n^2+4n+3}\right\}$.

For each positive integer n, we see that

$1 \le 2n^2+4n+3 \le 9n^2$ also works

$n^2 \le 2n^2+4n+3 \le 9n^2$

$\left\{2n^2+4n^2+3n^2\right\}$ is bigger

and thus

$$\sqrt[n]{n^2} \le \sqrt[n]{2n^2+4n+3} \le \sqrt[n]{9n^2}.$$

Since the sequences $\left\{\sqrt[n]{2n^2}\right\} = \left\{\sqrt[n]{2}\cdot\sqrt[n]{n}\cdot\sqrt[n]{n}\right\}$ and $\left\{\sqrt[n]{9n^2}\right\} = \left\{\sqrt[n]{9}\cdot\sqrt[n]{n}\cdot\sqrt[n]{n}\right\}$ both converge to 1, the sequence $\left\{\sqrt[n]{2n^2+4n+3}\right\}$ converges to 1 by the squeeze theorem.

4. Turn in a solution to problem 4 in Section 3.3. Once again, use the squeeze theorem.

Suppose $0 < a < b$. Find the limit of $\left\{(a^n+b^n)^{1/n}\right\}$.

For each positive integer n, we see that

$$b^n \le \overset{a>0}{a^n+b^n} \le \overset{b>a}{2b^n} \iff b \le \sqrt[n]{a^n+b^n} \le b\sqrt[n]{2}.$$

Since $\{b\}$ and $\left\{b\sqrt[n]{2}\right\}$ both converge to b, the sequence $\left\{(a^n+b^n)^{1/n}\right\}$ converges to b by the squeeze theorem.

as an example $\left\{\sqrt[n]{2^n+10^n}\right\}$

when n is large 10^n is much much bigger than 2^n

$(2^n+10^n)^{1/n} \sim (10^n)^{1/n} = 10$ guess limit is 10