Name: due November 2 by 3:00 pm Pacific

Math 126 **Homework Assignment 25** **Fall 2020**

1. Determine whether or not the series $\sum_{k=1}^{\infty}\left(\frac{2k}{3k+\sqrt[k]{k}}\right)^k$ converges.

We will use the Root Test.

$$l = \lim_{k\to\infty} \sqrt[k]{\left(\frac{2k}{3k+\sqrt[k]{k}}\right)^k} = \lim_{k\to\infty} \frac{2k}{3k+\sqrt[k]{k}} = \frac{2}{3}.$$

Since $l<1$, the series converges.

recall that $\{\sqrt[k]{k}\}$ converges to 1

C and AC are the same when all terms are positive.

2. Determine whether or not the series $\sum_{k=1}^{\infty}\frac{(-2)^k k!}{4\cdot 7\cdot 10\cdot\dots\cdot(3k+1)}$ converges.

We will use the Ratio Test:

$$l = \lim_{k\to\infty} \frac{2^{k+1}(k+1)!}{4\cdot 7\cdot 10\cdots(3k+1)(3k+4)} \cdot \frac{4\cdot 7\cdot 10\cdots(3k+1)}{2^k\, k!}$$

$$= \lim_{k\to\infty} \frac{2(k+1)}{3k+4} = \frac{2}{3}.$$

Since $l<1$, the series converges (absolutely).

$(-1)^k$ is "removed" by the absolute values

went straight to the invert and multiply step

3. Determine if the series $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^3}{3^k}$ converges absolutely, converges conditionally, or diverges.

We will use the Root Test:

$$\ell = \lim_{k\to\infty} \sqrt[k]{\left|(-1)^{k+1}\frac{k^3}{3^k}\right|} = \lim_{k\to\infty} \frac{\left(\sqrt[k]{k}\right)^3}{3} = \frac{1}{3}.$$

Since $\ell < 1$, the series converges absolutely.

4. Find all values of x for which the series $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k}{3^k}(x-1)^k$ converges absolutely. *Hint*: Express ℓ as a function of x.

We will use the Ratio Test: (just to illustrate)

$$\ell = \lim_{k\to\infty} \left|\frac{(k+1)(x-1)^{k+1}}{3^{k+1}} \cdot \frac{3^k}{k(x-1)^k}\right|$$

$$= \lim_{k\to\infty} \frac{k+1}{3k}|x-1| = \frac{|x-1|}{3}.$$

We need $\ell < 1$ for absolute convergence. Hence, the series converges absolutely for all x that satisfy $|x-1| < 3$.

this corresponds to the interval $(-2, 4)$