Name: due November 4 by 3:00 pm Pacific

Math 126 **Homework Assignment 26** **Fall 2020**

1. Find the interval of convergence for the power series $\sum_{k=0}^{\infty} \frac{4}{(k+2)3^k}(x-1)^k$. Be certain to check whether or not the series converges at the endpoints.

By the Root Test, we have

$$l = \lim_{k \to \infty} \frac{\sqrt[k]{4}}{3 \cdot \sqrt[k]{k+2}} |x-1| = \frac{|x-1|}{3}.$$

The series is absolutely convergent when $|x-1| < 3$ which is equivalent to $x \in (-2, 4)$.

radius of convergence is 3

$x = -2$ gives $\sum_{k=0}^{\infty} \frac{4(-1)^k}{k+2}$, which converges by the AST

$x = 4$ gives $\sum_{k=0}^{\infty} \frac{4}{k+2}$, which diverges by the CT

compare with harmonic series

The interval of convergence is $[-2, 4)$.

2. Give an example of a power series that has $[2, 8)$ as its interval of convergence.

The center is $\frac{2+8}{2} = 5$ and radius is $\frac{8-2}{2} = 3$. The series $\sum_{k=0}^{\infty} \frac{1}{3^k(2k+1)}(x-5)^k$ has the desired properties.

at 2, $\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}$ C by AST; at 8, $\sum_{k=0}^{\infty} \frac{1}{2k+1}$ D by CT

3. Referring to Exercise 7 in Section 3.10, find a simple expression for the function represented by the power series $\sum_{k=0}^{\infty} \frac{(-1)^k}{4^{k+1}}(x-3)^k$. In addition, determine both the radius of convergence and the interval of convergence for this series, noting that you should NOT need to use the Root or Ratio Test to do so.

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{4^{k+1}}(x-3)^k = \sum_{k=0}^{\infty} \frac{1}{4}\left(-\frac{x-3}{4}\right)^k$$

This is a geometric series with $r = -\frac{x-3}{4}$.

It converges when $\left|-\frac{x-3}{4}\right| < 1 \iff |x-3| < 4$.

The radius of convergence is thus 4 and the interval of convergence is $(-1, 7)$. The function represented by the power series is

$$\frac{\frac{1}{4}}{1-\left(-\frac{x-3}{4}\right)} = \frac{1/4}{1+\frac{x-3}{4}} = \frac{1}{x+1}.$$

use $\frac{a}{1-r}$ for a geometric series