Calculus I questions

1. Carefully state the Mean Value Theorem, then find the point c guaranteed by this theorem for the function $g(x) = \dfrac{x^2+3x}{x-1}$ on the interval $[-1, 0]$.

2. Determine the (x, y) coordinates of all inflection points that lie on the graph of the function $f(x) = 4e^{-x^2/10}$.

3. Find an equation for the line that is tangent to the curve $y = x\sqrt{3x+1}$ at $x = 1$.

4. Find an equation of a line that is tangent to the curve $y = x^2$ and passes through the point $(0, -8)$.

5. Give an example of a function f such that $\displaystyle\lim_{x\to\infty} \frac{3x^2+4x-600}{\sqrt{f(x)}} = \frac{3}{4}$. Show that your choice of f has the above property. Is f unique? Explain.

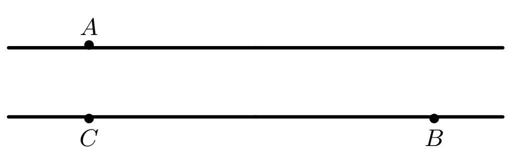
6. A rectangular box with a square base is to be constructed using materials that cost one dollar per square foot for the top, two dollars per square foot for the sides, and four dollars per square foot for the base. Find the volume of the largest box that can be constructed for eighty dollars.

7. Suppose that a particle is moving in a straight line with a velocity of 120 ft/sec. At time $t = 0$, the object begins to decelerate at the rate of $-6t$ ft/sec^2. How far does this particle travel before coming to a stop?

8. Let f be the function defined by $f(x) = 1/x$ and let $[a, b]$ be an interval with $a > 0$. Find the point c, in simplified form, guaranteed by the Mean Value Theorem for f on $[a, b]$.

9. Any line with negative slope that passes through the point $(5, 2)$ forms a triangle with the positive x and y axes. Among all such lines, find the line that gives the triangle with the least area. Include an argument that shows that you have actually found the minimum area.

10. An underground cable must connect point A with point B (see the figure). The perpendicular distance from A to C, which is across a marshy bog, is 120 feet while the distance from C to B, which is level solid ground, is 400 feet. It costs \$100 per foot to lay the cable in the bog and \$50 per foot to lay the cable in solid ground. What path for the cable will minimize the total cost?



11. A balloon leaves the ground 200 feet from a person standing at ground level. When the balloon is at a height of 80 feet (assume that this distance is relative to the eye level of the person), it is rising at the rate of 30 ft/min. How fast is the person's angle of observation increasing at this instant? Include the units of your answer.

12. For the curve defined by $x^2 + 3xy - 2y^2 - x + 5y = 11$, find $\left.\frac{dy}{dx}\right|_{(2,1)}$.

13. Evaluate $\lim_{x \to 0} \frac{\ln(1+x) - \sin x}{1 - \cos^2 x}$, a slight variation on the problem that appears in the movie *Mean Girls*.

14. Consider the graph of the parabola $y = x^2/4$. Let P be the point on the parabola corresponding to $x = 6$, let T be the point of intersection of the line tangent to the parabola at P and the line $y = -3$, and let N be the point of intersection of the line normal to the parabola at P and the x-axis. Find the area of the triangle with vertices P, T, and N.

15. Find an equation for the line tangent to the curve $y = \ln(x^2 + 2x - 2)$ when $x = 1$.

16. Use the definition of the derivative to find the derivative of the function $f(x) = 1/x$.

17. Find the maximum value and the minimum value (give exact answers in simplified form) of the function $g(x) = 30x - x^3$ on the interval $[1, 4]$.

18. A hot air balloon is ascending at the rate of 24 feet per second at a height 160 feet above the ground at the instant a rock is dropped over the side. How long does it take for the rock to reach the ground? With what speed does it hit the ground? Use 32 ft/sec^2 for the acceleration due to gravity.

19. There are two points on the curve $x^3 + y^3 = 6xy$ at which the tangent line is horizontal. One of these points is the origin. Find the (x, y) coordinates of the other such point. (You should give simplified values for the coordinates.)

20. Find values for a and b so that the function g defined by $g(x) = \begin{cases} x^2 - a, & \text{if } x < 2; \\ bx^3 + 3, & \text{if } x \geq 2; \end{cases}$ is both continuous and differentiable at $x = 2$.

21. Use the definition of derivative to prove that the derivative of $\sqrt{f(x)}$ is $\frac{f'(x)}{2\sqrt{f(x)}}$, where f is any nonnegative differentiable function.

22. Find the minimum distance from the origin to a point on the graph of $y = 2/x^2$.

23. A fence for a rectangular garden costs twice as much per meter for one side as it does for the other three sides. If the area of the garden is 40 square meters, what dimensions for the garden will minimize the cost?

24. The area of a circle is increasing at the rate of 4 cm^2/sec. Find the rate at which the circumference of the circle is increasing when the area of the circle is 25 cm^2.