Calculus II questions

1. A cylindrical hole of radius a is bored through the center of a sphere of radius $r > a$. Find the volume of the solid that remains.

2. Find the total area of the region bounded by the curve $y = x(x-2)(x-5)$ and the x–axis.

3. Evaluate $\displaystyle\int \frac{x^3+1}{x^2+4}\,dx$.

4. State both versions of the Fundamental Theorem of Calculus. Write down a function f such that $f'(x) = 2^x/x$ and $f(1) = 0$.

5. Let R be the region that is under the graph of $y = \cos x$ and above the x-axis on the interval $[0, \pi/2]$. Find the volume of the solid that is generated when R is revolved around the y-axis.

6. Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n+1} - 2^n}{4^{n-1}}$.

7. Find the radius of convergence **and** the interval of convergence for the power series $\displaystyle\sum_{n=1}^{\infty} \frac{5}{n3^n}(x-2)^n$. Then let f be the function defined by this power series on its interval of convergence and find $f^{(20)}(2)$.

8. Evaluate any two of the following integrals. Choose wisely.
   (i) $\displaystyle\int x\sin(x^2)\,dx$ (ii) $\displaystyle\int x^2\sin(x^2)\,dx$ (iii) $\displaystyle\int x\sin(x)\,dx$

9. Evaluate $\displaystyle\int_1^{\sqrt{2}} \frac{\sqrt{x^2-1}}{x^4}\,dx$.

10. Find the limit of the sequence $\left\{k\left(\sqrt[k]{10} - 1\right)\right\}$.

11. Evaluate $\displaystyle\int e^{2x}\tan^2(e^{2x})\,dx$.

12. Define a function f on $(0, \pi)$ by $f(x) = \displaystyle\sum_{k=1}^{\infty}\left(1 - \sin^2 x\right)^k$. Find $f(2\pi/3)$.

13. The base of a solid is the region enclosed by the curves $y = \sin x$ and $y = \cos x$ over the interval $[0, \pi/4]$. Cross-sections of the solid perpendicular to the x-axis are squares. Determine the volume of the solid.

14. Suppose that the function f satisfies the equation $\displaystyle\int_0^{2x} f(t)\,dt = 4\sin x + x$. Evaluate $f(\pi/3)$.

15. Find the area of the region that lies between the parabolas $y = x^2$ and $y = 2x^2$ and below the curve $y = 12\sqrt{x}$.

16. Evaluate $\displaystyle\int_0^{\pi} \frac{\sin t}{(3 + 2\cos t)^3}\,dt$.

17. Evaluate $\displaystyle\int \frac{2x+3}{x^2 - 4x - 5}\,dx$.

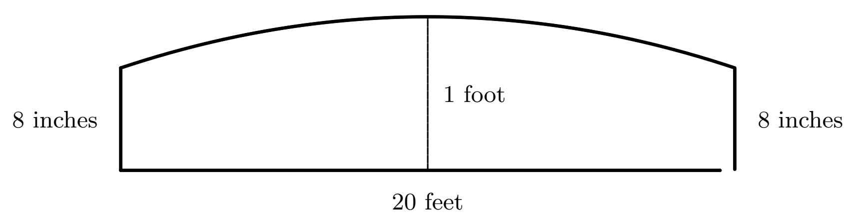
18. Evaluate $\displaystyle\int_1^{\infty} \frac{4x-3}{x^4}\,dx$.

19. Find the limit of the sequence $\left\{\left(\dfrac{n}{n+2}\right)^n\right\}$.

20. Find the first four terms of the Taylor series for $f(x) = \sqrt{1+x}$ centered at $x = 3$.

21. Suppose that $v(t) = 5t - t^3$ gives the velocity in meters per second of a particle at time t seconds. Find the total distance traveled by the particle from time $t = 0$ to time $t = 4$ seconds.

22. Determine (to the nearest cubic yard) the number of cubic yards of crushed rock necessary to make a roadbed 5400 feet long with cross section (not to scale) shown below.



Assume that the crown of the roadbed is a parabola. (There are 12 inches in a foot and 3 feet in a yard.)

23. Determine whether or not the given series converge. You must provide clear justification along with complete details for each of your conclusion. Be certain to mention which test you are using and pay careful attention to your notation.

$$\sum_{k=1}^{\infty} \frac{5\cdot 8\cdot 11\cdot \cdots \cdot (3k+2)}{2^k\, k!}, \qquad \sum_{k=1}^{\infty} \frac{(-1)^{k+1}(3k^2+1)}{k^4 + 8k - 3}, \qquad \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{4k-1}, \qquad \sum_{k=1}^{\infty} \frac{1}{\sqrt[k]{4k}}, \qquad \sum_{k=1}^{\infty} \left(\frac{k}{3k+1}\right)^k$$

24. Let R be the region in the first quadrant bounded by the curves $y = \sqrt[3]{x}$ and $y = x/4$. For each of the following, write an integral expression that represents the volume of the solid. Do **NOT** evaluate the integrals.

a. the solid that is generated when R is revolved around the x-axis;

b. the solid that is generated when R is revolved around the y-axis;

c. the solid that is generated when R is revolved around the line $x = 8$;

d. the solid whose base is R and each cross-section of the solid taken perpendicular to the y-axis is a square.