Calculus III questions

1. Consider the points A $(1, 0, 1)$, B $(2, 1, 3)$, C $(0, 4, -1)$, and D $(3, 4, 10)$. Find the point on the plane containing A, B, and C that is closest to the point D.

2. Consider the curve C defined by the parametric equations $x = t^3 - t$ and $y = t^2 + 8$.

   a) Find an equation for the line tangent to C when $t = 2$.

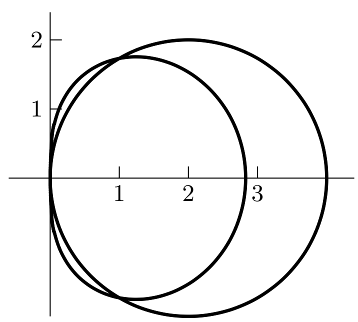
   b) A quick sketch of C reveals that the graph has a loop. Find the area of the loop.

3. Find parametric equations for the circle $(x - 2)^2 + (y - 4)^2 = 9$ for which the circle is traversed once in a clockwise manner for $0 \le t \le 1$ beginning at $(5, 4)$.

4. Find an equation for the plane tangent to the surface $z = x^2 + 2y^3 - 4xy$ at the point $(1, -1, 3)$.

5. Let $f(x, y, z) = \dfrac{x + 2y}{3z - y}$. Find the directional derivative of f at the point $(2, 4, 1)$ in the direction from $(2, 4, 1)$ toward the point $(3, 5, 0)$.

6. Find the area of the region that lies inside the polar curve $r = 4\cos\theta$ but outside the polar curve $r = \sqrt{8\cos\theta}$. (You may find the unlabeled graph helpful.)



7. Evaluate $\displaystyle\int_C xy\, ds$, where C is the line segment from $(0, 1, 2)$ to $(4, 2, 3)$.

8. Evaluate $\displaystyle\iiint_S 3xy\, dV$, where S is the solid in the first octant bounded by the planes $y = 0$ and $z = 0$ and by the surfaces $z = 4 - x^2$ and $y = \sqrt{x}$.

9. Evaluate $\displaystyle\int_C \sqrt{1 + x^3}\, dx + 2xy\, dy$, where C is the triangular path that goes from $(0, 0)$ to $(1, 0)$ to $(1, 3)$ then back to $(0, 0)$.

10. Find the work done by the force $\mathbf{F}(x, y, z) = yz\,\mathbf{i} + xz\,\mathbf{j} + (xy + z)\,\mathbf{k}$ in moving a particle along the helix given by $x = \cos(\pi t)$, $y = \sin(\pi t)$, and $z = t$ for $0 \le t \le 7$.

11. Find the mass of the solid S bounded by the paraboloid $z = 8 - 2x^2 - 2y^2$ and the xy-plane if the density function for S is given by $\rho(x, y, z) = 3x^2$.

12. Evaluate the double integral $\displaystyle\int_0^{\pi} \int_{y/2}^{\pi/2} \frac{\sin x}{x}\, dx\, dy$.

13. The temperature T in degrees Celsius at a point (x, y) on a metal plate in the shape of an ellipse is given by $T(x, y) = \sqrt{20 - x^2 - 7y^2}$.

a) Find the rate of change of the temperature at the point $(2, 1)$ in the direction of the origin.

b) Find a unit vector in the direction for which the temperature decreases most rapidly at $(2, 1)$.

14. Find an equation for the plane that contains the point $(1, 0, 1)$ and the line with parametric equations $x = 1 - t,\ y = 2 + t,\ z = t$.

15. Let $\mathbf{F}$ be the vector field defined by $\mathbf{F}(x, y, z) = y\mathbf{i} + x\mathbf{j} + xyz\mathbf{k}$ and let C by the path that consists of the straight line from $(0, 0, 0)$ to $(1, 1, 0)$ followed by the vertical line from $(1, 1, 0)$ to $(1, 1, 1)$. Find the work done by the field $\mathbf{F}$ in moving a particle along the path C.

16. Evaluate $\displaystyle\int_{-\sqrt{2}}^{\sqrt{2}} \int_0^{\sqrt{2-x^2}} \sqrt{x^2 + y^2}\, dy\, dx$.

17. Find the maximum value of the function $f(x, y, z) = 2x - 2y + z$ subject to the condition $x^2 + 2y^2 + 3z^2 = 114$.

18. Let $\mathbf{F}$ be the vector field defined by $\mathbf{F}(x, y, z) = (3y^3 - 10xz^2)\,\mathbf{i} + 9xy^2\,\mathbf{j} - 10x^2z\,\mathbf{k}$ and let C be the path given by $\mathbf{r}(t) = \langle 1 + \sin(\pi t), 2 + \cos(3\pi t), 3 - 4t\rangle$ for $0 \le t \le 1$. Find the work done by the field $\mathbf{F}$ in moving a particle along the path C.

19. Find the volume of the solid bounded by the planes $y = 0$, $x = 2$, $y = x$, $z = 0$, and $z = 2x + 2y + 5$.

20. Find the distance between the parallel planes $2x - y + 2z = 4$ and $2x - y + 2z = 13$.

21. Find a point on the surface $z = x^2y - 4xy + 5x - 8$ where the tangent plane is perpendicular to the line $x = 1 + 6t,\ y = -3 - 10t,\ z = 2t$.

22. A flat circular plate has the shape of the region $\{(x, y) : x^2 + y^2 \le 1\}$. The plate is heated so that the temperature in degrees Celsius at any point (x, y) is $T(x, y) = x^2 + 2y^2 - x + 100$. Find the hottest and coldest points on the plate and the temperature at these points.

23. Find the work done by the force field $\mathbf{F}(x, y) = \langle -y^2, 2x\rangle$ in moving a particle from $(2, 0)$ to $(-2, 0)$ along the upper half of the circle $x^2 + y^2 = 4$.

24. Evaluate $\displaystyle\iiint_S z\, dV$, where S is the upper half of the sphere $x^2 + y^2 + z^2 = 4$.

25. Verify that the Divergence Theorem is valid for the vector field $\mathbf{F}(x, y, z) = \langle x, y, y^2\rangle$ on the region E that is bounded by the surfaces $\{(x, y, z) : x^2 + y^2 + z^2 = 1, z \ge 0\}$ and $\{(x, y, z) : x^2 + y^2 \le 1, z = 0\}$.