Nonroutine problems are challenging because it is not always clear how to get started. When given a problem in a textbook, the section in which the problem is found gives an implicit hint as to how to proceed and there may very well be an example to imitate. Problems that present themselves in the "real world" do not come in this way. You have (or will soon acquire) the necessary knowledge to solve the problems given below. However, determining what previous knowledge you need and how to organize these ideas into a solution strategy takes time and practice. Do not give up easily on these problems and remember that it can be helpful to think about them for a while and then return to them another day. Once you have a solution, carefully write up the details and provide a clear discussion of the methods (including any appropriate technology or other resources) that you used and I will be happy to read your solutions.

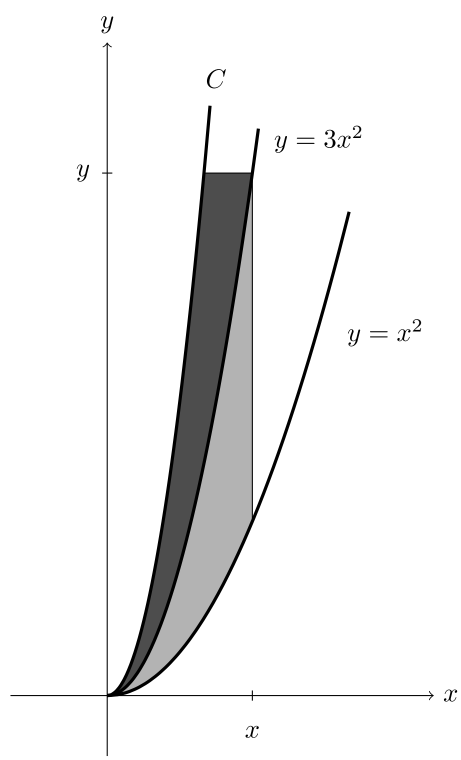
1. Evaluate $\lim_{n\to\infty}\left(\frac{1}{n}\cdot\ln\left(\frac{(2n)!}{n!n^n}\right)\right)$.

2. Let f be a linear function and consider the function F defined by $F(x) = x\int_1^{2x+3} f(t)\,dt$. Given that $F(1) = 2$ and $F'(0) = -10$, find the function $f(x)$.

3. Let R be the region bounded by the curves $y = 1/x^2$, $y = x$, and $y = 8x$. Find a value for c so that the line $y = cx$ splits the region R into two parts of equal area.

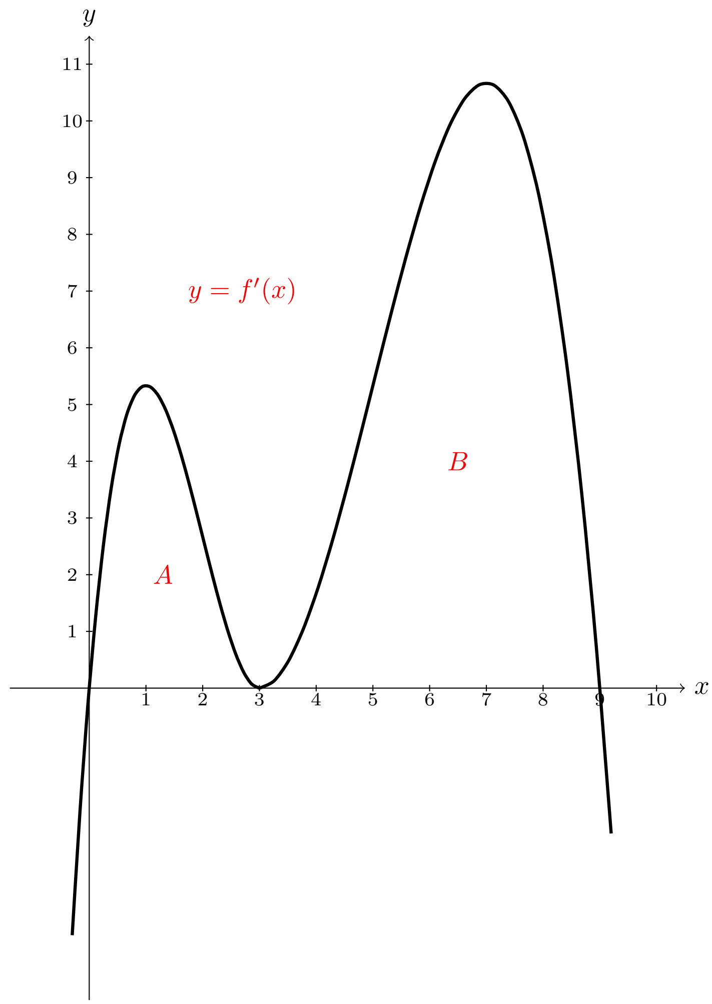
4. Consider the curves $y = 2a^2 - x^2$ and $y = ax$, where a is a positive constant. Find a value of a so that the area of the region bounded by these two curves is 90 square units.

5. Referring to the figure below, we want to find a curve C so that the area of the lightly shaded region equals the area of the darkly shaded region for each value of $x > 0$. Assuming that the curve C has the form $y = ax^2$, find the value of a.



6. A large wine cask that is six feet tall has a diameter of six feet at its top and bottom and a diameter of eight feet in its middle. The sides of the cask are parabolic in shape.
   i. Find the total volume of the cask. Give your answer to the nearest gallon.
   ii. Suppose that the liquid in the cask (when standing upright) is four feet deep. Determine the number of gallons of liquid in the cask.
   iii. Assuming that the cask (when standing upright) is 25% full, how deep is the liquid? Give your answer for the depth to the nearest one hundredth of a foot.

7. Consider the function f whose derivative is graphed below. The function f is defined for all real numbers and the graph continues in both directions in the predictable manner indicated by the ends of the graph that are shown. The area of region A is 9 and the area of region B is 36.



   a. Find the x-coordinates at which the graph of $y = f(x)$ has a relative extrema.
   b. Find the x-coordinates at which the graph of $y = f(x)$ has an inflection point.
   c. On what fraction of the interval $(0, 9)$ is the graph of $y = f(x)$ both increasing and concave up?
   d. Given that $f(3) = 20$, find $f(0)$ and $f(9)$