Miscellaneous problems to try before the final exam

Since the answers are given right under the problem, you need to be careful and avoid looking at the answer to get a hint for how to start a problem as this does not mimic a testing situation. For the record, these solutions have not yet been carefully vetted. Proceed without technology if at all possible.

1. Find a simple formula, one that does not involve summation notation, for $\sum_{k=1}^{n}(2k-1)(2k+1)$.

   $$\sum_{k=1}^{n}(2k-1)(2k+1) = \frac{n}{3}\big(4n^2+6n-1\big).$$

2. Find a simple formula, one that does not involve summation notation, for $\sum_{i=1}^{2n-1}\big((i+1)^7 - i^7\big)$.

   $$\sum_{i=1}^{2n-1}\big((i+1)^7 - i^7\big) = (2n)^7 - 1.$$

3. Evaluate the limit: $\displaystyle\lim_{n\to\infty}\frac{n^4+n^2+1}{3^3+6^3+9^3+\cdots+(3n)^3}$.

   $$\lim_{n\to\infty}\frac{n^4+n^2+1}{3^3+6^3+9^3+\cdots+(3n)^3} = \frac{4}{27}.$$

4. Find the area of the region bounded by the graph of $y = x^4/4$, the tangent line to this curve when $x = 2$, and the x-axis.

   The area of this region is $3/5$.

5. Find the area under the curve $y = 7 - |2x-3|$ and above the x-axis on the interval $[0, 5]$.

   The area under the curve is $41/2$.

6. Evaluate the limit: $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\left(5+\frac{3i}{n}\right)^2\frac{3}{n}$.

   You should recognize this limit as $\displaystyle\int_5^8 x^2\,dx$ and thus obtain 129.

7. Use the definition of the integral to evaluate $\displaystyle\int_0^2 (x^3-4x)\,dx$.

   Using sums and limits (this is crucial), you should obtain -4.

8. Evaluate $\displaystyle\int_{-1}^{2}(2x^2-5x+3)\,dx$.

   $$\int_{-1}^{2}(2x^2-5x+3)\,dx = \frac{15}{2}.$$

9. Evaluate $\displaystyle\int_0^3 (4x+2|x-1|)\,dx$.

   $$\int_0^3 (4x+2|x-1|)\,dx = 23.$$

10. Evaluate $\int_0^1 \left(2t - 3 + 2\sqrt{1-t^2}\right)\,dt$.

$$\int_0^1 \left(2t - 3 + 2\sqrt{1-t^2}\right)\,dt = \frac{\pi}{2} - 2.$$

11. Find, with explanation, upper and lower bounds for the value of $\int_1^8 \frac{1}{4 + 3\sqrt[3]{x}}\,dx$.

Using the max/min values of the integrand, the value is between 0.7 and 1.

12. Without evaluating them, explain which of the integrals $\int_0^1 \sin^4(\pi x)\,dx$ or $\int_0^1 \sin^5(\pi x)\,dx$ is larger.

$\int_0^1 \sin^4(\pi x)\,dx \geq \int_0^1 \sin^5(\pi x)\,dx$ since $\sin^4(\pi x) \geq \sin^5(\pi x)$ for all $x \in [0,1]$.

13. Use an appropriate function to approximate $\int_1^2 \frac{1}{\sqrt{4x^6 - 1}}\,dx$. Is your estimate high or low?

$$\int_1^2 \frac{1}{\sqrt{4x^6-1}}\,dx > \int_1^2 \frac{1}{2x^3}\,dx = \frac{3}{16}.$$

14. Find the derivative the function F defined by $F(x) = \int_0^{x^2} t\sqrt{t^3+4}\,dt$.

Using the FTC and the Chain Rule, we find that $F'(x) = 2x^3\sqrt{x^6+4}$.

15. Evaluate $\lim_{x\to 0} \frac{1}{x^3} \int_0^x \left(1 - e^{-2t^2}\right)\,dt$.

The value of the limit is 2/3.

16. Find an integral expression for a function f such that $f(\pi) = 0$ and $f'(x) = \cos^2(3x^2)$.

By the FTC, the function $f(x) = \int_\pi^x \cos^2(3t^2)\,dt$ has the desired properties.

17. Evaluate $\int_1^8 \frac{x+2}{\sqrt[3]{x}}\,dx$.

$$\int_1^8 \frac{x+2}{\sqrt[3]{x}}\,dx = \frac{138}{5}.$$

18. Evaluate $\int_0^2 \left(e^{x/2} - x\right)\,dx$.

$$\int_0^2 \left(e^{x/2} - x\right)\,dx = 2e - 4.$$

19. Evaluate $\int_1^4 \frac{1}{3x-2}\,dx$.

$$\int_1^4 \frac{1}{3x-2}\,dx = \frac{1}{3}\ln 10.$$

20. Suppose that $v(t) = 3t - t^3$ gives the velocity in meters per second of a particle at time t seconds. Find the distance traveled by the particle for the time interval $0 \leq t \leq 4$.

The particle travels 44.5 meters during this time period.

21. Evaluate $\int \left(2\sqrt{x}+1\right)^2 dx$.

$$\int \left(2\sqrt{x}+1\right)^2 dx = 2x^2 + \frac{8}{3}\, x^{3/2} + x + C.$$

22. Evaluate $\int \frac{3x}{(2x^2+5)^3}\, dx$.

$$\int \frac{3x}{2x^2+5)^3}\, dx = \frac{-3}{8(2x^2+5)^2} + C.$$

23. Evaluate $\int_0^2 \frac{x}{4+x^2}\, dx$.

$$\int_0^2 \frac{x}{4+x^2}\, dx = \frac{1}{2}\, \ln 2.$$

24. Evaluate $\int_0^2 \frac{1}{4+x^2}\, dx$.

$$\int_0^2 \frac{1}{4+x^2}\, dx = \frac{\pi}{8}.$$

25. Evaluate $\int_0^2 \frac{x}{\sqrt{4+x^2}}\, dx$.

$$\int_0^2 \frac{x}{\sqrt{4+x^2}}\, dx = 2\left(\sqrt{2}-1\right).$$

26. Evaluate $\int_0^2 \frac{x}{\sqrt{4+x}}\, dx$.

$$\int_0^2 \frac{x}{\sqrt{4+x}}\, dx = \frac{32}{3} - 4\sqrt{6}.$$

27. Evaluate $\int \frac{12x}{(3x-1)^2}\, dx$.

$$\int \frac{12x}{(3x-1)^2}\, dx = \frac{4}{3}\left(\ln|3x-1| - \frac{1}{3x-1}\right) + C.$$

28. Evaluate $\int_0^9 \frac{1}{1+\sqrt{x}}\, dx$.

Evaluate $\int_0^9 \frac{1}{1+\sqrt{x}}\, dx = 6 - \ln 16.$

29. Evaluate $\int x \sec^2 x\, dx$.

$$\int x \sec^2 x\, dx = x\tan x + \ln|\cos x| + C.$$

30. Evaluate $\int \arctan x\, dx$.

$$\int \arctan x\, dx = x\arctan x - \frac{1}{2}\, \ln(1+x^2) + C.$$

31. Evaluate $\int x\, e^{-x/2}\, dx$.

$$\int x\, e^{-x/2}\, dx = -2(x+2)e^{-x/2} + C.$$

32. Evaluate $\displaystyle\int_1^{\infty} \frac{8}{(2x+5)^3}\, dx$.

$$\int_1^{\infty} \frac{8}{(2x+5)^3}\, dx = \frac{2}{49}.$$

33. Evaluate $\displaystyle\int_2^{\infty} \frac{6}{2x+5}\, dx$.

The integral diverges.

34. Evaluate $\displaystyle\int_0^{\infty} \frac{6+e^{2x}}{e^{3x}}\, dx$.

$$\int_0^{\infty} \frac{6+e^{2x}}{e^{3x}}\, dx = 3.$$

35. Find the area of the region bounded by the curves $x^2 y = 90$ and $40x + y = 130$.

The area is 40 square units.

36. Find the area of the region bounded by the curves $y = 2\sqrt{x}$ and $y = x^3/16$.

The area is 20/3 square units.

37. Find the volume of the solid that is generated when the region bounded by the curves $y = 4x$ and $y = x^2$ is revolved around (a) the x-axis (b) the y-axis.

The volume is $\dfrac{2048}{15}\pi$ around the x-axis and $\dfrac{128}{3}\pi$ around the y-axis.

38. Suppose that the base of a solid is the part of the parabola $y = 8 - 0.5x^2$ that lies above the x-axis and that each cross section perpendicular to the y-axis is a semicircle. Find the volume of this solid.

The volume of the solid is 8π cubic units.

39. Find the volume of the solid that is generated when the region that lies below the curve $y = \ln x$ and above the x-axis on the interval $[1, e]$ is revolved around the y-axis.

The volume of the solid is $(e^2 + 1)/2$ cubic units.

40. Find the volume of the solid that is generated when the region that lies above the x-axis and below the curve $y = \sqrt{\dfrac{14(3-x)}{(x+1)(7-x)}}$ on the interval $[0, 3]$ is revolved around the x-axis.

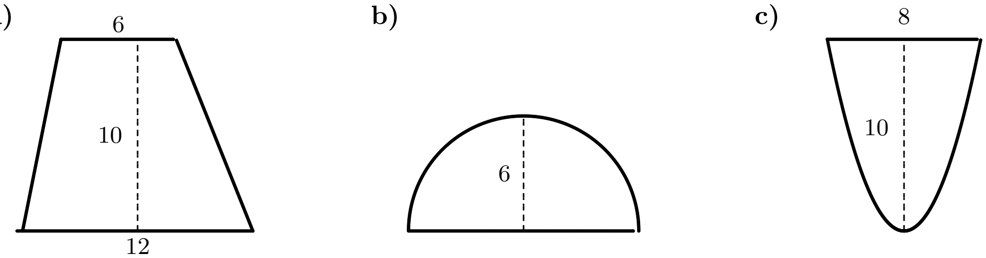
The volume of the solid is $7\pi(\ln 16 - \ln 7)$ cubic units.

41. Find the volume of the solid that is generated when the portion of the parabola $y = (x-1)(3-x)$ that lies above the x-axis is revolved around the y-axis.

The volume of the solid is $16\pi/3$ cubic units.

42. Find the force exerted by a liquid with weight density w on one side of each vertically submerged plate. The units on the figures are feet and the top of each plate is six feet beneath the surface of the liquid.

a) **b)** **c)**



trapezoid semicircle parabola

The forces are $1040w$, $(216\pi - 144)w$, and $1600w/3$ pounds, respectively.

43. Find the center of mass of the region that lies below the curve $y = x^4/10$ and above the x-axis on the interval $[0, 3]$.

The center of mass is $\left(\frac{5}{2}, \frac{9}{4}\right)$.

44. Find the center of mass of the region bounded by the curves $y = 2\sqrt{x}$ and $y = x^3/16$.

The center of mass is $\left(\frac{48}{25}, \frac{12}{7}\right)$.

45. Find the center of mass of the solid that is generated when the region bounded by $y = 4 - x^2$ in the first quadrant is revolved around the x-axis.

The center of mass is on the x-axis and the x-coordinate is $5/8$.

46. Find the center of mass of the solid that is generated when the region below the curve $y = 4e^{-x/4}$ and above the x-axis on the interval $[0, \infty)$ is revolved around the x-axis.

The center of mass is on the x-axis and the x-coordinate is 2.

47. Find the length of the curve $y = 4x^{3/2}$ on the interval $[0, 10]$.

The length of the curve is 127 units.

48. Find the length of the curve $y = \frac{4}{5}x^{5/4}$ on the interval $[0, 9]$.

The length of the curve is $232/15$ units.

49. Evaluate $\displaystyle\int \frac{3x+1}{\sqrt{12x - x^2}}\,dx$.

$$\int \frac{3x+1}{\sqrt{12x - x^2}}\,dx = -3\sqrt{12x - x^2} + 19\arcsin\left(\frac{x-6}{6}\right) + C.$$

50. Evaluate $\displaystyle\int \frac{3x+1}{\sqrt{13 - 12x - x^2}}\,dx$.

$$\int \frac{3x+1}{\sqrt{13 - 12x - x^2}}\,dx = -3\sqrt{13 - 12x - x^2} - 17\arcsin\left(\frac{x+6}{7}\right) + C.$$

51. Evaluate $\displaystyle\int \frac{3x+8}{x^2 + 4x + 6}\,dx$.

$$\int \frac{3x+8}{x^2 + 4x + 6}\,dx = \frac{3}{2}\ln\left(x^2 + 4x + 6\right) + \sqrt{2}\arctan\left(\frac{x+2}{\sqrt{2}}\right) + C.$$

52. Evaluate $\displaystyle\int \frac{4x-7}{2x+1}\,dx$.

$$\int \frac{4x-7}{2x+1}\,dx = 2x - \frac{9}{2}\ \ln|2x+1| + C.$$

53. Derive the reduction formula for tangent.

You do not need integration by parts; use $\tan^2 x = \sec^2 x - 1$.

54. Use a table of integrals to find $\displaystyle\int \sqrt{4-e^x}\,dx$.

$\displaystyle\int \sqrt{4-e^x}\,dx = 2x + 2\sqrt{4-e^x} - 4\ln\bigl(2+\sqrt{4-e^x}\,\bigr) + C.$ (You may need to do some algebra.)

55. Use a table of integrals to find $\displaystyle\int_0^1 \frac{x^2}{\sqrt{1+4x^2}}\,dx$.

$$\int_0^1 \frac{x^2}{\sqrt{1+4x^2}}\,dx = \frac{1}{16}\bigl(2\sqrt{5} - \ln\bigl(2+\sqrt{5}\,\bigr)\bigr).$$

56. Use trigonometric substitution to derive $\displaystyle\int \frac{\sqrt{a^2-u^2}}{u^2}\,du = -\frac{\sqrt{a^2-u^2}}{u} - \arcsin(u/a) + C.$

You should end up with something like $\displaystyle\int \bigl(\csc^2\theta - 1\bigr)\,d\theta.$

57. Use trigonometric substitution to derive $\displaystyle\int \frac{\sqrt{a^2+u^2}}{u^2}\,du = -\frac{\sqrt{a^2+u^2}}{u} + \ln\bigl|u + \sqrt{a^2+u^2}\bigr| + C.$

You should end up with something like $\displaystyle\int \left(\frac{\cos\theta}{\sin^2\theta} - \sec\theta\right)d\theta.$

58. Use trigonometric substitution to derive $\displaystyle\int \frac{\sqrt{u^2-a^2}}{u^2}\,du = -\frac{\sqrt{u^2-a^2}}{u} + \ln\bigl|u + \sqrt{u^2-a^2}\bigr| + C.$

You should end up with something like $\displaystyle\int \bigl(\sec\theta - \cos\theta\bigr)\,d\theta.$

59. Evaluate $\displaystyle\int \frac{\sqrt{x^2+4}}{x^4}\,dx$.

$$\int \frac{\sqrt{x^2+4}}{x^4}\,dx = \frac{-(x^2+4)^{3/2}}{12x^3} + C.$$

60. Evaluate $\displaystyle\int_1^{\sqrt{2}} \frac{\sqrt{x^2-1}}{x^4}\,dx$.

$$\int_1^{\sqrt{2}} \frac{\sqrt{x^2-1}}{x^4}\,dx = \frac{\sqrt{2}}{12}.$$

61. Evaluate $\displaystyle\int \frac{4x-1}{x^2+2x-15}\,dx$.

$$\int \frac{4x-1}{x^2+2x-15}\,dx = \frac{11}{8}\ln|x-3| + \frac{21}{8}\ln|x+5| + C.$$

62. Evaluate $\displaystyle\int \frac{x^2+2x+4}{x^3+x^2+x+1}\,dx$.

$$\int \frac{x^2+2x+4}{x^3+x^2+x+1}\,dx = \frac{3}{2}\ln|x+1| - \frac{1}{4}\ln(x^2+1) + \frac{5}{2}\arctan x + C.$$

63. Evaluate $\displaystyle\int \frac{3x-7}{2x^2+7x-9}\,dx$.

$$\int \frac{3x-7}{2x^2+7x-9}\,dx = \frac{41}{22}\ln|2x+9| - \frac{4}{11}\ln|x-1| + C.$$

64. Use the trapezoid rule and Simpson's rule with $n = 4$ to approximate $\displaystyle\int_0^1 e^{-x^2/2}\,dx$ to four decimal places.

The approximations are 0.8526 and 0.8557, respectively.

65. Use the trapezoid rule and Simpson's rule with $n = 6$ to approximate $\displaystyle\int_1^2 \sqrt{1+x^3}\,dx$ to four decimal places.

The approximations are 2.1320 and 2.1299, respectively.

66. Suppose that the following table represents the velocity of a particle moving in a straight line.

t	(sec)	0	1	2	3	4	5	6
v	(m/sec)	0	5	12	15	14	6	0

Use Simpson's rule to approximate the distance traveled by the particle.

The particle travels approximately 48 meters according to Simpson's rule.

67. Prove that $1^3 + 2^3 + 3^3 + \cdots + n^3 = \dfrac{n^2(n+1)^2}{4}$ for each positive integer n.

Use mathematical induction.

68. Prove the following statement: for each positive integer n, the integer $2^{5n-4} + 5^{2n-1}$ is divisible by 7.

Use mathematical induction.

69. Define a sequence $\{c_n\}$ by $c_1 = 2$ and $c_n = 1/(3 - c_{n-1})$ for $n \geq 2$. Prove that $0 < c_n \leq 2$ for all positive integers n and that $c_{n+1} < c_n$ for all positive integers n. Conclude that $\{c_n\}$ converges and find its limit.

The limit of the sequence is $\left(3 - \sqrt{5}\right)/2$.

70. Prove that $f_1f_2 + f_2f_3 + f_3f_4 + \cdots + f_{2n-1}f_{2n} = f_{2n}^2$ for each positive integer n, where f_n represents the nth Fibonacci number.

Use mathematical induction.

71. Give an example of a sequence that converges to 5 but is not monotone.

The sequence $\left\{5 + \dfrac{(-1)^n}{n}\right\}$ is one of many examples.

72. Give an example of a monotone sequence of negative numbers that does not converge.

    The sequence $\{-n^2\}$ is one of many examples.

73. Give an example of a bounded sequence that does not converge.

    The sequence $\{(-1)^n\}$ is one of many examples.

74. Find the limit of the sequence $\left\{\dfrac{k}{\sqrt{3k^2+4k+1}}\right\}$

    The limit of the sequence is $1/\sqrt{3}$.

75. Find the limit of the sequence $\left\{\sqrt{k^2-7k+15}-k\right\}$

    The limit of the sequence is $-7/2$.

76. Find the limit of the sequence $\left\{k\left(\sqrt[k]{10}-1\right)\right\}$

    The limit of the sequence is $\ln 10$.

77. Find the limit of the sequence $\left\{\left(1-\dfrac{2}{3n}\right)^n\right\}$.

    The limit of the sequence is $e^{-2/3}$.

78. Find the limit of the sequence $\left\{\dfrac{4^n+n^2}{2^{2n-3}+n^7}\right\}$.

    The limit of the sequence is 8.

79. Find the limit of the sequence $\left\{\sqrt[n]{4n^2+n+3}\right\}$.

    The limit of the sequence is 1.

80. Use the squeeze theorem to prove that the sequence $\{5^n/n!\}$ converges to 0.

    For $n \geq 5$, the nth term is positive and less than $5 \cdot 3 \cdot 2 \cdot 2 \cdot \dfrac{5}{n}$.

81. For each positive integer n, let $x_n = \dfrac{1}{n+1} + \dfrac{1}{n+2} + \dfrac{1}{n+3} + \cdots + \dfrac{1}{2n}$. Prove that the sequence $\{x_n\}$ converges.

    Show that the sequence is bounded and monotone.

82. Define a sequence $\{x_n\}$ by $x_1 = 5$ and $x_{n+1} = 4 - (1/x_n)$ for $n \geq 1$. Prove that $1 \leq x_n \leq 5$ for all n, then prove that $\{x_n\}$ is a decreasing sequence. Conclude that $\{x_n\}$ converges and find its limit.

    The limit of the sequence is $2+\sqrt{3}$.

83. Let $x_1 = 5$ and let $x_{n+1} = \sqrt{5+x_n}$ for $n \geq 1$. Suppose you know that the sequence $\{x_n\}$ converges. Find its limit.

    The limit of the sequence is $\left(1+\sqrt{21}\right)/2$.

84. Determine whether or not the series $\displaystyle\sum_{k=1}^{\infty} \frac{2}{\sqrt[k]{10}}$ converges.

    The series diverges by the Divergence Test.

85. Find the sum of the series $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}3^k}{4^{k-1}}$.

The sum of the series is 12/7.

86. Find the sum of the series $\displaystyle\sum_{k=1}^{\infty} \frac{3^k + 5^k}{7^k}$.

The sum of the series is 13/4.

87. Let $\sum_{k=1}^{\infty} a_k$ be an infinite series and suppose that $s_n = \dfrac{n+1}{1-3n}$ for all $n \geq 1$, where $\{s_n\}$ is its corresponding sequence of partial sums. Find a_1, a_2, a_{10}, and the sum of the series.

The values are -1, $2/5$, $2/377$, and $-1/3$, respectively.

88. Use the Integral Test to show that $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k \ln k}$ diverges.

The integral can be evaluated with a simple substitution.

89. For each positive integer n, let $a_n = 1 + \dfrac{1}{2} + \dfrac{1}{3} + \cdots + \dfrac{1}{n} - \displaystyle\int_1^n \frac{dx}{x}$. Prove that $\{a_n\}$ is a decreasing sequence.

Use ideas related to the proof of the integral test. (The limit is known as Euler's constant.)

90. Determine whether or not the series $\displaystyle\sum_{k=1}^{\infty} \frac{12}{3k+2}$ converges.

The series diverges by the Comparison Test.

91. Determine whether or not the series $\displaystyle\sum_{k=1}^{\infty} \frac{4k-1}{k^2+5k+2}$ converges.

The series diverges by the Limit Comparison Test.

92. Determine whether or not the series $\displaystyle\sum_{k=1}^{\infty} \frac{2k^2+3}{k^4+7k-1}$ converges.

The series converges by the Limit Comparison Test.

93. Determine whether or not the series $\displaystyle\sum_{k=1}^{\infty} \frac{5^k}{2^k+6^k}$ converges.

The series converges by the Comparison Test.

94. Determine whether or not the series $\displaystyle\sum_{k=1}^{\infty} \frac{3^k \sin k}{4^k}$ is absolutely convergent.

The series is absolutely convergent.

95. Determine whether or not the series $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}k^2}{k^4+3k^2+10}$ is absolutely convergent.

The series is absolutely convergent.

96. Determine whether or not the series $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{5k+4}$ converges.

The series converges by the Alternating Series Test.

97. Prove carefully that the series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}k}{k^2+1}$ is conditionally convergent.

Use BOTH a comparison test and the Alternating Series Test.

98. Determine whether or not the series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt[k]{5}}$ converges.

The series diverges by the Divergence Test.

99. Determine whether or not the series $\sum_{k=1}^{\infty} \left(\frac{k}{3k+1}\right)^k$ converges.

The series converges by the Root Test.

100. Determine whether or not the series $\sum_{k=1}^{\infty} \frac{(-3)^k k!}{3 \cdot 7 \cdot 11 \cdot \cdots \cdot (4k-1)}$ converges.

The series converges absolutely by the Ratio Test.

101. Find the radius of convergence of the power series $\sum_{k=1}^{\infty} \frac{1}{k^2 3^k}(x-4)^k$.

The radius of convergence is 3.

102. Find the interval of convergence of the power series $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)2^k}(x-1)^k$.

The interval of convergence is $(-1, 3]$. (Pay careful attention to the endpoints.)

103. Give an example of a power series with $[4, 10)$ as its interval of convergence.

One example is $\sum_{k=1}^{\infty} \frac{1}{k3^k}(x-7)^k$.

104. Find (in more familiar terms) the function represented by the power series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2^k}(x+1)^k$.

This is the power series for the function $f(x) = \frac{x+1}{x+3}$, valid on the interval $(-3, 1)$.

105. Find the Maclaurin series for the function $f(x) = \frac{1}{5-2x}$ and determine its interval of convergence.

The Maclaurin series is $\sum_{k=0}^{\infty} \frac{2^k}{5^{k+1}} x^k$, with interval of convergence $(-2.5, 2.5)$.

106. By differentiating an appropriate power series (see problem 3.11.2), find a the sum of the series $\sum_{k=1}^{\infty} k^3 x^k$.

$$\sum_{k=1}^{\infty} k^3 x^k = \frac{x+4x+x^3}{(1-x)^4}.$$

107. Use known series to find the Maclaurin series for the given function.

a) $f(x) = e^{-x/3}$ **b)** $g(x) = \sin(x^2)$ **c)** $h(x) = \dfrac{1-\cos x}{x}$

$$f(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{3^k k!}\, x^k, \qquad g(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!}\, x^{4k+2}, \qquad h(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(2k)!}\, x^{2k-1}$$

108. Use known Maclaurin series to determine in more familiar terms the given function.

a) $\displaystyle\sum_{k=0}^{\infty} \frac{1}{2^k k!}\, x^k$ **b)** $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!}\, x^{2k}$ **c)** $\displaystyle\sum_{k=0}^{\infty} \frac{(-9)^k}{(2k)!}\, x^{2k+1}$

The functions are $e^{x/2}$, $\sin x/x$, and $x\cos(3x)$, respectively.

109. Find the Maclaurin series for the function $f(x) = 1/\sqrt{1-x}$.

$$\frac{1}{\sqrt{1-x}} = 1 + \sum_{k=1}^{\infty} \frac{1\cdot 3\cdot 5\cdot \cdots \cdot (2k-1)}{2^k\, k!}\, x^k = \sum_{k=0}^{\infty} \frac{(2k)!}{4^k (k!)^2} \text{ for } |x| < 1.$$

110. Find the Taylor series for the function $f(x) = 1/(2x-1)$ centered at $a = 6$.

$$\frac{1}{2x-1} = \sum_{k=0}^{\infty} \frac{(-2)^k}{11^{k+1}}\, (x-6)^k \text{ with } \rho = 5.5.$$

111. Find the Taylor series for the function $f(x) = \sqrt{x}$ centered at $a = 9$.

$$\sqrt{x} = 3 + \frac{1}{6}(x-9) + \sum_{k=2}^{\infty} (-1)^{k+1}\, \frac{1\cdot 3\cdot 5\cdot \cdots \cdot (2k-3)}{2^k\, 3^{2k-1}\, k!}\, (x-9)^k \text{ with } \rho = 9.$$

112. Express the value of each of the following integrals as infinite series.

a) $\displaystyle\int_0^1 e^{-x^2}\, dx$ **b)** $\displaystyle\int_0^{0.5} \frac{\sin x}{x}\, dx$ **c)** $\displaystyle\int_0^1 \cos(x^2)\, dx$

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)\, k!}, \qquad \sum_{k=0}^{\infty} \frac{(-1)^k}{2^{2k+1}(2k+1)(2k+1)!}, \qquad \sum_{k=0}^{\infty} \frac{(-1)^k}{(4k+1)(2k)!}$$