

Review Problems for Calculus I

1. Find the x -intercept of the tangent line to $y = 3x + 4\sqrt{x}$ when $x = 4$.

When $x = 4$, we see that $y = 12 + 8 = 20$. To find the slope, we take the derivative:

$$\frac{dy}{dx} = 3 + \frac{2}{\sqrt{x}} \Rightarrow \left. \frac{dy}{dx} \right|_{x=4} = 3 + 1 = 4.$$

An equation for the tangent line is thus

$$y - 20 = 4(x - 4)$$

Set $y = 0$ to find the x -intercept

$$-20 = 4(x - 4) \Rightarrow -5 = x - 4 \Rightarrow x = -1$$

The x -intercept of the tangent line is $(-1, 0)$.

2. Evaluate $\lim_{x \rightarrow \infty} \frac{3x+1}{\sqrt{2x^2+5}}$.

The limit has the form $\frac{\infty}{\infty}$, which is indeterminate. L'Hopital's Rule is not a good choice here so we use algebra:

$$\lim_{x \rightarrow \infty} \frac{3x+1}{\sqrt{2x^2+5}} = \lim_{x \rightarrow \infty} \frac{3x+1}{\sqrt{2x^2+5}} \cdot \frac{1/x}{1/x}$$

$$= \lim_{x \rightarrow \infty} \frac{3 + \frac{1}{x}}{\sqrt{2 + \frac{5}{x^2}}}$$

$$= \frac{3}{\sqrt{2}}$$

15. Find and simplify the derivative of the function

$$f(x) = 4 \arcsin\left(\frac{x}{2}\right) + x\sqrt{4-x^2}$$

Using standard derivative formulas and the product rule (along with some algebra), we find that

$$f'(x) = 4 \cdot \frac{1}{\sqrt{1 - \left(\frac{x}{2}\right)^2}} \cdot \frac{1}{2} + \sqrt{4-x^2} + x \cdot \frac{-2x}{2\sqrt{4-x^2}}$$

$$= \frac{4}{\sqrt{4-x^2}} + \sqrt{4-x^2} - \frac{x^2}{\sqrt{4-x^2}}$$

$$= \frac{4-x^2}{\sqrt{4-x^2}} + \sqrt{4-x^2}$$

$$= \sqrt{4-x^2} + \sqrt{4-x^2}$$

$$= 2\sqrt{4-x^2}$$