Math 126: Calculus II **Fall 2010**

Russ Gordon Office: Olin 235 Hours: 3:00-4:00 pm MTThF

Calculus has its roots in two geometric problems: determining the areas of regions that have curved boundaries and finding tangents to curves other than circles. Using geometric techniques, the ancient Greeks were able to solve these two problems for the conic sections and some other easily defined curves. With the introduction of analytic geometry (the familiar x, y coordinate system which provides a strong link between algebra and geometry) in the middle of the seventeenth century, a vast array of new curves were considered, and algebraic techniques were developed to determine areas and tangents for these curves. Not long after this development, Newton and Leibniz independently discovered that the problems of areas and tangents were related to one another in a simple way; this discovery is considered to be the birth of calculus. The fact that many problems in the physical sciences can be reduced to finding areas or tangents has made calculus the cornerstone of the scientific revolution.

Mathematics in general, and calculus in particular, is often taught as a set of algorithms and/or skills to be mastered and the focus is usually on one skill at a time. Several sample problems are explained then a large collection of similar problems are given as exercises. These exercises can often be solved by imitation rather than understanding. While skills are certainly important and an essential part of problem solving, other crucial aspects of problem solving include knowing which skills to use and combining several skills in a multi-step problem. Learning to solve non-routine problems, those without an example to imitate and requiring more than one step, is one of the goals of this class. These sorts of problems require more time and effort and can be frustrating, but the satisfaction of solving such a problem is much greater than simply cranking out an answer to a plug and chug problem. You will be given ample opportunity to experience such satisfaction during the next four months.

A typical class period will take the following form: a brief overview of previous material, including a discussion of some of the homework problems, followed by a lecture on new material and, as time permits, a preview of what to expect on the next assignment. As an understanding of the material from the previous lecture is often needed in the next lecture, it is necessary to do the homework before the next class period. Note that this statement, combined with the meeting times for this class, has an impact on your plans for Thursday evening. There will usually be enough time to go over some of the homework problems but certainly not all of them. If you have further questions, seek assistance from other sources: students in class, my office hours, evening tutor sessions (7–9 PM, Sunday through Thursday in Olin 247), etc.

It is important that you come to class on time and be ready to think as class begins. (This is the educational equivalent of stretching and warming up prior to an athletic performance.) I know this is difficult for early morning classes, but give it your best shot. I will keep track of your attendance; more than three unexcused absences will negatively impact your grade. Also, be certain that your cell phones are turned off before entering the classroom. Finally, please refrain from eating in class; bringing your breakfast to class is not an option as it is a distraction for other students.

Homework problems will be assigned after every lecture and you should do them before the next class period; allow at least 2–3 hours for each assignment. It is important to realize that working on and struggling with problems is the best way to learn new material. We will go over some of the problems in class the next period, but you should seek extra help if you still have unresolved questions. I will only collect a few of the assigned homework problems (these will be noted in advance and these are the only problems I want turned in) and grade each assignment on a 9 point scale. In addition to the mathematical content of your homework, I will be checking for neatness and correct use of notation. Whenever relevant, you should use complete sentences in your homework solutions. I have high standards for written work and expect you to learn how to present technical material in a clear and concise manner (see the examples below). It is a good idea to recopy the solutions to the few problems that you are turning in; the solution should be somewhat polished as opposed to a recorded history of all your unsuccessful attempts.

The following two examples illustrate how homework solutions should be presented. First of all, you should copy the problem (or some variation of the problem), including the section number and problem number. Then give a full solution to the problem, using complete sentences when appropriate. For "purely mathematical" problems (such as the first one below), there may be no need to use sentences. Actually, an equation is a sentence, and this may be all that is required to present the solution to the problem. In other cases (see the second example), some technical writing, involving complete sentences and an explanation of the symbols and methods used, will be expected.

Problem 1.17.1m: Find the derivative of the function $f(x) = \tan^3(4x)$.

Solution: Using the derivative formula for $\tan x$ and the Chain Rule, we find that

$$f'(x) = 3\tan^2(4x)\sec^2(4x) \cdot 4 = 12\tan^2(4x)\sec^2(4x).$$

Problem 1.13.2m: Determine the interval(s) on which the function $g(x) = xe^{-2x}$ is increasing and those on which it is decreasing.

Solution 1: The sign of the derivative of the function g indicates whether g is increasing (g' is positive) or decreasing (g' is negative). We can find the derivative of g by using the product rule:

$$g'(x) = x(-2)e^{-2x} + e^{-2x} = (1-2x)e^{-2x}.$$

Note that $g'(x) = 0$ only when $x = \frac{1}{2}$ and that e^{-2x} is always positive. Since g' is positive on the interval $(-\infty, \frac{1}{2})$, the function g is increasing on $(-\infty, \frac{1}{2}]$. Since g' is negative on the interval $(\frac{1}{2}, \infty)$, the function g is decreasing on $[\frac{1}{2}, \infty)$.

Solution 2: We first need to find the values of x for which $g'(x) = 0$ or $g'(x)$ does not exist.

$$g(x) = xe^{-2x} \Rightarrow g'(x) = x(-2)e^{-2x} + e^{-2x} = (1 - 2x)e^{-2x};$$

$$g'(x) = 0 \Rightarrow x = 1/2;$$

$$\text{for } -\infty < x < 1/2,\ g'(x) > 0; \quad \text{for } 1/2 < x < \infty,\ g'(x) < 0.$$

Therefore, the function g is increasing on $(-\infty, \frac{1}{2}]$ and decreasing on $[\frac{1}{2}, \infty)$.

Either form of the solution for the second problem is fine; pick a style with which you are most comfortable. The important points are (i) copy the problem, (ii) write clear solutions using explanations and correct notation, and (iii) finish with a complete sentence (not a boxed number!) that identifies your answer.

Homework is due at the beginning of class and late homework will not be accepted. You should not be overly concerned about missing an occasional assignment as I typically discard two or three of the lowest homework scores during each portion of the course (the time period between exams), but you should turn in as many as possible. (If you know in advance that you will miss a class, have someone turn in your homework and be certain to get the new assignment.) I do not make elaborate comments on the homework—enough to point out your error. Come in for assistance if you are still stuck after looking at the problem again. Pay attention to your errors and learn from your mistakes. Do not use spiral notebook paper for the homework assignments since the torn edges make the papers difficult to shuffle.

Two other comments about the homework are worth noting. First of all, you are on your own to do the problems that are not collected. You will need to be disciplined enough to do these problems. One of the more common reasons why students do poorly on an exam is failure to do enough homework problems. Secondly, pay careful attention to your calculations as I will not be giving a lot of partial credit. It does not take that much extra effort to do things correctly the first time; we all expect this of the people we hire to do work for us. As far as the course as a whole is concerned, I strongly encourage you to spend a few minutes reading (and thinking about) a section before we discuss it in class. This will not only help improve your understanding of the lecture, but it will also give you practice reading technical material.

There will be three in-class exams and each exam will be worth 60 points. The dates for these are Sept. 30, Oct. 29, and Dec. 3. Rescheduling of exams will be arranged only in rare circumstances and I need to be notified in advance if such a situation arises. The real check of what a person has learned from a math class is how much they know at the end of the course. Consequently, the final exam will be comprehensive and be worth 80 points. The total of all of the homework scores will be converted to a 60 point scale. The grade for the course will be based on these 320 points; I will keep you posted on the grading scale during the semester.

Students often ask "Is Calculus II more difficult than Calculus I?" To be completely honest, I would say that it is definitely more difficult. Some more prerequisites from algebra and trigonometry are used (such as an increased emphasis on $\ln x$ and the inverse trigonometric functions) as well as much of the material from Calculus I. The key concepts, integration and infinite series, are more abstract than the derivative, and the key skill, antidifferentiation, is harder than differentiation. In addition, some of the applications are more sophisticated and there is a greater emphasis on understanding than imitation. On the positive side, the ideas and applications in Calculus II are more interesting than those of Calculus I.

Calculus is an abstract subject; time and effort are necessary to get a good grasp of its content. A fair amount of prerequisite knowledge, including but not limited to algebra and trigonometry, is required. The key concepts must be thought about carefully and understood before the mechanics make sense. Since all of this may sound rather intimidating, here are some guidelines. You may want to refer to these more than once during the semester.

(1) Study the book carefully. The sections are short enough that you should be able to read and understand every detail in the section. Think hard about the ideas and ask questions on anything that is not clear. You will be expected to do some thinking and reading on your own and to work on some problems without seeing an example first. Don't wait a week to seek help if you start having trouble.

(2) Keep up with the homework. In a class such as this you cannot get by with studying every once in a while or waiting until a few days before the test. Note that reading solutions and solving problems are different skills. Learning mathematics is an active process; observing others solve problems is not sufficient. Learn the techniques, don't just imitate examples. Be certain you understand the main ideas. If you choose to work with others, make sure that you are actively participating, especially paying attention to how to start problems on your own.

(3) Pay attention to notation and presentation. An inability to use correct notation is often a sign that the concepts have not yet been learned. Go back over the guidelines for writing up homework solutions and take them seriously. In particular, remember to recopy the problem and use a legitimate complete sentence to finish the problem.

(4) Use discretion while taking notes. You need not write down everything that appears on the board. In fact, this is usually a poor strategy for taking notes in a math class. It is better to watch and think rather than mindlessly transcribe. You should also be aware that I say some important things without ever writing them on the board.

In order to make them explicit, here are the goals of this course.

- to develop quantitative reasoning skills;
- to learn how to read technical material;
- to learn to write technical information correctly and clearly;
- to take pride in your work and to avoid errors;
- to learn how to solve non-routine problems;
- to appreciate/understand how mathematicians view mathematics;
- to comprehend some aspects of calculus.

Some of these may not be the goals you envisioned for this class, but I encourage you to view the class in this light. All of these goals will serve you well in the years ahead.