Math 260: Introduction to Higher Mathematics **Spring 2016**

Russ Gordon Office: Olin 235 Hours: 3:00-4:00 pm M–F

Math 260 is designed primarily for students planning to major in mathematics or a related field, but it is also a good course for anyone who is curious about why mathematics works the way it does or wants to know what mathematics beyond numbers and calculations might involve. The pathway to higher mathematics or "mathematical sophistication" is lined with abstract concepts, nonroutine problems, and rigorous proofs. Although it is essential for a mathematics major to grapple with abstract ideas, to understand basic logic, and to learn how to read and write mathematical proofs, many people in other fields can benefit from a careful study of deeper concepts as well as the construction and analysis of valid arguments. This course represents a first step in this process. You may initially find the material difficult, abstract, and frustrating, but with patience and hard work, you should be able to learn how to "do proofs" and, perhaps, even reach the point where you find proofs beautiful and enjoy creating them.

Every field of study requires proof or some sort of criteria to establish valid arguments. For instance, consider the following five statements.

(1) Climate change is a consequence of human activities.

(2) Smoking increases the risk of contracting lung cancer.

(3) The sum of the angles of a triangle equals two right angles.

(4) 12475 can be represented as a sum of two perfect squares.

(5) There are an infinite number of primes.

If you believe that a given statement is true or false, how can you convince someone that your point of view is correct? What sort of argument is sufficient? Is there such a thing as truth or Truth? Such questions lead into the realms of philosophy and psychology. We won't dig quite that deeply and simply accept the usual and well-known rules of logic as obvious truths. Mathematicians begin with these rules of logic, some undefined and defined terms, and a few axioms (statements accepted as true), then they build up a body of knowledge; knowledge that they are confident is true and that others operating with the same rules must also believe is true. As an aside, statement (3) above is only true if one accepts a certain axiom concerning parallel lines; there are legitimate geometries for which this statement is false.

The focus of this particular class will be on developing a sense for and appreciation of some basic ideas in abstract mathematics and learning to read and write coherent proofs. We will consider mathematical ideas that are familiar to you from other math classes (such as sets, functions, and integers) but look at them much more carefully and expand their treatment in new ways. The textbook *Introduction to Higher Mathematics* is available online at http://people.whitman.edu/~gordon/ by following the clearly (hopefully) indicated links.

We will typically cover a section of the text each class meeting and some problems will be assigned each class period. A few of these problems will be collected and graded on a 9 point scale. This particular scale initially arose from the collection of three problems per assignment with each problem graded on a 0-1-2-3 scale (no progress at all, a germ of a correct idea but minimal progress, substantial progress but lacking some details or clarity, a well-written and correct solution). I still find the scale useful even if I do not collect three problems. Do not convert each homework score to a traditional percentage as this is not the sort of grading scale I use. However, low scores are certainly an indication that something needs to change. For the record, mathematics is a precise language and words must be used carefully. I have high standards for neatness, clarity, and proper use of language and will expect that your writing and presentation of homework solutions will improve during the semester. The assigned problems that are not collected will be discussed (hopefully, in the true meaning of a discussion) in class, sometimes with students putting solutions on the board. It is important that each person participate in these discussions and be willing to give and take criticism/comments on presented work. Learning to explain your ideas to others in either written or verbal form is a valuable skill to acquire.

There will be three exams (Feb. 11, Mar. 10, and Apr. 28) and a comprehensive final exam. The regular exams will represent 45% of your grade (each one being 15%), homework (with some participation tossed in) 30%, and the final exam 25%. In addition to the usual homework, there may be occasional homework assignments in which you have more time (roughly a week) and less help (you must work on your own without any other resources). I will try to keep you posted on your homework score and "general progress" during the semester.

This class can often be the mathematical equivalent of "hitting the wall." Here are some suggestions for coping with this. First of all, don't give up and conclude that you are not good at mathematics. Be patient and don't get down on yourself. Do not expect to read a problem and then begin to immediately write a proof. You will need time to mull problems over and explore options. There is no algorithm for writing proofs so don't look for one. However, there are various problem-solving strategies that you will master over time and these can help point the way to a solution. Read proofs over and over again, looking for the main ideas, then see if you can recreate the proof on your own. Don't pretend that you understand something just because you "sort of got the gist of it." Do not expect to read a mathematics textbook quickly or easily; read slowly and work to understand every phrase. Past experience indicates that students either do not know when they are lost, are too afraid to look stupid in class and thus say nothing, or feel so lost they cannot even ask a question. You need to find some way to get beyond these issues and take control of your own learning. With time and effort, this material should eventually make sense and you can develop a much deeper understanding of your mathematical knowledge.

An “introduction to proofs class” developed across the United States in the early 1980’s. Teachers discovered that many potential math majors did well in calculus and differential equations then struggled with upper division courses such as linear algebra, abstract algebra, and real analysis. They felt that a course was needed to bridge the gap from computation-oriented classes to proof-oriented classes. There is still some disagreement as to whether or not these transition courses actually help students shift from one type of math course to another; some feel that the best way to learn abstract mathematics is in the context of a specific course. This is much like the “throw the kid in the pool and they will learn to swim” approach. While this approach does have some merits, it can create some problems or fears (or both) as well. In the 1980’s, many of the math majors at Whitman took Math 260 (designed by Pat Keef) followed by a year of algebra (as juniors) then a year of analysis (as seniors). The original set of notes was designed to help prepare students for abstract algebra. By the time they made it to real analysis, they had a year and a half of experience with abstract mathematics. Since we now alternate abstract algebra and real analysis, such a linear progression no longer occurs. However, the content of this intro course (modified several times in the past decade) is still beneficial and we feel that it does help our majors prepare for the work ahead.

I can briefly tell you my experience. When I took calculus, the course was split into three 5 credit courses (and the thousand page book cost $13.95 while the TI-10 calculator cost $70). For one of the semesters, we met at 8:00 am Tuesday through Saturday (and it was uphill both ways). Anyway, I took the first semester as a high school student then continued with the other two courses my freshman year. I started my sophomore year by taking two math courses: abstract algebra and modern geometry. I had always worked hard in my math classes (I still do not get it when students say math has never been hard for them), but for the first time in my life, I had the experience of not being able to do my homework. I did not understand what I was expected to do for the homework and thus did not know how to start the problems. After six weeks of agony (and pondering biology as a major—but there were those stupid labs), the ideas finally began to click and I could make progress on my homework once again. This involved reading the books (the same pages) over and over and solving lots of problems, sometimes the same ones over and over. It meant spending a lot of time just thinking about the ideas. (I remember lying on the floor and staring at the ceiling, imagining the floor as the domain of a function and the ceiling as the codomain, trying to grasp injective and surjective functions.) I want to emphasize that it was not time alone that made the difference; it was time and lots of effort thinking about the concepts and problems.

Making the transition to higher mathematics involves a lot of mental effort and perhaps some soul searching, but it is necessary to do so to reach the next level of more advanced topics in mathematics. You can go along for a while on a plateau and survey various types of math at that level, then you need to make a steep climb to reach a new level. This sort of transition occurs again and again if you continue with mathematics in graduate school. It is a very labor intensive discipline. However, putting forth this effort has its rewards because you get to explore strange new worlds and boldly go where few people in other civilizations have gone. You can attain an outlook where you truly see that “mathematics is beautiful.”