Math 260: Introduction to Higher Mathematics **Fall 2025**

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Math 260 is designed primarily for students planning to major in mathematics or a related field, but it is also a good course for anyone who is curious about why mathematics works the way it does or wants to know what mathematics beyond numbers and calculations might involve. The pathway to higher mathematics or "mathematical sophistication" is lined with abstract concepts, nonroutine problems, and rigorous proofs. While it is essential for a mathematics major to grapple with abstract ideas, to understand basic logic, and to learn how to read and write mathematical proofs, many students in other fields (such as pre-law, theoretical physics, or economics for example) can benefit from a careful study of deeper mathematical concepts as well as the construction and analysis of valid arguments. This course represents a first step in this process. You may initially find the material difficult, abstract, and frustrating, but with patience and hard work, you should be able to learn how to "do proofs" and, perhaps, even reach the point where you find proofs beautiful and enjoy creating them.

Every field of study requires proof or some sort of criteria to establish valid arguments. For instance, consider the following five statements.

(1) Climate change is a consequence of human activities.

(2) Smoking increases the risk of contracting lung cancer.

(3) The sum of the angles of a triangle equals two right angles.

(4) 12475 can be represented as a sum of two perfect squares.

(5) There are an infinite number of primes.

If you believe that a given statement is true or false, how can you convince someone that your point of view is correct? What sort of argument is sufficient? Is there such a thing as truth or Truth? Such questions lead into the realms of philosophy and psychology. We won't dig quite that deeply and simply accept the usual and well-known rules of logic as obvious truths. Mathematicians begin with these rules of logic, some undefined and defined terms, and a few axioms (statements accepted as true), then they build up a body of knowledge; knowledge that they are confident is true and that others operating with the same rules must also believe is true. As an aside, statement (3) above is only true if one accepts a certain axiom concerning parallel lines; there are legitimate geometries (non-Euclidean) for which statement (3) is false.

The focus of this particular class will be on developing a sense for and appreciation of some basic ideas in abstract mathematics and learning to read and write coherent proofs. We will consider mathematical ideas that are familiar to you from other math classes (such as sets, functions, and integers) but look at them much more carefully and expand their treatment in new ways. The textbook *Introduction to Higher Mathematics* is available online at `people.whitman.edu/~gordon/` by following the indicated links.

We will typically cover a section of the textbook for each scheduled class period on MWF, reserving the Tuesday period for exams, group work, oral presentations, and review. Some homework problems will be assigned for almost every MWF class period. A few of these problems will be collected and graded on a 10 point scale. Do not convert each homework score to a traditional percentage as this is not the sort of grading scale I use. However, low homework scores are certainly an indication that something needs to change. For the record, mathematics is a precise language and words must be used carefully. I have high standards for clarity, proper use of language/notation, and neatness, and I will expect that your writing and presentation of homework solutions will improve during the semester. The assigned problems that are not collected will be discussed in class. It is important that each student participate in these 'discussions' (in some fashion) and be willing to give and take criticism/comments on their presented work. Learning to explain your ideas to others in either written or verbal form and to respond to feedback are valuable skills to acquire. You will also be expected to learn (mainly on your own) how to create pdf files in LaTeX (see the end of the syllabus).

There will be four in-class exams (September 23, October 21, November 11, and December 9) and a comprehensive final exam (scheduled for the afternoon of Wednesday, December 17). The regular exams will represent 50% of your grade (each one being 12.5%), homework 30%, and the final exam 20%. In addition to the usual homework, there will be additional homework assignments in which you have more time (roughly a week) and less help (you must work on your own without any resources other than the textbook). I will try to keep you posted on your homework score and 'general progress' during the semester. It is important to emphasize at the outset that the workload for this four credit class is quite high; you need to allot plenty of time (plan on 12–15 hours per week) for attending class, reading the textbook, pondering the ideas, and working on the homework.

This course can often be the mathematical equivalent of "hitting the wall." Here are some suggestions for coping with this. First of all, don't give up and conclude that you are not good at mathematics. Be patient and don't get down on yourself. Do not expect to read a problem and then begin to immediately write a solution or proof. You will need time to mull problems over and explore options. There is no algorithm for writing proofs so don't look for one. However, there are various problem-solving strategies that you will master over time and these can help point the way to a solution. Read proofs over and over again, looking for the main ideas, then see if you can recreate the proof on your own. Don't pretend that you understand something just because you "sort of got the gist of it." Do not expect to read a mathematics textbook quickly or easily; read slowly and work to understand every single word and phrase. Past experience indicates that students either do not know when they are lost, are too afraid to appear unintelligent in class and thus say nothing, or feel so lost they cannot even ask a question. You need to find some strategies to get beyond these issues and take control of your own learning. With time and effort, this material should eventually make sense and you can develop a much deeper understanding of your mathematical knowledge.

Important: Please use discretion with AI and other Internet resources. Yes, they can give you answers, but they also can detract from your learning. For example, driving a car ten miles versus walking ten miles involves very different muscles and skills. You cannot use any of these resources for exams, so make sure you are actually learning the concepts and skills necessary to succeed in this course.

An “introduction to proofs class” developed across the United States in the early 1980’s. A number of professors discovered that many potential math majors did well in calculus and differential equations then struggled with upper division courses such as linear algebra, abstract algebra, and real analysis. Teachers felt that a course was needed to bridge the gap from computation-oriented classes to proof-oriented classes. There is still some disagreement as to whether or not these transition courses actually help students shift from one type of math course to another; some professors feel that the best way to learn abstract mathematics is in the context of a specific course. This approach is much like the “throw the kid in the pool and they will learn to swim” statement. While this teaching method does have some merits, it can create some problems or fears (or both) as well. In the 1980’s, many of the math majors at Whitman took Math 260 (designed by Pat Keef) followed by a year of algebra (as juniors) and then a year of analysis (as seniors). The original set of notes was designed to help prepare students for abstract algebra. By the time they made it to real analysis (a notoriously challenging course), they had a year and a half of experience with abstract mathematics. For most current math majors, this sort of linear course progression no longer occurs. However, the content of this intro course (modified several times over the past three decades) is still beneficial and we feel that it does help our majors prepare for the course work that lies ahead of them.

I can briefly tell you my experience as an undergraduate math major. When I took calculus, the course was split into three 5 credit courses (the thousand page book cost $14 while the TI-10 calculator cost $70). For one of the semesters, the class was scheduled for 8:00 am Tuesday through Saturday. I took the first semester as a high school student then continued with the other two courses my freshman year of college. I started my sophomore year by taking two math courses: abstract algebra and modern geometry. I had always worked diligently in my math classes (I still do not get it when students say math has never been hard for them), but for the first time in my life, I had the experience of not being able to do my homework. I did not understand what I was expected to do for the homework and thus did not know how to start the problems. After six weeks of agony (and pondering biology as a major—but there were those tedious and boring labs), the ideas finally began to click and I could make progress on my homework once again. This involved reading the books (the same pages) over and over and solving lots of problems, sometimes the same ones over and over. It meant spending a lot of time just thinking about the ideas. (I remember lying on the floor and staring at the ceiling, imagining the floor as the domain of a function and the ceiling as the codomain, trying to grasp injective and surjective functions.) I want to emphasize that it was not time alone that made the difference; it was time and lots of effort thinking about the concepts and problems.

Making the transition to higher mathematics involves a lot of mental effort and perhaps some soul searching, but it is necessary to do so to reach the next level of more advanced topics in mathematics. You can go along for a while on a plateau and survey various types of math at that level, then you need to make a steep climb to reach a new level. This sort of transition occurs again and again if you continue with mathematics in graduate school. It is a very labor intensive (mental rather than physical) discipline. However, putting forth this effort has its rewards because you get to explore strange new worlds and boldly go where few people in other civilizations have gone. You can attain an outlook where you truly see that “mathematics is beautiful.”

During the semester, I will most likely require some assignments to be typeset in LaTeX, a mathematical word processing language. You will need to learn this language on your own; there are tutorials and some templates available at https://www.overleaf.com/login. I can help with some aspects of this program, but my help will be limited since I still prefer to use Plain TeX and thus I am not up to speed with all of the features available in LaTeX or in Overleaf. However, as we will only be doing simple formatting, the task should not be too onerous. Hence, you do not need to spend hours learning LaTeX, but I want to give you a heads-up in case you decide to plan ahead. For practice, you could try typing up solutions for Exercises 1.1.5a and 1.1.5c just to see how long it takes. The code below is relevant for some concepts and notations that appear in later chapters of the textbook, but you can try putting this code in a LaTeX document now just to see how the corresponding page looks when it is compiled.

```
\documentclass{article}
\usepackage[utf8]{inputenc}
\title{Homework 1}
\author{Russ Gordon}
\date{August 28, 2024}
\begin{document}
\maketitle
\section{Problem 1}
Find the minimum value of the function $f$ defined by $f(x)=16x+x^{-2}$ for $x>0$.
\bigskip
\bigskip
Using the AM/GM Inequality, we find that
$$ f(x) = 16x+\frac{1}{x^2} = 8x+8x+\frac{1}{x^2}
\ge 3\root3\of{(8x)(8x)(1/x^2)} = 12. $$
It follows that the minimum value of $f$ for $x>0$ is $12$ and this value occurs when
$8x=4=1/x^2$, that is, when $x=\frac{1}{2}$.
\bigskip
${n \choose k}$ versus ${\displaystyle{n\choose k}}$
\bigskip
$a\equiv b\pmod n}$
\begin{eqnarray*}
 (x-2)^2 + 8x
 &=& (x^2-4x+4) + 8x \\
 &=& x^2+4x+4 \\
 &=& (x+2)^2
\end{eqnarray*}
\end{document}
```