Math 455: Real Analysis **Fall 2013**

Russ Gordon Office: 235 Olin Hall Hours: 3:00-4:00 pm M–F

Real analysis is the study of real numbers, sets of real numbers, and functions defined on sets of real numbers. It is a large branch of mathematics, important to both pure and applied mathematics, and enters realms you cannot even imagine at this point in time. Elementary real analysis is primarily concerned with the theory behind single variable calculus. Thus the focus of the course is on theorems and proofs rather than applications and numerical calculations. Most of the theorems will be familiar to you (such as the Mean Value Theorem and the Fundamental Theorem of Calculus), but learning and understanding their proofs will be a new experience. Due to the frequent appearance of quantifiers and the abstract nature of the material, many students find real analysis to be a challenging subject. Fortunately, there are some fascinating and beautiful results in analysis (as well as important applications to other fields) that justify the effort in learning its intricacies.

Class time will be spent in various ways including lectures on new material, interactive discussions of material you have read or exercises you have attempted, and solutions to further exercises in the textbook. It is important that you be intellectually involved during class and ask questions about concepts that are not clear; remaining silent for fear of asking a seemingly stupid question will do little to advance your learning. Since our class is much larger than any analysis class I have taught at Whitman, there will be some unique challenges that arise from this fact. We may need to make an occasional adjustment to how we do things in class to improve learning and participation from all students.

Homework will be assigned every class period. These assignments will include some reading from the text (R. Gordon, *Real Analysis, A First Course*, 2nd ed.) and some exercises. I will only be collecting a few of the assigned exercises; solutions for some of the other exercises will be discussed or presented in class. I encourage you to work together on these exercises as talking about them can further your understanding of the concepts. On the other hand, you must learn how to tackle problems on your own as well so do not rely too much on others to get your started. One assignment each week will involve one or two exercises that require more thinking or effort to solve. You must work alone on these special assignments, that is, no help from classmates, professors (including me), other books, or the Internet. I want to see what you can do on your own with these exercises. It may take you a while to adjust to this type of assignment since you are usually given a safety net. However, in time you will develop confidence in yourself and realize that you can solve problems and critique solutions on your own. Keep in mind that for trying to understand the reading and for the other assigned exercises, both exercises to turn in and exercises for class discussion, working together and/or coming to office hours is a good option.

I have high expectations for the clarity of written work, so pay particular attention to your writing and be certain (as much as possible) that your proof is correct and easy to follow. When attempting to solve an assigned problem, first do some work on scratch paper until you think you have a solution. Then sketch out a proof and see if it appears reasonable. Finally, write up a polished version of the proof using complete sentences. Although it is tempting to try to circumvent these steps, I highly recommend that you do not. Be certain you believe your proof; do not turn in proofs you know are flawed as this simply makes the professor think you are an idiot. If you cannot justify a step you need in a proof, make a parenthetical comment such as "I believe this to be true but am unable to prove it" or "I cannot figure out what to do at this point." It is extremely important that you learn to critique your own proofs. One difficulty students sometimes have with analysis is what to assume when solving a problem, that is, what statements can be used without proof and which ones must be proved. There is no definitive way to resolve this difficulty, but you should be able to figure it out after a few weeks of trial and error. For example, you cannot use calculus in the first three chapters of the book. If a problem in these chapters asks you to find a maximum value of a function, you cannot take the derivative and set it equal to zero. I will try to make some comments concerning what assumptions you can make as I assign various problems; you can always ask me if you are unsure. The key is to make certain that you can justify every statement that you make.

As you may surmise, the work load for this class will be rather heavy. You probably need to plan on several hours for each assignment. Some of the concepts in real analysis do not sink in quickly; you must be willing to invest time in contemplating definitions, examples, theorems, and types of arguments. Spending an hour reading one proof is not unusual nor is needing several hours (sometimes spread over several days) to solve a problem. It is therefore a good idea to start working on the homework problems well in advance of their due date. In upper level mathematics courses, definitions and theorems are extremely important. Just as you can rattle off, say, the derivative of x^2, you need to be able to recite, for instance, the definition of the limit of a function. In some cases, proofs of theorems are as important or more important than the statement of the theorem, so you need to be familiar with proofs as well. It is thus extremely important that you read the textbook carefully and do so multiple times. Be certain you understand every step, phrase, and equation, and be aware of/make note of those places where you do not follow the reasoning so you can ask questions later. Read the previous sentence again and let it sink in. You need to learn how to read a math textbook; this includes knowing what you do and do not understand. If you do not write out your questions, you will most likely forget them. The bottom line is that you may need to approach this course differently than other math courses you have taken.

There will be four exams during the semester (September 26, October 17, November 7, and December 5) and a comprehensive final exam (scheduled for 2–5 pm on Monday, December 16). Each of the exams represents 15% of your course grade and the final represents another 20% of your course grade. The regular graded homework counts as 10% and the special homework as 10% of your course grade.