Math 125: Calculus I **Fall 2016**

Russ Gordon Office: Olin 221 Hours: 3:00–4:00 pm M–F

Calculus has its roots in two elementary problems from geometry: determining the areas of regions with curved boundaries and finding tangents to curves other than circles. Using geometric techniques, the ancient Greeks were able to solve these two problems for the conic sections and some other easily defined curves. With the introduction of analytic geometry (which provides a strong link between algebra and geometry) in the middle of the seventeenth century, a vast array of new curves were considered, and algebraic techniques were developed to determine areas and tangents for these curves. Newton and Leibniz independently discovered that the problems of areas and tangents were related to one another in a simple way; this discovery is considered to be the birth of calculus. The fact that many problems in the physical sciences can be reduced to finding areas or tangents has made calculus the cornerstone of the scientific revolution.

Mathematics in general, and calculus in particular, is often taught as a set of algorithms and/or skills to be mastered and the focus is usually on one skill at a time. Several sample problems are explained then a large collection of similar problems are given as exercises. These exercises can often be solved by imitation (or typing the problem into a website such as Wolfram Alpha) rather than understanding. While skills are certainly important and an essential part of problem solving, an argument can be made that understanding the ideas behind the computations, knowing which skills to use in a given situation, and combining several skills in a multi-step problem are even more important. In addition, learning to solve non-routine problems, those without an example to imitate and requiring more than one step, is a valuable skill to acquire and more closely resembles the type of thinking one encounters in the 'real' world. These sorts of problems require more time and effort and can be frustrating, but the satisfaction of solving such a problem is much greater than simply cranking out an answer to a plug and chug problem. You will be given ample opportunity to experience such satisfaction during the next four months.

A typical class period will take the following form: a brief overview of previous material, including a discussion of some of the homework problems, followed by a lecture on new material and, as time permits, a preview of what to expect on the next assignment. The lecture often has two components; an explanation of the theory behind a concept along with some practical solutions to problems related to the concept. It is tempting to 'coast' during the theory and just try to mimic the problem solutions, but this is not the best approach at this level of mathematics. I encourage you to focus on the theory as well as the practice and to learn to think about mathematics in a deeper way. As an understanding of the material from the previous lecture is often needed in the next lecture, it is important to do the homework before the next class period. There will usually be enough time to go over some of the homework problems but certainly not all of them. If you have further questions, seek assistance from other sources; students in class, my office hours, evening tutor sessions (7–9 PM, Sunday through Thursday in Olin 205), the Academic Resource Center, etc.

It is important that you come to class on time and be ready to think as class begins. (This is the educational equivalent of stretching and warming up prior to an athletic or musical performance.) I know this is difficult for early morning classes, but give it your best shot. I will keep track of your attendance; more than three unexcused absences will negatively impact your grade. Be certain that your cell phones (and any other non-essential electronic devices) are turned off before entering the classroom. Finally, please refrain from eating during class, that is, do not bring breakfast or a snack to class as this can be distracting to other students.

Homework problems will be assigned after every lecture and you should do them before the next class period. Allow at least two to three hours for each assignment; look over your notes and the textbook, do all of the assigned problems, think about the key ideas in the section, and preview the next section. (For the record, I strongly encourage you to spend a few minutes reading a section before we discuss it in class. This will not only help improve your understanding of the lecture, but it will also give you practice reading technical material.) It is important to realize that working on and struggling with problems is the best way to learn new material. We will go over some of the problems in class the next period, but you should seek extra help if you still have unresolved questions. I will not be collecting any homework so it is up to you to have the discipline to complete it. One of the more common reasons why students do poorly on a quiz or exam is failure to do enough homework problems. There will be frequent quizzes, both take-home and in-class, with the latter usually, but not always, mentioned in advance. The problems on the quizzes will typically be similar to homework problems. Each quiz will be worth 20 points but overall all the quizzes during the semester will count as much as one exam. It will not be possible to make up missed quizzes—several quiz scores will be dropped to allow for absences due to illness, school events, or an 'off' day.

For the record, in addition to your final answers on quizzes and exams, I will be grading the mathematical content of your work. This includes brief explanations of your steps, clear presentations of your solutions, and the correct use of notation. Whenever relevant, you should use a complete sentence to finish a problem, realizing that a mathematical equation is often a sentence. I have high standards for written work and expect you to learn how to present technical material in a clear and concise manner. The following example illustrates what I am expecting in this regard (at least somewhat, more details will appear during the course of the semester). The first two solutions (E and D) are typical of beginning students, but I want you to move beyond this style. If all you are doing is playing around with problems to see if you know what to do or are trying to "get the answer in the back of the book," then this style is fine. However, if you hope to later understand what you did, such scratchwork is not very useful. Solution C is acceptable for in-class quizzes and exams; it simply asks you to indicate a little bit of your thought process and to use a complete sentence (rather than a box) to indicate your final answer. The last two solutions are provided for those of you who would like to work on your technical writing (and I highly recommend that you practice this at times). Solution B is really just Solution C with complete sentences rather than phrases; it takes very little extra work to do this and this level of detail will be expected for out of class quizzes. Solution A provides the reader with some explanation for the ideas behind the computations. For instance, it mentions rise over run (for computing slope), negative reciprocals (how to find the slope of a perpendicular line), and the point-slope equation for a line. The idea is that a person (either another person or yourself at a later date) can read the solution and understand the solution process. It provides good practice for writing technical material (a skill that requires some patience to acquire) and some students find that writing out the details really helps them understand the key ideas behind a problem.

Problem: Find an equation for the line that goes through the point $(1, 2)$ and is perpendicular to the line containing the points $(-1, 4)$ and $(3, 11)$.

Solution E: (sloppy and uninformative)

$$\frac{11-4}{3+1} = \frac{7}{4}, \qquad -\frac{4}{7} \qquad \boxed{y - 2 = -\frac{4}{7}(x-1)}$$

Solution D: (a little better since parts of the solution are identified)

$$\text{slope of given line } \frac{7}{4}, \qquad \perp \text{ slope } -\frac{4}{7}$$

$$\text{equation of line } \boxed{y - 2 = -\frac{4}{7}(x-1)}$$

Solution C: (this style would work well for an in-class quiz or exam solution)

$$\text{slope of given line is } \frac{11-4}{3-(-1)} = \frac{7}{4}, \quad \text{perpendicular slope is then } -\frac{4}{7}$$

$$\text{An equation for the perpendicular line is } \; y - 2 = -\frac{4}{7}(x-1).$$

Solution B: The slope of the line passing through $(-1, 4)$ and $(3, 11)$ is

$$\frac{11-4}{3-(-1)} = \frac{7}{4}.$$

The slope of any line perpendicular to this one is $-4/7$. An equation for the desired line is

$$y - 2 = -\frac{4}{7}(x-1).$$

Solution A: Let ℓ_1 be the line containing the points $(-1, 4)$ and $(3, 11)$ and let ℓ_2 be the line that goes through $(1, 2)$ and is perpendicular to ℓ_1. The slope of ℓ_1 is determined by the rise over the run:

$$\frac{11-4}{3-(-1)} = \frac{7}{4}.$$

To find the slope of ℓ_2, the perpendicular line, we take the negative reciprocal of this value and obtain $-4/7$. Using the point-slope equation for a line, we find that an equation for ℓ_2 is

$$y - 2 = -\frac{4}{7}(x-1) \qquad \text{or} \qquad 4x + 7y = 18.$$

There will be three in-class exams and each exam will be worth 60 points. The dates for these are Sept. 28, Nov. 2, and Dec. 7. Rescheduling of exams will be arranged only in rare circumstances and I need to be notified in advance if such a situation arises. The real check of what a person has learned from a math class is how much they know at the end of the course. Consequently, the final exam will be comprehensive and be worth 80 points. The total of all of the quiz scores (with some low scores dropped) will be converted to a 60 point scale. The grade for the course will be based on these 320 points. Since I do not have a predetermined grading scale, I will keep you posted on the grading scale during the semester.

Calculus is an abstract subject; time and effort are necessary to get a good grasp of its content. A fair amount of prerequisite knowledge, including but not limited to algebra and trigonometry, is required. The key concepts must be thought about carefully and understood before the mechanics make sense. Since all of this may sound rather intimidating, here are some guidelines. You may want to refer to these more than once during the semester.

(1) Study the book carefully. The sections are short enough that you should be able to read and understand every detail in the section. Think hard about the ideas and ask questions on anything that is not clear. You will be expected to do some thinking and reading on your own and to work on some problems without seeing an example first. Don't wait a week or so to seek help if you are having trouble.

(2) Keep up with the homework. In a class such as this you cannot get by with studying every once in a while or waiting until a few days before the test. It is important to realize that reading solutions and solving problems are different skills. Learning mathematics is an active process; observing others solve problems is not sufficient. Learn the techniques, don't just imitate examples. Be certain you understand the main ideas. If you choose to work with others, make sure that you are actively participating, especially paying attention to how to start problems on your own.

(3) Pay attention to notation and presentation. An inability to use correct notation is often a sign that the concepts have not yet been learned. Go back over the guidelines for writing up quiz/exam solutions and take them seriously. In particular, remember to use a legitimate complete sentence to finish a problem and avoid drawing boxes around your answers. When solving a problem, do your best to stay focused so as not to make 'stupid' mistakes.

(4) Use discretion while taking notes during class. You do not need to write down everything that appears on the board. In fact, this is usually a poor strategy for taking notes in a math class. It is better to watch and think rather than mindlessly transcribe. You should also be aware that I say some important things without ever writing them on the board.

In order to make them explicit, here are the primary goals for this course.

- to develop quantitative reasoning skills;
- to learn how to read technical material;
- to learn to write technical information correctly and clearly;
- to take pride in your work and to avoid errors;
- to learn how to solve non-routine problems;
- to appreciate/understand how mathematicians view mathematics;
- to comprehend some aspects of calculus.

Some of these may not be the goals you envisioned for this class, but I encourage you to view the class in this light. All of these skills will serve you well in the years ahead.