Math 240 **Linear Algebra** **Fall 2024**

Russ Gordon Office: 221 Olin Hall Hours: 3:00-4:00 pm TuTh, 4:00-5:00 pm MWF in Olin 305

Linear algebra has its roots in finding solutions to systems of linear equations. The simple techniques that can be used to solve two equations in two unknowns can be extended to solve thousands of equations in thousands of unknowns. Many current applications in a wide variety of fields can be reduced to huge linear systems and, as you can certainly guess, would be impossible to solve without the aid of computers. However, these types of problem are only the tip of the iceberg when it comes to the study of linear algebra. If you step back a little and look for similarities in the structures of certain types of problems in algebra (systems of equations), geometry (dimensions higher than 3D), and analysis (Fourier series), you begin to see a number of common ideas. Extracting the common elements leads to the concept of a vector space, which then opens the door to a panoramic view you would never have imagined existed. For example, certain aspects of theoretical physics make use of Hilbert spaces, which are vector spaces with some extra structure added. In addition, the language and ideas of linear algebra appear in many areas of both pure and applied mathematics, as well as computer science and statistics. Consequently, it is an important and interesting field to study.

After reading the opening paragraph, you might expect to be excited from day one for this class. This may certainly be the case for some of you, but for others, there may be a bit of a time lag before you see the merits of the subject matter. As we will soon see, the computations, even in the basic examples, are simple but quite tedious. Secondly, it is the concepts not the computations that play the crucial role. Most of you have not had a math class where the focus is on ideas rather than on computations and this will require some adjustment. Third, the concepts are abstract. This means that there will not always be a picture, graph, or example that makes the concept obvious. You will need to think hard about the concepts and struggle with the ideas. Finally, this is a mathematics class, not an economics, physics, biology, electrical engineering, or computer science class, etc. so we will not be doing many applications since time is limited and we all have varied backgrounds in fields that use mathematics.

Let me make a few comments about some of the items listed in the previous paragraph. The computations we will do are mainly arithmetic operations; add, subtract, multiply, and divide. But each problem will involve many of these and one mistake will magnify greatly. This means that you must take your time, write neatly, and double check your work. Use this as an opportunity to learn to do things right the first time; this is a skill that is applicable to any type of work. Of course, there are computer programs that will do all of these computations quickly and accurately. Our use of technology will be limited for two reasons. For the first, let me make an analogy. It is bad pedagogy to give second graders calculators to find things like 3×7; they should learn the times tables by rote and they will be handicapped later if they do not know these facts well. The same thing occurs at any level; you need to do some simple computations first before letting a black box take over. The second reason is that teaching technology takes time and sometimes the computer interface overwhelms the mathematics itself. As our time is limited and there are a number of mathematical ideas we must cover, we will be content to do simple examples by hand to illustrate the concepts and use just a limited amount of technology.

The concepts will build slowly and may seem like no big deal at first. However, all of basic linear algebra hinges on these ideas and if you do not master them early, your performance in the course will suffer dramatically. Since the concepts are abstract, you will have to spend time thinking about them and going over them repeatedly so that they begin to sink in. The ideas are actually quite simple once you see the big picture, but until that occurs you may feel as though you are running around in a fog. Do not let the easy calculations at the beginning lull you into a false sense of security; pay close attention to the ideas.

I know many of you are taking mathematics because of its applications. Being a perfectionist at heart, I have never been a fan of applications. You have to deal with the messy real world, make a number of simplifying assumptions, and (at least for applications that are not contrived) you have to work with some complicated (and often abstract) mathematical machinery. To do even an introduction to an application justice, we would need to spend at least a week on it, starting with the background of the problem and leading up to a solution. Due to limited time, varied backgrounds of the students, and minimal interest/expertise on my part, we will not be doing many applications. Our textbook touches briefly on several applications of linear algebra. If you are interested in learning some of these, I highly suggest that you take the time to read the appropriate pages of the book. Students sometimes feel afraid to read parts of the book not covered in class, but it is good practice to give some of these pages a try and you just might learn something interesting. If you get stuck, I may be able to help you get through the spots you find confusing. Of course, another option is to take it on faith that linear algebra has real world applications and focus exclusively on the mathematics which, if you give it a chance, you might find quite fascinating in and of itself.

Since some of the ideas and techniques of linear algebra are quite boring/tedious to present, I will be requesting that you read the textbook carefully. In addition, during the Spring 2021 semester, I made YouTube videos summarizing various sections of the textbook; links to these can be found on the course website. Perhaps these will prove useful for some of you. Finally, in an abstract course such as this, it is very important that you learn to speak up when you have questions; this can be intimidating, but it is one of the best ways to advance your understanding.

Homework problems will be assigned almost every class period with a few of them collected for grading; see the regularly updated assignments page on the website. Late homework will generally not be accepted. You may work together on these problems and/or get help from me. On occasion, however, I will give assignments that you must do on your own without working with other people; more details on these assignments will be given later. You should do all of the assigned problems before the next class period since most sections of the book rely on ideas from previous sections. When grading your work, I will pay attention to your use of notation and the presentation of your answers (neatness, organization, etc.). A correct answer is not acceptable if the line of reasoning leading up to it is unclear.

There will be three exams (October 2, November 6, and December 11) and a comprehensive final exam (scheduled for the morning of Thursday December 19). The regular exams will represent 45% of your grade (each one being 15%), homework 35%, and the final exam 20%. As mentioned above, there will be occasional homework assignments in which you have more time (roughly a week) and less help (you must work on your own). There may also be some quizzes scattered throughout the semester. I will try to keep you posted on your homework score and 'general progress' during the semester.