Math 456: Real Analysis **Spring 2016**

Russ Gordon Office: 235 Olin Hall Hours: 3:00-4:00 pm M–F

Real analysis is the study of real numbers, sets of real numbers, and functions defined on sets of real numbers. It is a large branch of mathematics, important to both pure and applied mathematics, and enters realms far beyond any material that you encounter in calculus. Elementary real analysis is primarily concerned with the theory behind single variable calculus. Thus the focus of the course is on theorems and proofs rather than applications and numerical calculations. Many of the theorems will be familiar to you (such as the Mean Value Theorem and the Fundamental Theorem of Calculus), but learning and understanding their proofs, as well as looking more closely at the concepts, will be a new experience. Due to the frequent appearance of quantifiers and the abstract nature of the material, many students find real analysis to be a challenging subject. Fortunately, there are some fascinating and beautiful results in analysis (as well as important applications to other fields) that justify the effort in learning its intricacies.

Since we are using a different textbook this semester, there may need to be some review of previous material or pauses to fill in unexpected gaps. We will begin by taking a deeper look at integration, which is an important topic in analysis, then consider sequences and series of functions (including such topics as uniform convergence and Taylor series). From there, we will move into some aspects of point-set topology and consider a few topics in the theory of metric spaces. In other words, we will be spending most of our time on Chapters 5, 7, and 8 in the text (R. Gordon, *Real Analysis, A First Course*, 2nd ed.).

Class time will be spent in various ways including lectures on new material, interactive discussions of material you have read or exercises you have attempted, and solutions to further exercises in the textbook. It is important that you be intellectually involved during class and ask questions about concepts that are not clear; remaining silent for fear of asking a seemingly stupid question will do little to advance your learning.

Homework will be assigned every class period. These assignments will include a fair amount of reading and some problems. I will only be collecting a few of the assigned problems (usually on Tuesdays and Fridays); solutions for some of the other problems will be discussed or presented in class. For these assignments, you may work together or come to office hours for assistance. I do encourage you to work together at times since talking about proofs and problems can further your understanding of the concepts. On the other hand, you must learn how to tackle problems on your own as well so do not rely too much on others to get you started. There will be several assignments during the semester, sometimes assigned a week in advance, for which you will be required to work alone, that is, no help from classmates, professors (including me), other books, or the Internet. I want to see what you can do on your own with these exercises.

I have high expectations for the clarity of written work, so pay particular attention to your writing and be certain (as much as possible) that your proof is correct and easy to follow. When attempting to solve an assigned problem, first do some work on scratch paper until you think you have a solution. Then sketch out a proof and see if it appears reasonable. Finally, write up a polished version of the proof using complete sentences. Although it is tempting to try to circumvent these steps, I highly recommend that you do not. Be certain you believe your proof; do not turn in proofs you know are flawed as this simply makes the professor think you are an idiot. If you cannot justify a step you need in a proof, make a parenthetical comment such as "I believe this to be true but am unable to prove it" or "I cannot figure out what to do at this point but here is what I tried." It is extremely important that you learn to critique your own proofs. One difficulty students sometimes have with analysis is what to assume when solving a problem, that is, what statements can be used without proof and which ones must be proved. There is no definitive way to resolve this difficulty, but you should be able to figure it out after a few weeks of trial and error. I will try to make some comments concerning what assumptions you can make as I assign various problems; you can always ask me if you are unsure. The key is to make certain that you can justify every statement that you make.

As you may surmise, the work load for this class will be rather heavy. You probably need to plan on several hours for each assignment. Some of the concepts in real analysis do not sink in quickly; you must be willing to invest time in contemplating definitions, examples, theorems, and types of arguments. Spending an hour reading one proof is not unusual nor is needing several hours (sometimes spread over several days) to solve a problem. It is therefore a good idea to start working on the homework problems well in advance of their due date. In upper level mathematics courses, definitions and theorems are extremely important. Just as you can rattle off, say, the derivative of x^2, you need to be able to recite, for instance, the definition of the limit of a function. In some cases, proofs of theorems are as important or more important than the statement of the theorem, so you need to be familiar with proofs as well. It is thus extremely important that you read the textbook carefully and do so multiple times. Be certain you understand every step, phrase, and equation, and be aware of/make note of those places where you do not follow the reasoning so you can ask questions later. Read the previous sentence again and let it sink in. You need to learn how to read a math textbook; this includes knowing what you do and do not understand. If you do not write out your questions, you will most likely forget them. The bottom line is that you may need to approach this course differently than other math courses you have taken.

There will be two exams during the semester (March 2 and April 20) and a comprehensive final exam (scheduled for the afternoon of Monday, May 16). Each of the hour exams represents 15% of your course grade and the final exam represents another 20% of your course grade. The regular graded homework counts as 30% and the special homework (the problems you must do on your own) as 20% of your course grade. I will make an attempt to keep you posted on your grade for the course as the semester progresses.