I certify that I followed all of the posted guidelines for this exam. *Signature*:

Math 126 **First Exam** **Fall 2020**

Write neat, concise, and accurate solutions to each of the problems below—I will not give partial credit for steps I cannot follow. Include all relevant steps, use correct notation, give sufficient details, and finish problems with a complete sentence; either in words or with an appropriate equation. Each problem is worth 6 points. If at all possible, keep your solutions to four pages or less.

1. Carefully state the definition of the integral.

The integral of a continuous function f on an interval $[a, b]$, denoted $\int_a^b f(x)\,dx$, is defined by

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(a + \frac{i(b-a)}{n}\right)\frac{b-a}{n}.$$

2. Evaluate $\displaystyle\int \left(2 + xe^{-x^2}\right) dx$.

$$\int \left(2 + xe^{-x^2}\right) dx = 2x - \frac{1}{2}e^{-x^2} + C.$$

[should not need u-sub here]

3. Evaluate $\displaystyle\int_0^1 \left(6(1-x)^2 + 4\sqrt{1-x^2}\,\right) dx$.

$$\int_0^1 \left(6(1-x)^2 + 4\sqrt{1-x^2}\right) dx$$

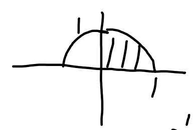
$$= \int_0^1 6(1-x)^2\,dx + 4\int_0^1 \sqrt{1-x^2}\,dx$$

$$= -2(1-x)^3\Big|_0^1 + 4\left(\frac{1}{4}\cdot\pi\cdot 1^2\right)$$

$$= 0 - (-2) + \pi$$

$$= \pi + 2.$$

second integral represents area of a quarter circle



$x^2 + y^2 = 1$

$y = \sqrt{1-x^2}$

$\int_0^1 \sqrt{1-x^2}\,dx = \frac{\pi}{4}$

4. Carefully state the version of the Fundamental Theorem of Calculus that involves a method for evaluating a definite integral.

If f is continuous on $[a,b]$, then $\int_a^b f(x)\,dx = F(b) - F(a)$, where F is any antiderivative of f.

5. Evaluate $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\cos(i\pi/(2n)) \cdot \frac{\pi}{2n}\right)$. Noting the definition of the integral,

$$\lim_{n\to\infty} \sum_{i=1}^{n} \cos\left(\frac{i \cdot \pi/2}{n}\right) \cdot \frac{\pi/2}{n} = \int_0^{\pi/2} \cos x\,dx$$

$$= \sin x \Big|_0^{\pi/2}$$

$$= 1.$$

6. Evaluate $\int_1^8 \frac{6}{\sqrt[3]{x^4}}\,dx$.

$$\int_1^8 \frac{6}{\sqrt[3]{x^4}}\,dx = \int_1^8 6x^{-4/3}\,dx$$

$$= -18x^{-1/3}\Big|_1^8$$

$$= -\frac{18}{\sqrt[3]{x}}\Big|_1^8$$

$$= -9 - (-18)$$

$$= 9.$$

(changed problem)

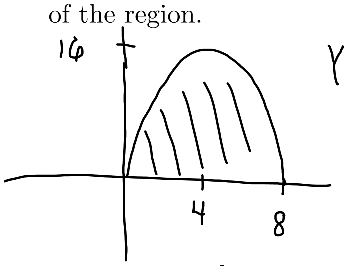
7. Evaluate $\int \cancel{\frac{1}{x(\ln x)^2}}\, dx$. $\int \frac{(\ln x)^2}{2x}\, dx$

$$\int \frac{(\ln x)^2}{2x}\, dx = \frac{1}{2}\int (\ln x)^2 \cdot \frac{1}{x}\, dx$$

$$= \frac{1}{6}(\ln x)^3 + C.$$

[mental guess and check]

8. Find the area of the region that lies below the curve $y = 8x - x^2$ and above the x-axis. Include a sketch of the region.



$y = 8x - x^2$
parabola

$$A = \int_0^8 (8x - x^2)\, dx = \left(8 \cdot \frac{1}{2}x^2 - \frac{1}{3}x^3\right)\Big|_0^8$$

$$= \frac{1}{2} \cdot 8^3 - \frac{1}{3}\, 8^3$$

$$= \frac{1}{6} \cdot 8^3$$

$$= \frac{512}{6}$$

$$= \frac{256}{3}$$

The area of the region is $\frac{256}{3}$ square units.

9. Consider the function f defined by $f(x) = 4x + 5 + \int_1^x \sqrt{t^3+8}\,dt$. Find an equation for the tangent line to the graph of this function when $x = 1$. (Think carefully here!)

$f(x) = 4x + 5 + \int_1^x \sqrt{t^3+8}\,dt$ $\qquad f(1) = 9$

$f'(x) = 4 + \sqrt{x^3+8}$ by FTC $\qquad f'(1) = 7$

$y - 9 = 7(x-1)$ point-slope form

An equation for the tangent line is $y = 7x + 2$.

10. Suppose that $v(t) = \dfrac{6\,t}{\sqrt{t+4}}$ gives the velocity in meters per second of a particle at time t seconds. Find the total distance traveled by the particle during the time interval $0 \le t \le 5$.

$$D = \int_0^5 |v(t)|\,dt = \int_0^5 \frac{6t}{\sqrt{t+4}}\,dt$$

let $u = t + 4$
then $du = dt$
$t = u - 4$

$$= \int_4^9 \frac{6(u-4)}{\sqrt{u}}\,du$$

$$= \int_4^9 \left(6u^{1/2} - 24u^{-1/2}\right) du$$

$$= \left(4u^{3/2} - 48u^{1/2}\right)\Big|_4^9$$

$$= 4(27-8) - 48(3-2)$$

$$= 4 \cdot 19 - 48$$

$$= 76 - 48$$

$$= 28$$

or

$$\left\{\begin{array}{l} u = \sqrt{t+4} \\ 2\,du = \dfrac{dt}{\sqrt{t+4}} \\ t = u^2 - 4 \\ \displaystyle\int_2^3 (12u^2 - 48)\,du \\ = \cdots \\ = 76 - 48 \\ = 28 \end{array}\right\}$$

The particle travels 28 meters during this time interval.